Debt Covenants and Competition among Investors

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Very preliminary ! Please do not circulate

Abstract

In this paper, we fully characterize the equilibrium outcomes of non-exclusive competition in credit markets subject to moral hazard, using the standard linear production setting popularized by Holmstrom and Tirole (1997, 1998). We consider two cases. When contracts are contingent on the state of nature only, the equilibrium outcome under non exclusive competition is unique, and allows investors to earn collectively monopoly profits. When contracts can include covenants contingent on total output or cash-flow, the equilibrium is indeterminate in the sense that all feasible allocations can be sustained at equilibrium. We show that this indeterminacy can be removed by creating an institution that controls accepted investments, and precludes strategic default. When such a credit bureau is available, the competitive outcome then emerges as the unique equilibrium allocation.

1 Introduction

The purpose of this paper is to study how contract design matters when investors compete to provide funds to a single firm that can trade several financial contracts at a time.

An important feature of non exclusive competition is that investors cannot control the number of contracts traded by their portfolio firm. Such non exclusivity creates an externality across investors, in the sense that additional contracts accepted can reduce the expected repayment of other

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contracts (Bizer and DeMarzo (1992)). Such features shape competition and can give rise to equilibrium allocations whereby investors collectively obtain positive profits, and total investment is reduced compared to some exclusive competition benchmark Parlour and Rajan (2001). One important restriction in the literature is to assume that investors' contracts can only take the form of simple debt contracts, without restrictive clauses. The main contribution of the paper is to considered that even when competition is non exclusive, contracts can be contingent on some observable financial results (cash-flows, assets...). This allows investors to specify covenants with some targeted investment level, and to design penalties when such covenants are violated. Designing penalties ex post is not equivalent to controlling the number of traded contracts ex ante, because the ability of investors to punish departures from targeted ratios is reduced by entrepreneurs' limited liability. In this paper, we investigate the consequences of introducing such covenants on investors' market power and credit rationing for firms. We also suggest some institutional framework to enhance competition among financial intermediaries.

The starting point of our analysis is the credit economy considered by Holmstrom and Tirole (1997, 1998) in which entrepreneurs need funds to invest in a project with a linear technology, and in which production is subject to moral hazard. Within this standard setting we model competition amongst financial intermediaries as an extensive form game. A main feature of competition is that financial contracts are taken to be non-exclusive. That is, none of the investors can make his proposal contingent on the offers that the entrepreneur is receiving from his rivals.

We first follow the route paved by previous studies and focus on plain debt contracts: Financial contracts are contingent on the success or failure state, but not contingent on total cash-flow. In such a context we show that competition delivers an extreme form of non-competitive result: The only aggregate allocation supported at equilibrium is the monopolistic one. None of the investors has a unilateral incentive to propose loans at a smaller rate, because this would always induce the entrepreneur to trade several contracts at a time and to select the low level of effort, which makes the deviation not profitable in the first place. In other words, with a linear production technology, the threat imposed by the externality across contracts is so powerful that it makes investors coordinate on a monopoly outcome. Positive profit equilibria have emerged in several alternative settings[1]. Our contribution to that literature is twofold: Firstly, our assumptions allow to support the monopolistic allocation irrespective of the number of investors. This is because it is always possible for each of the investors to profitably deviate, and reduce the entrepreneur's utility except when the latter is left with his reservation utility. Secondly, in the literature equilibria with strictly positive profit for the lenders usually rely on concave production functions. We show that a monopoly outcome also emerges when the production technology is linear. Interestingly, linearity exacerbates the market power of lenders and allows us to pin down monopoly as the unique equilibrium allocation.

This sharp result provides a natural framework to study contracts that can also be contingent on the level of cash-flows and not simply on the success or failure states. We interpret these contracts as debt contracts with financial covenants. In practice, covenants are designed to induce efficient decisions from the borrower and thereby reduce potential agency problems between borrowers and lenders. Such financial covenants are typically contingent on verifiable and contractible variables such as balance sheet, income statement or cash flow items[2]. The benefit of introducing covenants is for investors to set penalties in case the entrepreneur deviates from some targeted investment level. Because of entrepreneurs' limited liability, penalties in case of violation of covenants are bounded above. A consequence is that investors' ability to write covenants contingent on cash-flows ex post is not equivalent to controlling for the number of contracts accepted ex ante. An important result of the paper is that financial covenants can help investors to coordinate on any investment level. It follows that any feasible allocation can be sustained at equilibrium when contracts are contingent on the final cash-flow.

We next study how financial institutions can help restore competition among investors. The externality across contracts created by non exclusive competition may induce the entrepreneur to make "false" promises, i.e. to accept more contracts than he can afford. Some investors might have an interest to induce such default, and benefit from it if their claim is senior to that of others. We study the equilibrium outcome when such strategic default is ruled out. More precisely, we as-

[1]See for example Bizer and DeMarzo (1992), Parlour and Rajan (2001), Attar, Campioni, and Piaser (2006), Attar and Chassagnon (2009).

[2]See for example **?**, **?**.

sume that there exists an institution, such as a credit bureau, that registers accepted contracts, and that cancels all agreements if promised repayments are higher than maximum possible cash-flows. While such a institution does not affect the equilibrium outcome when only debt contracts can be used, it modifies sharply the nature of competition when contracts can include covenants on investment. In that case, we show that the only equilibrium allocation is the competitive one. Our analysis thus provides a rationale for the joint use of covenants in debt contracts and institutions that monitor the level of indebtness of firms.

2 The model

2.1 Agents and technology

We consider a production economy that lasts two periods. It is populated by a single representative entrepreneur and a finite number N of investors. The entrepreneur owns a project and can ask money to investors to expand the scale of her project. Following Holmstrom and Tirole (1997, 1998), production takes place through a linear technology which realizations are subject to uncertainty and which is subject to moral hazard. An investment of $I \in \mathbb{R}_+$ yields a final output (or cash-flow, normalizing the price of goods to one) GI, with $G \in \mathbb{R}_+$, in case of success and zero in case of failure. The final cash-flow is verifiable. The probability distribution over final outcomes depends on an unobservable effort $e = \{L, H\}$ chosen by the entrepreneur. Denote $(\pi_e, 1 - \pi_e)$ the probability distribution induced by the effort choice e, where π_e is the probability of success. We assume $\pi_H > \pi_L$. If the entrepreneur misbehaves, i.e. she selects $e = L$, she receives a private benefit $B \in \mathbb{R}_+$ per unit invested in the project.

In line with Holmstrom and Tirole (1998), we introduce the following assumptions:

$$\pi_H G > 1 \text{ and } \pi_L G + B < 1, \tag{1}$$

which guarantee that the investment project has a positive net present value if and only if the entrepreneur selects $e = H$. To ensure finiteness of the equilibrium investment level, we also assume:

$$0 < \pi_H G - \frac{\pi_H B}{\Delta\pi} < 1, \quad (2)$$

where $\Delta\pi = \pi_H - \pi_L$. [3]

2.2 Financial contracts

The entrepreneur is risk-neutral and protected by limited liability. She has an initial endowment of $A \in \mathbb{R}_+$ and can raise additional funds by trading financial contracts issued by competing investors. A specific feature is that the entrepreneur can trade several financial contracts at a time, and her decisions cannot be contracted upon. That is, financial contracts are non-exclusive. A financial contract proposed by investor i is therefore denoted $(I^i, R^i(\theta))$, where I^i is the credit and $R^i(\theta)$ the (menu of) repayments asked by the investor according to the final cash-flow realization $\theta \in \{0, GI\}$ $\forall I$. We assume for now that no repayment occurs in case of failure i.e. $R^i(0) = 0$. Because the entrepreneur is protected by limited liability, one cannot have $R^i(0) > 0$. We will see below that $R^i(0) < 0$ is not optimal as it is standard in moral hazard settings.

I^i cannot be contingent on the number of contracts accepted by the entrepreneur because competition is non exclusive. However, the final cash-flow can be contracted upon. To the extent that the final cash-flow perfectly reflects the total investment I in case of success, this amounts to contracting on I: Without loss of generality, the repayment will be denoted $R^i(I)$. This is not equivalent to contracting on the number of contracts accepted ex ante, because of the entrepreneur's limited liability. This limits possible punishment if the entrepreneur deviates from the targeted investment level.

Our assumption of cash-flow contingent contracts departs from the standard literature on non-exclusive competition, which assumes that R^i is contingent on the success state only.[4] In a complete contract setting, the latter assumption is conceivable if the entrepreneur can create several firms, each endowed with the same linear production technology. It is then impossible for each

[3] The assumption can be interpreted saying that if the entrepreneur chooses $e = H$, the unitary revenue from the project is smaller than the unitary agency cost (see Tirole (2006), p. 127).

[4] See e.g. Bizer and DeMarzo (1992), Parlour and Rajan (2001), **?**.

investor to contract on the total cash-flow, and R^i is contingent on the success state only. But if the entrepreneur founds a single firm, R^i can de facto be contingent on cash-flow. In the remainder of the paper, we consider both cases, and explore the consequences of expanding the contract space on the equilibrium allocation, and on investors' market power. A major insight of the paper is that by expanding the contract space, coordination among investors is easier to achieve, and more equilibria arise.

