Insurance under moral hazard and adverse selection: the case of pure competition*

A. Chassagnon† P.A. Chiappori‡

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Abstract

We consider a model of pure competition between insurers a la Rothschild-Stiglitz, where two types of agents privately choose an effort level, and where the effort costs and the resulting accident probabilities differ across agents. We characterize the set of possible separating equilibria, with a special emphasis on the case where the Spence-Mirrlees condition is not satisfied. We show, in particular, that several equilibria a la Rothschild-Stiglitz may coexist; that they are Pareto-ranked, only the best of them being an equilibrium in the sense of Hahn (1978); and that equilibria may take original forms (for instance, both revelation constraints may then be binding). Finally, we discuss the existence of an equilibrium in this context, and show that, though equilibria may fail to exist, conditions for existence may differ from those in the initial Rothschild-Stiglitz setting.

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† DELTA, 48 Bd Jourdan, 75014 Paris

‡ CNRS and DELTA, 48 Bd Jourdan, 75014 Paris - Corresponding author.

1. Introduction

1.1. Moral hazard and adverse selection in insurance

Following the seminal work by Arrow (1963), the notion of information asymmetry have by now been recognized as a cornerstone of modern insurance theory. Two focal cases have so far attracted particular attention from insurance economists. The concept of adverse selection refers to situations where, before the contract is signed, one party (in general the insured agent) has an information advantage upon the other. In most models, it is assumed that clients know better their own risk than insurance companies; the latter may then use deductible as a way of separating individuals with different riskiness. Moral hazard, on the other hand, occurs when the outcome of the relationship (here, the occurrence of an accident or a claim) depends, in a stochastic way, on a decision that is privately made by one party and not observable by the other. Typically, the insured party may choose to make an effort that is costly to her, but reduces her risk. In this context, full insurance generally leads to suboptimal outcomes, because it provides no incentive to reduce accident probabilities.

The effects of asymmetric information upon competition between insurers have been investigated in a number of papers, following the seminal contributions by Akerlof (1970), Rothschild and Stiglitz (1976) and Wilson (1977). Under adverse selection, equilibria a la Rothschild-Stiglitz may fail to exist; moreover, when they do, they may not be Pareto efficient, even among the subset of contracts that are compatible with the existing information asymmetry (second best efficiency). The properties of competitive equilibria under moral hazard, on the other hand, strongly depend on whether contracts are exclusive (i.e., the insurer may prohibit the acquisition of another contract by his clients) or not. With exclusivity, equilibria do exist in general, and are second best efficient, at least in a one commodity setting (see for instance Prescott and Townsend (1984)).

Surprisingly enough, however, these two polar cases are almost always taken as mutually exclusive. Models of insurance under moral hazard systematically suppose (implicitly in general) that all heterogeneity across agents is either public information, or unobservable by the agents as well. Conversely, in the adverse selection setting, it is assumed that accident probabilities are fixed, exogenously given, and cannot be affected by any incentive (such as the form of the insurance contract)[1]. Such limitations are in general justified by considerations of simplicity

[1]Only a few models in contract theory introduce moral hazard and adverse selection within

or analytical convenience. This, of course, does not imply they can be seen as 're-alistic' in any sense. Natural-born 'bad' drivers have more accidents; but, at the same time, accident probabilities do depend on the incentives provided by insurance contracts, as documented by various studies[2]. A plant may be more likely to suffer from fire because of insufficient prevention, and also because some specificities of its technology entail increased risk - a feature on which the entrepreneur's information is of much better quality than that of the insurance company. Workers usually have a better knowledge of their unemployment risk than (private or public) unemployment insurance schemes; but, in addition, the level of benefits they receive will typically influence job search, hence the expected duration of unemployment, in a typically non contractible way. In fact, one could argue that cases of pure moral hazard or pure adverse selection constitute the exception, rather than the rule. Most 'real life' situations entail at least some ingredient of each type of asymmetry[3].

The goal of this paper is precisely to analyze a simple model of competition a la Rothschild-Stiglitz under adverse selection and moral hazard. The framework we use, as described in the next section, is as elementary (and as basic) as possible. There are two states of nature (with or without an accident), two types of agents and two possible levels of effort. But, at the same time, our approach is general in several senses. First, moral hazard and adverse selection are modelled, in the most general way, as independent phenomena, each of which would still be present even if the other was assumed away[4]. Also, agents are taken to be risk-averse. This assumption is of course quite natural in the insurance context; but, again, several models that have considered moral hazard and adverse selection in the past did

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the same framework. Works by Laffont and Tirole (1986, 1992) or Guesnerie, Picard, Rey (1988) rely upon a risk neutrality assumption, hence can hardly be transposed to the case of insurance contracts. In other models, moral hazard can essentially be eliminated by a punishment scheme 'a la Mirrlees', because the occurence of some extreme performances reveals with probability one the effort choice. But, apart from these somewhat specific settings, the general case does not appear to have attracted much attention (see, however, Chiu and Karni (1994)).