2.3 Strategic default

With non-exclusive competition, $R^i(I)$ is not necessarily the payment received by investor i, because strategic default can occur. This happens if the entrepreneur commits to repay more than the cash-flow generated by the project in case of success. In other words, the entrepreneur can make 'false promises', and accept contracts even though all repayments cannot be honored in case of success. This does not happen with one investor only because there is a unique claimant on the final cash-flow. Strategic default is a natural consequence of non exclusive competition: If one cannot contract on contracts accepted with third parties, this raises the possibility that contracts involve conflicting prescriptions.

Financial contracts could in principle specify seniority rules to determine the order of repayments in case of default. However, such clauses do not always preclude conflicts. For instance, it can happen that two contracts are accepted and entail the same level of seniority.[5] For that reason, we do not treat priority rules as given[6] but assume that they are part of players' strategies.

[5] As anecdotal evidence, consider the example of ABN Amro and JPMorgan who disputed over their claim on Dutch investor Louis Reijtenbagh's art collection after he defaulted on both institutions' loans. Reijtenbagh apparently used his art collection as collateral in the two banks.

[6] This contrasts with **?**, who focus mainly on prorata rules, and with Bizer and DeMarzo (1992) who impose priority rules.

2.4 Preferences

If default does not occur in case of success, the entrepreneur's expected utility is given by:

$$U = \begin{cases} \pi_H\big(G(I+A) - R\big) - A & \text{if } \ e = H \\ \pi_L\big(G(I+A) - R\big) + B(I+A) - A & \text{if } \ e = L \end{cases}$$

where R and I denote the aggregate repayment and investment traded by the entrepreneur. The above equation implies that A is observable, so that if the entrepreneur chooses $e = L$, he has to invest A.[7] If the entrepreneur decides not to enter into a credit relationship, she is left with the option to carry out the production process using her endowment only. Given assumption 1, the corresponding payoff is:

$$U(0) = (\pi_H G - 1)A.$$

If (I, R) is an aggregate allocation traded by the entrepreneur, $e(I, R) \in \arg\max_e U(I, R, e)$ is the corresponding optimal effort choice.

Denote $\mathcal{H} = \{(I, R) \in \mathbb{R}_+^2 \ : \ e(I, R) = H\}$ the set of aggregate allocations inducing $e = H$ as the optimal effort choice and $\mathcal{L} = \{(I, R) \in \mathbb{R}_+^2 \ : \ e(I, R)\}$ its complement. Finally, $\Psi = \mathcal{H} \cap \mathcal{L} = \{I, R) \in \mathbb{R}_+^2 \ : \ \pi_H\big(G(I+A) - R\big) = \pi_L\big(G(I+A) - R\big) + B(I+A)\}$ is the set of aggregate allocations that make the entrepreneur indifferent between $e = H$ and $e = L$. We call Ψ the incentive frontier.

Given a contract $C_i = (I_i, R_i(I))$, if there is no default after success, investor i's utility is written:

$$V^i(I^i, R^i, e) = \pi_e R^i(I) - I^i \quad \text{with } \ e \in \{L, H\},$$

if the contract $(I^i, R^i(I))$ is traded, and zero otherwise. The reservation utility of each investor is zero.

[7] Assuming that the entrepreneur does not invest A if he exerts effort $e = L$ does not modify the results.

2.5 Competitive and monopoly allocations

We now define allocations that will be useful benchmarks to compare our equilibrium allocations to. A competitive allocation (I^c, R^c) maximizes the entrepreneur's utility subject to incentive and participation constraints. Given (1) and (2), (I^c, R^c) solves:

$$\begin{aligned} &(I^c, R^c) \in \arg\max_{(I,R)} U(I,R,e) \\ \text{s.t.} \quad & U(I,R,H) \geq U(I,R,L) \qquad (3) \\ & \pi_H R - I \geq 0, \qquad (4) \end{aligned}$$

where (3) is the entrepreneur's incentive compatibility constraint, and (4) the participation constraint of investors. At the optimum, (3) and (4) are binding. The optimal investment-repayment pair is characterized by the equations

$$R^c = G(I^c + A) - \frac{B(I^c + A)}{\Delta\pi} \qquad (5)$$

and

$$R^c = (1/\pi_H)I^c. \qquad (6)$$

Similarly, define the monopolistic allocation (I^m, R^m) as that prevailing when financial investors maximize their joint utility. It is determined by

$$\begin{aligned} &(I^m, R^m) \in \arg\max_{(I,R)} \ \pi_H R - I \\ \text{s.t.} \quad & U(I,R,H) \geq U(I,R,L) \\ & U(I,R,H) \geq U(0) \end{aligned}$$

which implies that

$$I^m = A\big(\frac{G\Delta\pi}{B} - 1\big) \qquad \text{and} \qquad R^m = GA\big(\frac{G\Delta\pi}{B} - 1\big). \qquad (7)$$

The next section explores the outcome of the non exclusive competition game when financial contracts are contingent on the success state only. We denote such contracts plain debt contracts.

3 Credit market equilibrium with plain debt contracts

In this section, we follow the literature on non exclusive competition in credit markets (see e.g. Bizer and DeMarzo (1992), Parlour and Rajan (2001) and **?**) and focus on contracts contingent on the success or failure state, but not contingent on total cash-flow. This allows us to compare directly our results with standard models. A plain debt contract specifies a loan amount I_i in exchange for a fixed repayment R_i. It is useful for the analysis to denote $p_i = \frac{R_i}{I_i}$ the price of this contract.

The timing of events is the following:

1. Investors simultaneously post a take-it or leave-it contract.
2. Having observed the array of offers, the entrepreneur selects which contracts to accept and chooses her optimal level of effort.

We derive below the set of subgame perfect equilibria of this extensive form game. Strategic default cannot occur in equilibrium in our setting: If strategic default arises in equilibrium, the entrepreneur optimally chooses $e = L$ because of his private benefit. But by assumption 1, $e = L$ cannot be an equilibrium strategy, otherwise some investors lose money. We therefore look for equilibria sustained by $e = H$.

We first emphasize a simple property of the entrepreneur's optimal choices. Denote $C = (I, R)$ the optimal aggregate investment-repayment pair chosen by the entrepreneur for a given array of investors' offers. Also denote $\tau(C)$ the entrepreneur's marginal rate of substitution evaluated at C. For a given level of effort, $\tau(C)$ reflects the maximum price that the entrepreneur is willing to pay for an additional unit of investment for his utility to remain constant. In particular, one gets $\tau(C) = G$ for every $C \in \mathcal{H}$ and $\tau(C) = G + \frac{B}{\pi_L}$ for every $C \in \mathcal{L}$ (if no default occurs). Any contract $C \in \mathcal{H}$ with a price G does not modify the entrepreneur's utility, if accepted. Similarly, any contract $C \in \mathcal{L}$ with a price $G + \frac{B}{\pi_L}$ does not modify the entrepreneur's utility.

Given the linearity in preferences, the entrepreneur always has an incentive to trade contracts which price is strictly smaller than her marginal rate of substitution. This is formalized in the following:

Lemma 1 *Let $\{C_1 = (I_1, R_1), \cdots, C_n = (I_n, R_n)\}$ be any array of offers such that the corresponding investment-repayment pair $C = (I, R)$ optimally chosen by the entrepreneur does not lie on the incentive frontier Ψ. Then, if not default occurs, all offers C_i such that $p_i < \tau(C)$ are traded, and all offers C_i such that $p_i > \tau(C)$ are rejected. If default occurs, any offer C_i is accepted by the entrepreneur.*

Lemma 1 has a natural interpretation. Because the marginal rate of substitution reflects the maximum price that the entrepreneur is willing to pay per unit of investment to maintain his utility, any price higher than the marginal rate of substitution decreases her utility. It is therefore rejected. Conversely, any contract which price is strictly smaller than the entrepreneur's marginal rate of substitution increases her utility. It is therefore accepted. See that if default occurs, the entrepreneur accepts contracts at any price because accepting an additional contract does not increase the entrepreneur's repayment in case of success (he obtains a zero cash-flow anyway) and increases his private benefit.

Lemma 1 has two implications. Firstly, because the marginal rate of substitution is higher for contracts in $\mathcal{L}$ than in $\mathcal{H}$, any contract accepted if $e = H$ is chosen is also accepted if $e = L$ is chosen. Secondly, at the equilibrium, a contract which price is strictly smaller than G cannot remain passive: it is necessarily accepted.

The following proposition first establishes that the monopoly allocation can always be achieved in equilibrium.

Proposition 1 *The competing investors game always has an equilibrium in which one of the investors proposes (I^m, R^m) and all other investors are inactive. At equilibrium the entrepreneur accepts the offer (I^m, R^m) and selects $e = H$.*

Proposition 1 shows that there always exists an equilibrium which delivers the monopolistic allocation (I^m, R^m). To fix ideas, assume that this allocation is achieved with one investor offering (I^m, R^m) and the others offering the null contract $(0, 0)$. When assumption 2 holds, i.e. when the expected project return is smaller than the expected agency rent (per unit of investment), if (I^m, R^m) is offered by one investor, any additional contract with a positive level of investment induces the entrepreneur to accept all contracts and exert low effort. Indeed, the entrepreneur always achieves a higher utility by accepting all contracts and shirk, rather than by accepting only the deviating lender's offer.

The next question is whether inducing the entrepreneur to accept all contracts and shirk is a profitable deviation for a passive investor. A specific feature of our modeling is that an investor offering $(0, 0)$ may have an incentive to induce strategic default (and thus to induce the entrepreneur to choose $e = L$), if he is repaid before the others. Whether inducing strategic default constitutes a profitable deviation thus depends on the repayment rule in case of default. To rule out such deviations, the repayment rule in case of default must not be too favorable towards the deviating investor. We show in the appendix that the monopoly outcome (I^m, R^m) is sustained if the active investor who offers (I^m, R^m) is repaid first. It is also sustained in case investors are repaid according to some pro rata rule. Again, no passive investor has an interest to deviate because he cannot make money at the expense of the others: In other words, if he wants to obtain a large repayment, he has to lend a large amount of money. This prevents him from earning positive profits when the entrepreneur chooses $e = L$. If however, repayment rules give priority to the deviating investor, the latter is induced to offer a small loan at a high price. He then recoups his investment even when the entrepreneur exerts low effort. When such deviations are possible, the monopoly equilibrium disappears.

The result of proposition 1 is strikingly different from that arising in an exclusive competition setting for the following reason. In standard exclusive competition in price à la Bertrand, if one investor offers a monopoly price, any competitor has an incentive to offer a lower price and capture the whole demand. Here, because competition is non exclusive, the entrepreneur cannot commit to choose a unique investor. When the moral hazard problem is severe, as illustrated by assumption

2, the entrepreneur's threat to accept all contracts and shirk, thereby reducing the probability of repayment and investors' expected profits, is strong. Passive investors have no incentive to deviate from the null contract. Effective competition cannot take place, and total surplus is reduced.

That a monopoly allocation can be sustained in equilibrium has already been emphasized in other contexts (see e.g. Parlour and Rajan (2001)). The novelty here is to show that it can emerge in a linear production technology setting. The next paragraphs provide a full characterization of the aggregate allocations supported at equilibrium when investors play pure strategies. To characterize such equilibrium allocations we first look for contracts that *cannot* be offered at equilibrium. Propositions 2 and 3 provide necessary conditions for an aggregate allocation $C = (I, R) \in \mathcal{H}$ to be supported at equilibrium. It will follow that no allocation different from the monopolistic one can be supported at equilibrium.

Proposition 2 *If $C = (I, R) \in \mathcal{H}$ is an equilibrium allocation, then:*

(i) The set of contracts offered by investors is such that the entrepreneur can achieve the allocation $\tilde{C} = (\tilde{I}, \tilde{R}) \in \mathcal{L}$ defined by $U(C, e = H) = U(\tilde{C}, e = L)$.

(ii) The set of contracts offered by investors is such that the entrepreneur can achieve the allocation $\bar{C} = (\bar{I}, \bar{R}) \in \Psi$ defined by $U(C, e = H) = U(\bar{C}, e = H)$.

Proposition 2 identifies two important necessary conditions for any equilibrium allocation. Property (i) is necessary for any investor not to deviate and propose a higher level of investment. Intuitively, if the allocation $\tilde{C}$ is not available, any investor can deviate to increase his profit as well as the entrepreneur's utility. This is because additional investment can be made without triggering low effort $e = L$. In other words, if the entrepreneur cannot achieve the allocation $\tilde{C}$ given all contracts offered, active investors can deviate and increase total surplus without inducing the entrepreneur to choose $e = L$. This is a standard property of equilibrium allocations in a non exclusive setting. The threat of misbehaving or reducing the probability of repayment of loan contracts (which is materialized here by choosing $e = L$) must be strong enough to prevent deviations.

Property ii) specifies how the allocation $\tilde{C}$ is achieved. From any equilibrium $C \in \mathcal{H} - \Psi$, $\tilde{C}$ is achieved when the entrepreneur accepts contracts with a price $p \geq G$. Proposition 2 states that an allocation $\bar{C}$ must exist. It can be interpreted as follows. If $\bar{C}$ is not available, $\tilde{C}$ can only be achieved if (some) contracts such that $p \in (G, G + \frac{B}{pi_L})$ are offered. By lemma 1, such contracts remain inactive at equilibrium. However, we show in the appendix that any investor that proposes such a contract has an incentive to deviate: He is strictly better off lowering the contract price to be accepted by the entrepreneur. Therefore, this cannot be an equilibrium situation. The existence of $\bar{C}$ guarantees that $\tilde{C}$ is achieved with contracts whose prices do not lie in $(G, G + \frac{B}{pi_L})$.

An important consequence of proposition 2 is thus to characterize contracts that cannot be offered at equilibrium. Because of properties i) and ii), investors never offer contracts such that $p_i \in (G, G + \frac{B}{\pi_L})$. They can however offer contracts with a price equal to G, or with a price larger than or equal to $G + \frac{B}{\pi_L}$ to achieve allocations $\bar{C}$ and $\tilde{C}$. Next proposition states that contracts with a price $p_i < G$ cannot be offered either.

Proposition 3 *If the offer $C_i = (I_i, R_i)$ is accepted at an equilibrium, then necessarily $p_i = \frac{R_i}{I_i} = G$.*

Proposition 3 states that at any equilibrium, investors offer contracts such that they capture all surplus from the project, i.e. such that $p_i = G$. The consequence is that at any equilibrium the entrepreneur earns her reservation utility. The reason is the following. If an investor offers a contract such that $p_i < G$, this contract is accepted by the entrepreneur (from lemma 1). However, this investor can deviate and offer a slightly more expensive contract that still induces the entrepreneur to exert effort $e = H$. This is formally proved in the appendix.

The equilibrium contracts property emphasized in proposition 3 implies that only allocations such that investors earn positive profit can be supported at equilibrium. This is the consequence of considering a linear production technology. Nothing can prevent an active investor from deviating and offering a higher-price contract that is accepted by the entrepreneur *without* inducing $e = L$. This is not always the case with a concave production technology. In that case, such deviations can sometimes be blocked because the entrepreneur can maintain his equilibrium utility even if one active investor withdraws his offer. The latter cannot offer a more expensive contract that is accepted by the entrepreneur. Here, this can only be the case when offered contracts have a price $p_i = G$: The entrepreneur is left with her reservation utility, and no additional deviation can occur. We are now in a position to argue that only the monopolistic allocation (I^m, R^m) can be supported at equilibrium.

Corollary 1 *No allocation $C \notin \Psi$ can be supported at equilibrium in the competing investors game.*

The intuition of corollary 1 is the following. Given that all offered contracts are such that $p_i = G$, if the equilibrium allocation is not on the incentive frontier, there must exist contracts that remain passive. Otherwise, the allocation $\bar{C}$ is not available. But any investor offering a contract that remains passive has an incentive to propose a cheaper contract that the entrepreneur is willing to accept. The only way to prevent such deviations is when the equilibrium C is on the incentive frontier. No allocation different from the monopolistic one can therefore be supported at equilibrium.

Corollary 2 *The monopoly allocation (I^m, R^m) is the unique equilibrium allocation of the competition game.*

Corollary 2 simply results from propositions 3 and 1, and from corollary 1. Proposition 3 and corollary 1 state that any equilibrium must deliver the monopoly allocation (I^m, R^m): This is the only allocation on the incentive frontier Ψ that can be attained when the price of all contracts offered at equilibrium is equal to G.

The fact that the competition game equilibrium is unique *and* delivers the monopoly allocation is not standard in the literature. Parlour and Rajan (2001) and more recently **?** have already emphasized that the monopoly allocation can emerge at equilibrium. However, the monopoly allocation is not unique in their setting. One contribution of this paper is to offer a setting where the monopoly outcome arises naturally. When the production function is linear, the number of equilibria is dramatically reduced, and the market power of investors is exacerbated. The fundamental reason why the monopoly equilibrium is unique in our setting is exposed in proposition 3. Any active investor has an incentive to raise the contract price to increase his profit because the entrepreneur cannot do better than accepting this contract, except in the extreme case where the entrepreneur is left with her reservation utility.

4 Credit market equilibrium with covenants contingent on cash-flows

A feature of non exclusive competition is that contracts cannot be contingent on other contracts accepted by an agent. In credit markets, this has often translated into assuming that financial contracts cannot be contingent on total investment, or total assets (see for instance Bizer and DeMarzo (1992) or **?**). This is the route we followed in section 3. In this section, we depart from these models by assuming that financial contracts can be contingent on the entrepreneur's total cash-flow which is itself related to the total amount invested initially. The reason is that while it may not be possible to control for the number and type of contracts accepted at the time they are negotiated by the agent, the consequences of such contracts are sometimes contractible. For instance, a firm's cash-flows or assets are verifiable and contractible.

The benefit of expanding the contract space is for investors to set punishments, or coercive clauses, in case the entrepreneur deviates from some targeted investment level. It is thus natural to interpret such contingent contracts as debt contracts with covenants. The question is whether one's ability to write covenants contingent on cash-flows ex post is equivalent to one's ability to control for the number of contracts accepted ex ante, as it would be the case in a setting of exclusive competition. Intuitively, the two might differ because investors' ability to "punish" departures from targeted investment levels is limited. In our model, the entrepreneur's limited liability sets an upper bound on penalties that investors can impose.

Considering contracts contingent on total cash-flow amounts to conditioning repayment R_i on total investment I rather than on the single investment I_i proposed by investor i. In contrast, exclusive competition amounts to conditioning I_i on total investment I before investment is actual sunk.

Next proposition states that introducing cash-flow-contingent covenants dramatically changes the equilibria that can be sustained in our model.