[2]A typical example is provided by Québec, where the switch to "no-fault" policies led to a considerable increase in the number of accidents. See for instance Cummins and Weiss (1992), Gaudry (1992) or Devlin (1992).

[3]The only possible example of pure adverse selection may be life insurance; even there, however, fraud does exist, as illustrated by a considerable number of novels and movies.

[4]This contrasts with several models in the literature that introduce moral hazard and adverse selection in particular frameworks where, although both the agent type θ and the effort level e are unobservable, their sum $\theta + e$ is public information - so that the knowledge of θ would immediatly reveal e, and conversely (see for instance Laffont and Tirole (1994)).

rely on risk neutrality assumptions, a feature that generate particular results[5]. Finally, we do not restrict attention to specific subcases; all possible situations are studied, including the non-standard ones (a claim that will be made more precise below). In particular, we consider in details the new types of equilibria that may appear in this context, and the consequences upon the conditions for existence of an equilibrium.

Restricting oneself to situations of pure moral hazard or pure adverse selection, as is usually done in the litterature, may not be an innocuous strategy. The robustness of the results is not guaranteed - and, as a matter of fact, may be quite dubious in many cases. The conclusions derived independently from each type of model be incorrect in a context where the two phenomena coexist; even the basic intuitions drawn from our knowledge of the standard cases may reveal quite misleading. In fact, the structure of equilibria in our framework turns out to be much richer and much more complex than the separate analysis of adverse selection and moral hazard might suggest. For instance, taking the (first best) perfect information setting as a benchmark, the introduction of adverse selection is known to decrease welfare of all agents but the risky ones; the intuition being that the latter impose a negative 'externality' upon agents with lower risk. When the initial situation entails moral hazard, this intuition does not hold. It may be the case that all agents create an externality - in which case they all loose from the introduction of adverse selection; or, conversely, no externality may be generated, so that no agent is made worse off. As another example, take the conclusion that an equilibrium a la Rothschild and Stiglitz exists if and only if there are 'enough' high risk agents. Again, when adverse selection and moral hazard coexist, this result is no longer true in general. Depending on the parameters, an equilibrium may exist whatever the proportion of agents of different types; or existence may require enough 'bad risks' *and* enough 'good risks' to be present; equilibria may even fail to exist whatever the respective proportions.

1.2. Multiple crossing

Besides its possible realism, the introduction of a moral hazard component within a standard adverse selection framework has another interest : it helps understanding how, and to what extend, some standard assumptions restrict the scope and the consequences of insurance models. A typical example is the 'Spence-Mirrlees' single-crossing condition - a feature that characterizes not only Rothschild and

[5]See for instance Guesnerie, Picard and Rey (1988)

Stiglitz's initial contribution, but, as a matter of fact, most papers dealing with competition under adverse selection. In Rothschild and Stiglitz's model, differences in risk are represented in a very simple way : each agent is characterized by some constant accident probability. As a consequence, whenever both agents face the same contract, and *whatever the latter may be,* it is always the same agent who is more risky. With identical risk aversion (another standard assumption of the litterature), this implies that *indifference curves of different agents can cross only once.* As it is well-known, this single-crossing property plays a key role in the derivation of many results.

In our case, however, although we keep identical preferences[6], the introduction of moral hazard deeply modifies the picture. Here, accident probabilities are no longer exogenous, but depend on the effort level selected by the agents; technically, accident probabilities must thus be expressed as *functions.* This fact has various consequences. One is that the mere definition of 'high risk' agents in this context is less obvious, since it involves a comparison of functions instead of numbers. A natural criterion, however, is the following : agent A is said to be more risky than agent B if, *for any given effort level*, A's accident probability is higher than B's; that is, whenever A and B adopt the same effort, then A is more likely to have an accident than B.

Throughout the paper, we shall use an assumption of this kind; i.e., one agent will be considered as the 'high risk' agent in the sense just defined. Two things must however be stressed at this stage. First, this assumption is by no means needed for our results to hold. It is made only for the sake of convenience; indeed, it considerably simplifies the interpretation of the basic theoretical insights. Secondly, this assumption, restrictive as it may seem, does not alter in fact the qualitative conclusions we obtain. All the diversity that one can get when considering arbitrary risk functions is preserved under this particular assumption. In other words, the increase in pedagogy is not paid by a restriction in the scope of the results.