Proposition 4 *Take any aggregate allocation $C = (I, R) \in \mathcal{H}$ that is feasible, i.e. $R \in [\frac{1}{\pi_H} I, GI]$. If repayments R_i can be contingent on the total investment $I = \sum_{i \in \mathcal{I}} I_i$ chosen by the entrepreneur, any such allocation can be supported at equilibrium as long as the number of investors N is large enough and the private benefit of shirking B is large enough, in the sense that $G < \frac{1}{\pi_H}(1 + B)$.*

Proposition 4 establishes that any feasible allocation can be sustained at equilibrium with covenants contingent on the final cash-flow. This result holds under some assumptions regarding the number of agents in the economy, and regarding the severity of the moral hazard problem. More precisely, the strategy profiles of agents that sustain these equilibria are the following. A set of investors offer contracts that collectively grant the entrepreneur his equilibrium utility: These contracts are active, i.e. accepted at equilibrium. To achieve this, each offer is formulated so that the entrepreneur obtains his equilibrium utility when accepting all (or all but one) contracts: If the entrepreneur accepts only a subset of these contracts, or a larger set of contracts, he is punished by having to repay all the cash-flow. Such contracts can be interpreted as debt contracts, with covenants specifying a targeted investment level. When a covenant is violated, lenders capture the firm's assets.[8] An important feature is that any equilibrium utility can be sustained by this set of active contracts. Another set of investors offer high price contracts that are not accepted at equilibrium. Such passive contracts ensure that the entrepreneur would obtain his equilibrium utility, were he to accept all contracts and exert effort $e = L$.

Given these strategies, we show in the appendix that no agent has an interest to deviate from his equilibrium strategy. The informal argument goes as follows. Firstly, it is clear that the entrepreneur maximizes his utility by accepting all active contracts (he cannot obtain a higher utility by accepting less, or more contracts). Secondly, no individual investor has an incentive to deviate. This is particularly striking given that some investors make positive profits, and others make

[8] In our model, asset value is simply the final realized cash-flow.

zero profits. The intuition is the following. A passive investor cannot offer a contract that the entrepreneur is willing to accept alone: For such a contract to attract the entrepreneur, the investment proposed has to be rather large, which induces the entrepreneur to accept all contracts and shirk, given the other contracts offered. A passive investor cannot offer either a contract that becomes part of the set of accepted contracts. This is because covenants contingent on cash-flows help active investors to deter such deviations. Last, an active investor has no incentive to deviate for analogous reasons.

An important assumption needed to sustain these equilibria is that the number of passive investors is large. This is because each passive investor's individual investment has to be small, but the sum of investment offered by passive investors has to be large. This allows to deter deviations by a passive investor. Indeed, for a deviation to be considered by the entrepreneur, a passive investor has to invest substantially more, which in turn triggers low effort $e = L$. Another assumption is that B has to be large. This is because all equilibria rely on the threat that any deviation triggers low effort: this threat has to be credible, i.e. the private benefit of shirking has to be larger than the per unit monetary return of the project. Last, the equilibria are constructed so that a deviating investor cannot make profits if the entrepreneur chooses $e = L$. This is done by assuming, as in section 3 that the deviating investor is repaid after the others, in case of strategic default. The result also holds when investors are repaid according to a pro rata rule. With no doubt, priority rules matter for equilibria to exist. If one investor could deviate and make his contract senior to any other, such deviations would be hard to deter.

The above discussion sheds light on the role of covenants in the determination of credit market equilibria. When the contract space is reduced, as it is the case when only the state of nature can be contracted upon, a unique equilibrium can be sustained that achieves the monopoly allocation. By contrast, expanding the contract space creates an indeterminacy that can be explained as follows. On the one hand, introducing cash-flow contingent covenants increases the ability of investors to punish deviations, which should enhance competition. On the other hand, cash-flow contingent contracts make coordination easier, and deter entry of passive investors, which renders all feasible allocations sustainable.

The indeterminacy stated in proposition 4 has important consequences for credit market efficiency. While section 3 determines a unique efficient equilibrium corresponding to the monopoly allocation, inefficient equilibria can be sustained here (i.e. equilibria such that the allocation $C = (I, R)$ is not on Ψ). This is the consequence of enlarging the ability of active investors to coordinate. Active investors can deter deviations of both passive and active investors, even when the allocation is inefficient, by offering contracts that trigger low effort in case total investment departs from the equilibrium one. This result calls for the emergence of institutions to remove such indeterminacy, or simply increase efficiency.

5 Institutional constraints to support the competitive outcome

One feature of non exclusive competition is that the entrepreneur is able to make "false " promises, i.e. to accept more contracts than he can afford. In those cases, even before investment is realized, the entrepreneur knows that he will be bankrupt with certainty. The entrepreneur's ability to default strategically influences the nature of competition, because some investors might have an interest to induce such default, and benefit from it if their claim is senior to that of others.

We now consider that there exists an institution that prevents the entrepreneur from accepting contracts that induce strategic default. Specifically, we assume that accepted contracts must be such that $\sum_{i\in\mathcal{I}}(GI_i - R_i) + GA \geq 0$. This means that the entrepreneur cannot accept contracts such that the sum of repayments is larger than the maximum achievable cash-flow. One interpretation is that there exists a credit bureau that registers accepted contracts, and that intervenes after contracts are accepted, but before production takes place, to cancel all agreements if the above condition is not satisfied. Together with investors' ability to write cash-flow contingent contracts, the existence of such an institution modifies the nature of competition among investors, as stated in the following proposition.

Proposition 5 *The following holds.*

(i) If repayments R_i are contingent on the total investment $I = \sum_{i\in\mathcal{I}} I_i$ chosen by the entrepreneur, and if strategic default is precluded in the sense that accepted contracts are such

that $\sum_{i\in\mathcal{I}}(GI_i - R_i) + GA \geq 0$*, then the competitive allocation* $C^c = (I^c, R^c)$ *is the unique equilibrium allocation.*

(ii) If only plain debt contracts are allowed, then the monopoly allocation (I^m, R^m) *is the unique equilibrium allocation.*

Ruling out strategic default when contracts are contingent on the final cash-flow removes the equilibrium indeterminacy. Also, compared to the standard case where contracts are contingent on the success state only, it allows to sustain the competitive allocation at the equilibrium. The reason is the following: Introducing contingent covenants and precluding strategic default allows to restore the basic mechanism of price competition. A cash-flow contingent contract allows a investor to deviate and offer a contract that is uniquely accepted by the entrepreneur. Precluding strategic default protects the deviator by preventing the entrepreneur from accepting all contracts and choosing $e = L$. This situation cannot happen in the case with strategic default because any investor's attempt to offer a contract that is uniquely accepted by the entrepreneur is ruled out by the entrepreneur's incentive to accept all contracts and shirk. Similarly, such an equilibrium cannot arise in the case where contracts are contingent on the state only, because each active investor has an incentive to increase his profit by reducing the investment he offers.

6 Conclusion

to be completed

Appendix

Proof of Lemma 1

The proof is developed by contradiction. Let $C_i = (I_i, R_i)$ be a contract not traded by the entrepreneur at her optimal choice C. If $p_i < \tau(C)$, then one can directly check that

$$U\big(I + I_i, R + R_i, e(I,R)\big) > U\big(I, R, e(I,R)\big).$$

That is, the entrepreneur always has an incentive to trade the C_i contract, without changing her effort choice. This contradicts that C is an optimal choice for the entrepreneur. A similar argument can be used to show that all offers C_i with $p_i > \tau(C)$ are not traded by the entrepreneur.

■

Proof of Proposition 1.

Let suppose that $(I_1, R_1) = (I^m, R^m)$ and that $(I_i, R_i) = (0, 0)$ for $i = 2, \ldots, n$. Then, it is a best reply for the entrepreneur to trade the contract (I_1, R_1) and to select $e = H$. This in turn provides her with the reservation utility $U(I_1, R_1, e = H) = \pi_H GA - A$. We now argue that none of the investors has a unilateral incentive to deviate. Since investor 1 is earning a monopoly profit, only deviations from inactive investors must be considered. Let (I_2', R_2') be a deviation of any of these investors, say investor 2.

• Consider first that this deviation induces the entrepreneur to select $e = H$. In this case, for the contract (I_2', R_2') to yield a strictly positive profit it must also be $R_2' > \dfrac{1}{\pi_H} I_2'$. We show that following any such deviation the entrepreneur has indeed an incentive to select $e = L$. Let K be the aggregate allocation optimally selected by the entrepreneur at the deviation stage, and assume that $K \in \mathcal{H}$.

We first show that if $K \in \mathcal{H}$, then the only contract traded by the entrepreneur at the deviation stage must necessarily be (I_2', R_2'). Assume on the contrary that the entrepreneur buys both contracts (I_1, R_1) and (I_2', R_2'). Because (I_1, R_1) belongs to the set Ψ, we have:

$$G(I_1 + I_2' + A) - (R_1 + R_2') = \frac{B(I_1 + A)}{\Delta\pi} + \big(G\, I_2' - R_2'\big).$$

By (2), $\frac{R_2'}{I_2'} > \dfrac{1}{\pi_H} > G - \frac{B}{\Delta\pi}$. It follows that:

$$G(I_1 + I_2' + A) - (R_1 + R_2') < \frac{B(I_1 + A)}{\Delta\pi} + G\, I_2 - (G - \frac{B}{\Delta\pi})\, I_2 = \frac{B(I_1 + I_2 + A)}{\Delta\pi},$$

i.e. $(I_1 + I_2', R_1 + R_2') \in \mathcal{L}$. One hence gets: $K = (I_2', R_2') \in \mathcal{H}$.

By continuity, it is always possible to find a $\mu \in [0, 1[$ such that:

$$\pi_H\big(G(\mu I_1 + I_2' + A) - (\mu R_1 + R_2')\big) = \pi_L\big(G(\mu I_1 + I_2' + A) - (\mu R_1 + R_2')\big) + B(\mu I_1 + I_2' + A). \quad (8)$$

That is, if the entrepreneur took a loan of $(\mu I_1 + I_2')$ paying back the amount $(\mu R_1 + R_2')$ to the aggregate of investors, she would be indifferent between selecting $e = L$ and $e = H$ (recall that $\mu < 1$ because $(I_1 + I_2', R_1 + R_2') \in \mathcal{L}$).

Since $p_1 = \dfrac{R_1}{I_1} < G + \dfrac{B}{\pi_L}$, one gets:

$$U(I_1 + I_2', R_1 + R_2', e = L) > U(\mu I_1 + I_2', \mu R_1 + R_2', e = L) = U(I_2', R_2', e = H).$$

It follows that $K = (I_2', R_2') \in \mathcal{H}$ will not be an optimal choice. The entrepreneur selects $e = L$ which contradicts our assumption.

• Consider next that this deviation induces the entrepreneur to choose $e = L$ (and thus to accept all contracts) *and* to default in the success state. Assume first that in case of default after success, investor 1 is repaid first.

Consider the following deviation by investor $i = \{2, \ldots, n\}$. He proposes the array $(I_2', R_2') = (\epsilon, R_2'(\epsilon))$, where $\epsilon > \frac{\pi_L}{\Delta\pi}(I^m + A)$ is chosen to satisfy inequality

$$B(I^m + A + \epsilon) > \pi_H(G(I^m + A) - R^m) = \frac{\pi_H B}{\Delta\pi}(I^m + A). \quad (9)$$

Under inequality (9) the entrepreneur earns a utility strictly greater than the equilibrium one by selecting $e = L$, accepting both contracts and defaulting even in the success state. Fix then $\epsilon = \frac{\pi_L}{\Delta\pi}(I^m + A)(1 + \frac{\eta}{G\pi_L})$, with η strictly positive, so that (9) is indeed satisfied. The corresponding total output is:

$$G(I^m + A + \epsilon) = (I^m + A)\frac{1}{\Delta\pi}(\pi_H G + \eta). \tag{10}$$

Now, if investor 1 is repaid first, it is impossible for the deviator to break-even, conditional on the entrepreneur choosing $e = L$. Formally, the deviator breaks even iff:

$$\pi_L \left(G(I^m + A + \epsilon) - R^m\right) > \epsilon \quad \Longleftrightarrow \quad B > (1 + \frac{\eta}{G\pi_L})(1 - \pi_L G), \tag{11}$$

which is impossible since $B < 1 - \pi_L G$.

Observe that the same argument holds if defaulted loans are repaid according to some *pro rata* rule. Equation (11) becomes $\pi_L G(I^m + A + \epsilon)(1 - \frac{I^m+A}{I^m+A+\epsilon}) > \epsilon \Longleftrightarrow \pi_L G > 1$ which is never satisfied since $\pi_L G + B < 1$. This completes the proof of Proposition 1..

Note that it is however possible to sustain a deviation by investor i in case he is repaid *before* investor 1. This illustrates how the specification of seniority affects the existence of the equilibrium. More precisely, if the deviating investor i is repaid first, it is always possible to choose R_2' large enough to guarantee that the entrepreneur defaults in case of success. Then the deviation is profitable whenever the expected total output is greater than ϵ. That is:

$$\pi_L G(I^m + A + \epsilon) > \epsilon \quad \Longleftrightarrow \quad \pi_H G - 1 < \eta\left(\frac{1}{\pi_L G} - 1\right), \tag{12}$$

which is verified for η small enough. In that case, the allocation $(I_m; R_m)$ cannot be sustained in equilibrium.

■

Proof of Proposition 2

Proof of assertion (i).

If $C \in \Psi$, the requirement is satisfied taking $\tilde{C} = C$. Consider now that $C \notin \Psi$ and that for any allocation $\tilde{C}$ in $\mathcal{L}$, $U(C, e = H) \neq U(\tilde{C}, e = L)$. Then, because C is an equilibrium allocation, we have that $U(C, e = H) > U(K, e = L)$, where K is the aggregate allocation optimally chosen by the entrepreneur when $e = L$ is selected. We now show that any investor whose offer is accepted has an incentive to deviate from his equilibrium offer.

Let C_i be the equilibrium offer of any active investor i, and suppose he deviates and offers $C_i' = (I_i + \epsilon, R_i + \frac{\epsilon}{\pi_H} + \epsilon^2)$, for some strictly positive number ϵ. The deviation, if accepted, increases the aggregate level of investment. It is also profitable for the investor if the offer C_i is accepted and the effort $e = H$ is selected. Indeed we have that $\pi_H(R_i + \frac{\epsilon}{\pi_H} + \epsilon^2) - I_i - \epsilon > \pi_H R_i - I_i$. Let us prove that for $\epsilon > 0$ small enough, the entrepreneur has an incentive to accept contract C_i' and choose $e = H$. To ease notations, denote $C = \sum C_j = \sum_{j \neq i} C_j + C_i$ and $C' = \sum_{j \neq i} C_j + C_i' = C + C_i' - C_i$. Write also $C' = (I', R')$ with $I' = I + \epsilon$ and $R' = R + \frac{\epsilon}{\pi_H} + \epsilon^2$. Now, since $C \in \mathcal{H} - \Psi$ we have:

$$\pi_H(G(I + A) - R) > \pi_L(G(I + A) - R) + B(I + A). \tag{13}$$

See that $C' \in \mathcal{H} - \Psi$ if and only if:

$$\pi_H(G(I+A)-R)+\pi_H(G\epsilon-\frac{\epsilon}{\pi_H}-\epsilon^2) > \pi_L(G(I+A)-R)+\pi_L(G\epsilon-\frac{\epsilon}{\pi_L}-\epsilon^2)+B(I+A+\epsilon). \tag{14}$$

Given (13), a sufficient condition for (14) to hold is:

$$\epsilon[\Delta\pi G - \epsilon - B] \geq 0.$$

Recalling that $\Delta\pi G - B > 0$, the above condition is true if ϵ is small enough, and we have

$C' \in \mathcal{H} - \Psi$. We next show that C' is chosen by the entrepreneur. See that:

$$\begin{aligned} U(C + C'_i - C_i, e = H) &= U(\sum_{i \neq j} C_i + C'_i, e = H) \\ &= \pi_H(G(I + A) - R) - A + \pi_H(G(I'_i - I_i) - (R'_i - R_i)) \\ &= \pi_H(G(I + A) - R) - A + \pi_H(G\epsilon - \frac{\epsilon}{\pi_H} - \epsilon^2) \\ &> \pi_H(G(I + A) - R) - A \end{aligned}$$

where the last inequality holds for ϵ sufficiently small since $\pi_H G > 1$. Therefore, $U(C', e = H) > U(C, e = H)$. Last, since $U(C, e = H) > U(K, e = L)$ it follows that for ϵ sufficiently small $U(C + C'_i - C_i, e = H) > U(K - C_i + C'_i, e = L)$. Therefore the offer C'_i constitutes a profitable deviation for investor i, which contradicts the claim that C is an equilibrium allocation.

Proof of assertion (ii).

We proceed by way of contradiction. Suppose that $\bar{C} \in \Psi$ does not exist. In a first step we show that the set J of investors who are inactive at equilibrium necessarily contains a investor j who has proposed an offer $C_j = (I_j, R_j)$ with $G < p_j < G + \frac{B}{\pi_L}$. In a second step we show that there exists a profitable deviation for investor j such that the entrepreneur accepts the deviating contract and chooses $e = H$. This contradicts the fact that C is an equilibrium allocation. Formally, assertion (ii) of Proposition 2 results from the two following lemmas.

Lemma 2 *Assume $\bar{C}$ does not exist. Let J be the set of investors who are inactive at equilibrium and consider $J_2 = \{j \in J \,:\, p_j \in (G, G + \frac{B}{\pi_L})\}$. Then we have that $J_2 \neq \emptyset$.*

Lemma 3 *Assume $\bar{C}$ does not exist and take a investor $s \in J_2$. Consider the contract $C'_s = (\epsilon, \frac{\epsilon}{\pi_H} + \epsilon^2)$ where ϵ is strictly positive and sufficiently small. C'_s is a profitable deviation for investor s and the entrepreneur has an incentive to trade the deviating contract C'_s and to select $e = H$.*

Proof of Lemma 2. First note that, if $\bar{C} \in \Psi$ is impossible then we also have $C \notin \Psi$ (if not $\bar{C} = C \in \Psi$). Now let J be the set of the investors who are inactive at equilibrium. By assertion (i), if $C \notin \Psi$, J is not empty. Indeed, we know that there exists $\tilde{C} \in \mathcal{L}$ such that $U(C, e = H) = U(\tilde{C}, e = L)$ and that all contracts accepted when $e = H$ are also accepted when $e = L$. If J is empty, $C = \tilde{C}$ and in turn $C \in \Psi$. This contradicts $C \notin \Psi$. We also know from lemma 1 that each C_j contract, with $j \in J$, must be such that $p_j = \frac{R_j}{I_j} \geq G$. In particular, $J = J_1 \cup J_2 \cup J_3$ with:

1. $J_1 = \{j \in J : p_j = G\}$
2. $J_2 = \{j \in J : p_j \in (G, G + \frac{B}{\pi_L})\}$
3. $J_3 = \{j \in J : p_j \geq G + \frac{B}{\pi_L}\}$,

Thus $C + \sum_{j \in J_1} C_j + \sum_{j \in J_2} C_j$ is an allocation which utility is maximum in $\mathcal{L}$ and from (i) one gets that $C + \sum_{j \in J_1} C_j + \sum_{j \in J_2} C_j = \tilde{C}$ (by lemma 1, a contract such that $p_i < G + \frac{B}{\pi_L}$ is always accepted by the entrepreneur with effort $e = L$). In words, to achieve the allocation $\tilde{C}$ all contracts offered at a price smaller than $G + \frac{B}{\pi_L}$ must be accepted. It follows that if $\bar{C}$ does not exist, J_2 is necessarily non-empty. By way of contradiction, assume that J_2 is empty, then $\tilde{C} = C + \sum_{j \in J_1} C_j$. By (i), we have: $U(C, e = H) = U(\tilde{C}, e = L)$. But we also have $U(C, e = H) = U(C + J_1, e = H) = U(\tilde{C}, e = H)$. This implies that $\tilde{C} = \bar{C}$ and yields a contradiction.

Proof of Lemma 3. Let us consider a investor $s \in J_2$ and suppose he deviates and offers $C'_s = (\epsilon, \frac{\epsilon}{\pi_H} + \epsilon^2)$ for some strictly positive ϵ. The deviation has been constructed to be profitable if the offer C'_s is accepted and the effort $e = H$ is selected by the entrepreneur. First see that the entrepreneur accepts contract C'_s. Indeed,

$$U(C + C'_s, e = H) > U(C, e = H) \Leftrightarrow G > \frac{1}{\pi_H} + \epsilon,$$

which holds for ϵ not too large. We next show that the entrepreneur prefers to select $e = H$ given the deviation C'_s.

Consider the limit case $\epsilon = 0$ that is $C'_s = (0,0)$. This means that investor s removes his initial offer. Recall that $\tilde{C} = C + \sum_{j \in J_1} C_j + \sum_{j \in J_2} C_j$. We thus have:

$$U(C + \sum_{j \in J_1} C_j + \sum_{j \in J_2, j \neq s} C_j, e = L) < U(\tilde{C}, e = L).$$

That is, if investor s withdraws his offer, the allocation $\tilde{C}$ is no more available for the entrepreneur. If, on the contrary, the entrepreneur chooses $e = H$, she can still achieve her equilibrium payoff $U(C, e = H)$. By continuity, for $\epsilon > 0$ small, when investor s offers $C'_s = (\epsilon, \frac{\epsilon}{\pi_H} + \epsilon^2)$, we have:

$$U(C + C'_s + \sum_{j \in J_1} C_j + \sum_{j \in J_2, j \neq s} C_j, e = L) < U(\tilde{C}, e = L) = U(C, e = H).$$

Recall that $U(C + C'_s, e = H) > U(C, H)$ to establish:

$$U(C + C'_s, e = H) > U(C + C'_s + \sum_{j \in J_1} C_j + \sum_{j \in J_2, j \neq s} C_j, e = L) > U(C + C'_s, e = L),$$

where the last inequality comes from the fact that contracts in J_1 and J_2 are always accepted when $e = L$. In words, following a unilateral deviation to C'_s, the entrepreneur has an incentive to trade the deviating contract and to select $e = H$, which guarantees that the deviation is indeed profitable.

A consequence of properties i) and ii) is that J_2 is empty. If not, we cannot have $U(C, e = H) = U(\bar{C}, e = H) = U(\tilde{C}, e = L)$.

■

Proof of Proposition 3

Suppose that Proposition 3 is not true. Let C_i be a contract of price $p_i < G$ which is traded at equilibrium. This indeed guarantees that the entrepreneur's payoff is strictly greater than her reservation utility $U(0)$. We now show that an active investor proposing C_i can profitably deviate and reduce the entrepreneur's utility. Suppose that investor i deviates to $C'_i = \big(I_i - \epsilon, R_i - \epsilon(G - \frac{B}{\Delta\pi}) - \epsilon^2\big)$ for some strictly positive ϵ. Under assumption (2), if the deviation is accepted and the entrepreneur chooses $e = H$, such a deviation increases investor i's profit for ϵ sufficiently small. We check below that the deviation is accepted and the entrepreneur chooses $e = H$.

By Lemma 1, the entrepreneur always has an incentive to trade the C'_i contract if $p(C'_i) < G$. This is equivalent to

$$
\begin{aligned}
& R_i - \epsilon\left(G - \tfrac{B}{\Delta\pi}\right) - \epsilon^2 < G(I_i - \epsilon) \\
\Leftrightarrow \quad & R_i - GI_i + \epsilon\left(\tfrac{B}{\Delta\pi} - \epsilon\right) < 0
\end{aligned}
$$

Since $R_i - GI_i < 0$, the above condition holds when ϵ is small enough.

Next, we show that in every continuation game following the deviation to C'_i the entrepreneur selects $e = H$.

Given the offers $\{C_1, \cdots, C'_i, \cdots C_N\}$, we take $\mathcal{I} = \{i \in N \backslash J_1\}$ to be the set of active investors at the equilibrium.

Then, the agent maximizes his utility in $\mathcal{L}$ when buying all contract of price less than G, plus all contracts of price G. Indeed, by Proposition 2, J_2 is empty: No contract can be offered with $p \in (G, G + \frac{B}{\pi_L})$. The set of such contracts is

$$
\sum_{i \in \mathcal{I}} C_i + \sum_{j \in J_1} C_j = \bar{C} - C_i + C'_i.
$$

See that

$$
\begin{aligned}
& U(\bar{C} - C_i + C'_i, e = H) > U(\bar{C} - C_i + C'_i, e = L) \\
\Leftrightarrow \quad & \pi_H \left\{G(\bar{I} - \epsilon) - (\bar{R} - \epsilon(G - \tfrac{B}{\Delta\pi}) - \epsilon^2)\right\} > \pi_L \left\{G(\bar{I} - \epsilon) - (\bar{R} - \epsilon(G - \tfrac{B}{\Delta\pi}) - \epsilon^2)\right\} + B(\bar{I} - \epsilon) \\
\Leftrightarrow \quad & \epsilon^2 > 0.
\end{aligned}
$$

The entrepreneur's maximal utility when $e = L$ is strictly lower than his utility when $e = H$. Any other feasible allocation, by adding contracts of J_3 and by subtracting contracts in J_1 or in $\mathcal{I}$ reduces the entrepreneur's utility. Therefore, following the deviation C'_i, the entrepreneur strictly prefers to choose $e = H$: No contract with $p < G$ can be offered at equilibrium.

■

Proof of Corollary 1.

The proof is developed by contradiction. Given the equilibrium offers' array $\{C_1, \cdots, C_N\}$, we take $C = (I, R)$ to be the corresponding loan-repayment pair supported at equilibrium. It follows from proposition 1 that there is a non-empty set of investors J_1 proposing contracts of price G which are not traded at equilibrium. Let $s \in J_1$ be one of these investors, and suppose he deviates towards the alternative offer $C'_s = (\epsilon, \frac{\epsilon}{\pi_H} + \epsilon^2)$ for some strictly positive ϵ. The deviation has been constructed to be profitable if the offer C'_i is accepted and the effort $e = H$ is selected. Following the same reasoning developed in Lemma 3 one can show that there is indeed a profitable deviation.

■

Proof of Proposition 4.

Let $C = (I, R) \in \mathcal{H}$ be a feasible aggregate allocation. Define $\mathring{I} \in \mathbb{R}_+$ as the investment level such that

$$\pi_H[G(I + A) - R] = B\big(I + \mathring{I} + A\big). \tag{15}$$

According to equation (15), the entrepreneur obtains the equilibrium utility if he borrows $\mathring{I}$ in addition to the equilibrium investment I, chooses $e = L$ and is left with his private benefit only. Recall that $R \leq GI$ and see that $\mathring{I} > 0$:

$$B\,\mathring{I} = \pi_H[G(I + A) - R] - B\,(I + A) \geq \pi_L[G(I + A) - R] \geq \pi_L GA > 0 \;\; \forall (I, R) \in \mathcal{H}.$$

The equilibrium we construct involves a large number of both active and inactive investors. Consider in particular the following profile of strategies. investors $1, 2, ..., M$ offer:

$$\begin{cases} (\frac{I}{M}, \frac{R}{M}) & \text{if} \quad \text{the total investment is } I \\ (\frac{I}{M}, \frac{\mathring{R}}{M-1}) & \text{if} \quad \text{the total investment is } \frac{(M-1)I}{M} \\ (\frac{I}{M}, G(\hat{I} + A)) & \text{if} \quad \text{the total investment is } \hat{I} \notin \{\frac{(M-1)I}{M}, I\} \end{cases} \tag{16}$$

where $\mathring{R}$ is defined by:

$$\pi_H[G(I+A)-R] = \pi_H[G(\frac{M-1}{M}I+A) - \mathring{R}], \tag{17}$$

which implies $\mathring{R} = R - \frac{GI}{M}$.
Each of the investors $M+1,\ldots,N$ proposes the pair $\left(\frac{\mathring{I}}{N-M}, G(\hat{I}+A)\right)\ \forall \hat{I} \in \mathbb{R}_+$.
Let us show that these strategies constitute an equilibrium in which the entrepreneur accepts the offers of investors $1,..,M$, rejects those of investor $M+1,\ldots,N$ and selects $e=H$. Therefore, only M investors are active. To prove that this is indeed an equilibrium we postulate that following any deviation, the entrepreneur keeps her equilibrium choices whenever indifferent. In addition, we specify that if default takes place, she first repays all non-deviating investors. Last we assume that the moral hazard problem is severe enough in the sense that:

$$G < \frac{1}{\pi_H}(1+B). \tag{18}$$

1. We first show that given the investors' offers, it is a best reply for the entrepreneur to accept the proposals of investors $1,\ldots,M$ and to reject those of investors $M+1,\ldots,N$. Following the equilibrium strategy, the entrepreneur obtains the payoff $\pi_H[G(I+A)-R] \geq \pi_H GA$. See that no other portfolio choice associated to $e=H$ provides the entrepreneur with a strictly higher payoff: indeed, accepting $M-1$ contracts yields the same utility for the entrepreneur. Accepting any of the $N-M$ contracts triggers $e=L$. Accepting less than $M-1$ contracts also triggers $e=L$. Next, see that the entrepreneur cannot increase her utility by choosing $e=L$ and accepting all N contracts. She then obtains an aggregate loan of $I+\mathring{I}$ and a utility level of $B\big(I+\mathring{I}+A\big) = \pi_H[G(I+A)-R]$, by (15).
2. We next show that given the equilibrium strategies, none of the inactive investors $M+1,\ldots,N$ can profitably deviate.
• Consider an inactive investor j. Let us establish that he cannot propose a deviation (I'_j, R'_j) such that only his offer is accepted out of equilibrium. Define (i,r) as the investment-repayment

pair that lies at the intersection between the zero-profit line (of investors) of slope $\frac{1}{\pi_H}$ and the entrepreneur's equilibrium indifference curve given (I, R). Check that $i = \dfrac{\pi_H(GI - R)}{\pi_H G - 1}$ and $r = \dfrac{i}{\pi_H}$. Denote also (i_I, r_I) the intersection between the entrepreneur's indifference curve and Ψ, and (i_P, r_P) the intersection between the zero-profit line and Ψ. Any deviation (I'_j, R'_j) that (weakly) increases the profit of investor j and (weakly) increases the utility of the entrepreneur if only (I'_j, R'_j) is accepted must lie in the triangle defined by (i, r), (i_I, r_I) and (i_P, r_P). We show below that any such deviation induces the entrepreneur to accept all contracts and exert $e = L$ if N is sufficiently large.

To do so, we prove that, if $N - M$ is large enough then the function

$$F(I'_j, R'_j; I, R) = \pi_H \left(G(I'_j + A) - R'_j\right) - B\left(I'_j + I + \mathring{\mathrm{I}}\, \frac{N - M - 1}{N - M} + A\right)$$

is negative at (i, r). This will imply in turn that F is negative at any point in the triangle $(i, r), (i_I, r_I), (i_P, r_P)$ which corresponds to the set of admissible deviations (I'_j, R'_j). Consider therefore that investor j deviates and proposes the pair (i, r), the entrepreneur prefers to choose $e = L$ if:

$$\begin{aligned} & \pi_H \left(G(i + A) - r\right) - B\left(i + I + \mathring{\mathrm{I}}\, \tfrac{N-M-1}{N-M} + A\right) \leq 0 \\ \Longleftrightarrow \quad & B\left(i - \tfrac{\mathring{\mathrm{I}}}{N-M}\right) \geq 0. \end{aligned} \tag{19}$$

We use then the definitions of i and $\mathring{\mathrm{I}}$ to rewrite (19) as:

$$(N - M)\frac{\pi_H(GI - R)}{\pi_H G - 1} - \pi_H \frac{G(I + A) - R}{B} + (I + A) \geq 0. \tag{20}$$

If $N - M$ is large enough, then (20) is satisfied for all feasible allocations (I, R): F is negative at the point (i, r). Next, see that for a given I'_j, the minimum value of R'_j such that the deviation is in the admissible triangle $(i, r), (i_I, r_I), (i_P, r_P)$ is defined by $R'_j = \frac{I'_j}{\pi_H}$ and observe that

$$\left.\frac{dF}{dI'_j}\right|_{R'_j = \frac{I'_j}{\pi_H}} = \pi_H G - 1 - B < 0. \tag{21}$$

This yields that $F(I'_j, R'_j; I, R) < 0$ for any pair $(I'_j, \frac{I'_j}{\pi_H})$ with $I'_j > i$. Finally, because F is also decreasing in R'_j one gets that F is negative at any point in the triangle $(i, r), (i_I, r_I), (i_P, r_P)$. Thus, for every feasible allocation (I, R) there always exists a sufficiently high number $(N - M)$ of inactive investors such that none of the inactive investors has a unilateral incentive to deviate as long as the entrepreneur accepts only his offer out of equilibrium.

• Let us now establish that no inactive investor j has an incentive to deviate so that the entrepreneur trades several contracts out of equilibrium. For a deviation to be profitable, the entrepreneur must choose $e = H$ following the deviation. This is because the deviating investor is repaid after the others in case of default. This can only happen if the aggregate investment selected by the entrepreneur at the deviation stage is either I, or $\frac{(M-1)I}{M}$. In all other cases, the entrepreneur optimally chooses $e = L$ since all the cash-flow is paid to investors.

It follows that only a subset of the active investors' offers will be traded out of equilibrium (and no offer from passive investors). Let (I'_j, R'_j) be a unilateral deviation by investor j, and let $m < M$ be the number of additional contracts which are traded following the deviation.

We first consider the case where the aggregate level of investment is equal to I, we must have:

$$I = I'_j + \frac{m}{M} I \iff I'_j = \frac{M - m}{M} I \tag{22}$$

If the deviation (I'_j, R'_j) is profitable then,

$$\pi_H \left[G(I + A) - R'_j - \frac{m}{M} R \right] > B \left(I + \mathring{I} + A + I'_j - \frac{\mathring{I}}{N - M} \right), \tag{23}$$

which, given (15) and (22), corresponds to

$$\pi_H \left[(1-\frac{m}{M})R - R'_j\right] > B\left(\frac{M-m}{M}I - \frac{\mathring{\mathrm{I}}}{N-M}\right)$$

$$\Leftrightarrow \quad B\frac{\mathring{\mathrm{I}}}{N-M} > \frac{M-m}{M}\left(BI - \pi_H R + \pi_H R'_j \frac{M}{M-m}\right) \tag{24}$$

The deviation (I'_j, R'_j) is admissible for the inactive investor j and therefore satisfies $R'_j \pi_H \geq I'_j$. Thus, if (24) holds then we have necessarily

$$B\frac{\mathring{\mathrm{I}}}{N-M} > \frac{M-m}{M}\left(BI - \pi_H R + I'_j \frac{M}{M-m}\right)$$

which, using (22), yields

$$B\frac{\mathring{\mathrm{I}}}{N-M} > \frac{M-m}{M}\left(I(1+B) - \pi_H R\right). \tag{25}$$

Since R cannot be greater than GI, it follows from (18) that $I(1+B) - \pi_H R > 0$. Thus, If $N - M$ is high enough, (25) does not hold, this in turn contradicts (23) and there exists no profitable deviation.

We now consider the situation where the aggregate investment chosen at the deviation stage is equal to $\left(\frac{M-1}{M}\right)I$. The corresponding I'_j is such that:

$$\left(\frac{M-1}{M}\right)I = I'_j + \frac{m}{M}I \iff I'_j = \left(\frac{M-m-1}{M}\right)I \tag{26}$$

with $m < M - 1$. A necessary condition for the deviation to (I'_j, R'_j) to be profitable is

$$\pi_H\left[G\left(\frac{M-1}{M}I + A\right) - R'_j - \frac{m}{M-1}\,\mathring{\mathrm{R}}\right] > B\left(I + \mathring{\mathrm{I}} + A + I'_j - \frac{\mathring{\mathrm{I}}}{N-M}\right), \tag{27}$$

which, given (15), corresponds to

$$B\frac{\mathring{I}}{N-M} > BI'_j - \pi_H \frac{M-m-1}{M-1}\left(R - \frac{GI}{M}\right) + \pi_H R'_j \tag{28}$$

Using $\pi_H R'_j \geq I'_j$ and (26) together with the fact that $\frac{M-m-1}{M-1}\left(R - \frac{GI}{M}\right) \leq \frac{M-m-1}{M} GI$ because $R \leq GI$, we deduce from (28) that

$$B\frac{\mathring{I}}{N-M} > \frac{M-m-1}{M} I\left(1 + B - \pi_H G\right). \tag{29}$$

The (RHS) of (29) is positive because of (18) from which it follows that if $N-M$ is large enough, (27) is violated and there is no profitable deviation. This concludes the analysis of deviations by passive investors.

3. We now turn to the proof that no active investor $(1, \ldots, M)$ can profitably deviate. Consider any of the investors who is active at equilibrium, say the k-th one.

• Consider first the situation where only the offer (I'_k, R'_k) of investor k is accepted out of equilibrium. Let us reformulate (i, r) as the investment-repayment pair that lies at the intersection between investor k's profit line and the entrepreneur's equilibrium indifference curve. Similarly, define (i_I, r_I) the intersection between the entrepreneur's indifference curve and Ψ, and (i_P, r_P) the intersection between investor k's profit line and Ψ. See that:

$$\begin{cases} \pi_H\left[G(I+A) - R\right] = \pi_H\left[G(i+A) - r\right] \\ \pi_H r - i = \pi_H \frac{R}{M} - \frac{I}{M} \end{cases} \tag{30}$$

It follows that $i = \frac{GI - R + (1/M)(R - I/\pi_H)}{G - 1/\pi_H}$. As before, we need to show that the function $F(I'_k, R'_k; I, R) = \pi_H\left(G(I'_k + A) - R'_k\right) - B\left(I'_k + I + \mathring{I} - \frac{I}{M} + A\right)$ is negative at each point in the triangle $(i, r), (i_I, r_I), (i_P, r_P)$. To prove this, we first show that F is negative at (i, r).

Assume that investor k deviates and offers (i, r), the entrepreneur prefers to choose $e = L$ if:

$$\pi_H \left(G(i + A) - r\right) - B\left(I + \mathring{I} + A + i - \frac{I}{M}\right) = -B\left(i - \frac{I}{M}\right) \leq 0. \tag{31}$$

Given the definition of i, (31) can be rewritten as:

$$\frac{(GI - R)\left(1 - \frac{1}{M}\right)}{G - \frac{1}{\pi_H}} \geq 0, \tag{32}$$

which is always satisfied since $GI \geq R$.

Next, see that for a given I'_k, the minimum value of R'_k such that the deviation is in the admissible triangle $(i, r), (i_I, r_I), (i_P, r_P)$ is defined by $R'_j = \frac{I'_k + V}{\pi_H}$, where V represents the equilibrium profit of investor k. See that:

$$\left.\frac{dF}{dI'_k}\right|_{R'_k = \frac{I'_k + V}{\pi_H}} = \pi_H G - 1 - B < 0. \tag{33}$$

This yields that $F(I'_k, R'_k; I, R) < 0$ for any pair $(I'_k, \frac{I'_k + V}{\pi_H})$ with $I'_k > i$. Note also that F is decreasing in R'_k. This yields that F is negative at any point in the triangle $(i, r), (i_I, r_I), (i_P, r_P)$. It follows that none of the active investors has a unilateral incentive to deviate if the entrepreneur accepts only his offer out of equilibrium.

• Consider next the situation where an active investor k deviates and the entrepreneur trades several contracts out of equilibrium. As before, for a deviation to be profitable, default should necessarily be avoided. This can only happen if the aggregate investment selected by the entrepreneur at the deviation stage is either I, or $\frac{(M-1)I}{M}$.

We first consider the situation where such aggregate investment is set equal to I. Let m be the number of additional contracts which are traded following the deviation, we have: $I'_k = \dfrac{M - m}{M} I$. A necessary condition for the deviation to be profitable is

$$\pi_H \left[G(I + A) - R'_k - \frac{m}{M} R\right] > B(I + \mathring{I} + A + I'_k - \frac{I}{M})$$

or equivalently using (15),

$$\pi_H \left[-R'_k + \frac{M-m}{M} R \right] > B \frac{M-m-1}{M} I. \tag{34}$$

The deviation (I'_k, R'_k) is admissible for the active investor k and thus satisfies $\pi_H R'_k - I'_k \geq \pi_H \frac{R}{M} - \frac{I}{M}$. It follows that, if (34) holds then, necessarily

$$\frac{M-m-1}{M} \pi_H R > \frac{M-m-1}{M} (B+1) I,$$

which is never satisfied given (18).

Finally, if the aggregate investment chosen at the deviation stage is equal to $\left(\frac{M-1}{M}\right) I$, we have $I'_k = \frac{M-m-1}{M} I$. A necessary condition for the deviation to be profitable is

$$\pi_H \left[G \left(I + A - \frac{I}{M} \right) - R'_k - \frac{m}{M-1} \mathring{\mathrm{R}} \right] > B(I + \mathring{\mathrm{I}} + A + I'_k - \frac{I}{M}),$$

equivalently, using (15) and the definition of $\mathring{\mathrm{R}}$,

$$\pi_H \left[-(M-m-1) \frac{GI}{M(M-1)} + \frac{M-m-1}{M-1} R - R'_k \right] > BI \left(\frac{M-m-2}{M} \right) \tag{35}$$

The deviation (I'_k, R'_k) satisfies $\pi_H R'_k - I'_k \geq \pi_H \frac{\mathring{\mathrm{R}}}{M-1} - \frac{I}{M}$. It follows that if (35) holds then necessarily,

$$\pi_H \left[R - \frac{1}{M} GI \right] \frac{M-m-2}{M-1} > (B+1) I \left(\frac{M-m-2}{M} \right). \tag{36}$$

Observe that the (LHS) of (36) is lower than $\pi_H \frac{M-m-2}{M} GI$, so that given (18), the inequality (35) cannot be satisfied. This completes the proof that any allocation $C = (I, R) \in \mathcal{H}$ such that $R \in [\frac{1}{\pi_H} I, GI]$ can be supported at equilibrium.

Remark: The above proof therefore shows that there exists a seniority rule that supports the equilibrium allocation (16). Interestingly, the same result holds if defaulted bonds are repaid *pro rata*.

To prove this let us first consider an inactive lender j who deviates and thus proposes $(\epsilon, R'_j(\epsilon))$ such that the borrower earns a utility strictly greater than the equilibrium one by selecting $e = L$ accepting all contracts and defaulting even in the success state. Formally ϵ satisfies

$$B(I + A + \frac{N - M - 1}{N - M} \mathring{I} + \epsilon) > \pi_H(I + \mathring{I} + A)$$

or equivalently, using (15)

$$\epsilon > \frac{\mathring{I}}{N - M}.$$

In a *pro rata* environment, the deviation is profitable for the lender iff

$$\pi_L \left(G \left(I + A + \frac{N - M - 1}{N - M} \mathring{I} + \epsilon \right) \left(1 - \frac{I + A + \frac{N-M-1}{N-M} \mathring{I}}{I + A + \frac{N-M-1}{N-M} \mathring{I} + \epsilon} \right) \right) > \epsilon$$

that is iff $\pi_L G > 1$. This is however never satisfied since $\pi_L G + B < 1$.
Second let consider an active lender j who deviates and proposes (I'_j, R'_j). Observe that, necessarily, $I'_j > I_j = \frac{I}{M}$. Indeed, if it was not the case, because of Equation (15), the borrower's utility at the deviation stage would be lower than her equilibrium utility. Formally,

$$B \left(\frac{M - 1}{M} I + A + \mathring{I} + I'_j \right) < B(I + A + \mathring{I}) = \pi_H(G(I + A) - R).$$

It thus follows that $I'_J = \frac{I}{M} + \epsilon$ with $\epsilon > 0$. This in turn implies that, under a *pro rata* rule, the deviating lender breaks even iff $\pi_L G > 1$. As already emphasized this latter inequality is impossible.

■

Proof of Proposition 5.

The proof follows the standard logic of price competition. Let $C \neq C^c$ be a equilibrium allocation where the entrepreneur selects $e = H$. Consider first the situation where investors are earning a strictly positive profit at equilibrium, which implies that $R > \frac{1}{\pi_H} I$. Take any of the investors earning the smallest profit at equilibrium (the same logic goes through if one considers a deviation by a passive investor), say the i-th one, and suppose he deviates proposing:

$$\begin{cases} (I_i', R_i') &= (I, R-\varepsilon) & \text{if the aggregate level of investment is } I \\ (I_i', R_i') &= (I, G(2I - I_i + \mathring{\text{I}} + A) & \text{if the aggregate level of investment is} \neq I, \end{cases}$$

where ϵ has been taken to be small enough. The deviation guarantees that investor i earns (almost) the aggregate equilibrium payoff. The entrepreneur has a clear incentive to accept the offer, since she can achieve a payoff strictly greater than the equilibrium one, and to accept only investor i's offer: any alternative choice would indeed induce her to reject the offer of investor i (because strategic default is ruled out). Also, the entrepreneur clearly selects $e = H$ at the deviation stage, which guarantees that the deviation is profitable in the first place.

If aggregate profits are zero at equilibrium, i.e. if $R = \frac{1}{\pi_H}I$, it is always possible for any of the investors, say the i-th one, to profitably deviate proposing:

$$\begin{cases} (I_i', R_i') &= (I+\epsilon, R+G\epsilon^2) & \text{if the aggregate level of investment is } I \\ (I_i', R_i') &= (I+\epsilon, G(2I+\epsilon - I_i + \mathring{\text{I}} + A)) & \text{if the aggregate level of investment is} \neq I, \end{cases}$$

where ϵ has been taken to be small enough. The deviation guarantees that investor i earns a strictly positive profit if $e = H$ is chosen. The entrepreneur has a clear incentive to accept the offer, since she can achieve a payoff strictly greater than the equilibrium one. In addition, the optimal level of investment she will be trading at the deviation stage is exactly I_i': any alternative choice would indeed induce her to reject the offer of investor i. It follows that $e = H$ is selected at the deviation stage, which guarantees that the deviation is profitable in the first place. Assertion (i) is proven. Assertion (ii) directly follows from the proof of Proposition 1.

■

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