To understand why this is the case, one point must be emphasized. The assumption establishes a link between each agent's *effort level* and her accident probability. But, of course, effort itself is endogenous, and depends on the contract the agent is facing. Different agents will in general choose different efforts, even when facing the same contract. Now the key remark is that, as we shall see,

[6] Allowing for differences in risk and risk aversion would introduce bi-dimensional adverse selction, hence considerably complexify the analysis. For a careful analysis of a setting of this kind (but without moral hazard), see Villeneuve (1996).

there is no direct link between absolute riskyness (in the sense just defined) and effort choice. It is not the case, for instance, that high risk agents necessarily choose lower effort levels. The intuition is that the effort level induced by a given contract does not depend on the *absolute value* of the accident probability, but rather on its *derivative* - i.e., the magnitude of the drop in accident probability resulting from a given effort increment. It may well be the case that the high-risk agent is easier to incite, because, in his case, a marginal effort of given cost is much more efficient in terms of risk reduction. Assume for a moment this is the case. Then for any given contract - except for those with full coverage - which agent is actually more risky is not clear, because agents with a higher 'natural' risk level are also more eager to reduce risk through prevention. In other words, though one can still, *ex ante*, make a clear-cut separation between 'low risks' and 'high risks' agents, it is not necessary the case that the former always exhibit lower *ex post* accident probability. Riskiness is now endogenous to the contract at stake (which, after all, is the main intuition of the moral hazard literature); and agents of a given type will typically be more risky for some contracts but less risky for others.

The technical consequence is that *the single-crossing property does not hold in general*, because the indifference curve of one agent may be steeper or flatter than that of the other agent, depending on the particular contract at stake. Analyzing a model with adverse selection and moral hazard thus leads in a very natural way to consider an adverse selection setting with multiple crossing - an issue that is investigated in some details in the paper.

Incidentally, it could be argued that multiple crossing is by no means a pathology. There are many other contexts in which single crossing cannot be expected to hold true. Assume, for instance, that agents differ by their riskiness *and* risk aversion; then high risk agents do not necessarily exhibit steeper indifference curves, and multiple crossing may obtain[7]. In fact, one could argue that single crossing is a very specific property, while multiple crossing could be viewed as a general case. Still, surprisingly enough, little attention has been devoted so far to models of competition under adverse selection and multiple crossing.. Again, which conclusions of the standard setting are robust to a relief of this hypothesis is an interesting issue that is considered in this paper. Chassagnon (1996) provides a general investigation of this problem; in the present paper, we concentrate upon the example of moral hazard and adverse selection.

[7] Other reasons include multiple risks, as in Fluet and Pannequin (1996) and Villeneuve (1996); non expected utility (Chiu and Karni 1994); and others.

1.3. The structure of the paper

Our main results can be summarized as follows. First, as in Rothschild and Stiglitz, equilibria can only be separating : different agents receive different equilibrium contracts. In addition, higher deductible still are associated with lower (ex post) riskiness. Hence, what probably constitutes the two main insights of Rothschild and Stiglitz's initial contribution are preserved.

However, the other standard conclusions of adverse selection models under single crossing can be seriously altered. For instance :

- several equilibria a la Rothschild and Stiglitz may coexist. When this is the case, they are always Pareto-ranked. As a consequence, whether firms are allowed to propose only one contract (as in Rothschild and Stiglitz) or a menu of contracts (a possibility that is evoked by Rothschild and Stiglitz and explicitly studied by Hahn (1978)) becomes an important issue. For instance, one can find robust examples where equilibrium allocations à la Rothschild and Stiglitz and à la Hahn coexist but do not coincide, because some equilibria à la Rothschild and Stiglitz fail to be equilibria à la Hahn.

- standard Rothschild and Stiglitz equilibria are characterized by the fact that only one type of agents - the 'bad risks' - face a binding revelation constraint. This needs not be true in our more general context. One may get equilibria where no agent's constraint is binding, or where both types face a binding constraint. In the latter case, in particular, no agent receives the contract he/she would get in the absence of adverse selection; everyone looses from the unobservability of accident probabilities. The paper provides a general characterization of the various types of equilibria that may obtain in this context.

- in the standard framework, the existence of an equilibrium depends on the proportion λ of high risk agents; precisely, there exist some limit value $\bar{\lambda}$ such that an equilibrium exists if and only if λ is larger than $\bar{\lambda}$. Again, this is no longer the case in our general context. We show that, depending on the type of the equilibrium, existence may obtain whatever the proportion of good and bad risks, or only when agents of one type are numerous enough; it may also require 'enough' agents of *each* type, and equilibria may even fail to exist whatever the respective proportions. Again, we provide a precise characterization of the various situations, in relation to the type of equilibria.

The structure of the paper is as follows. The model is described in Section 2. Section 3 presents our basic toolkit. Section 4 gathers our main results concerning the form of separating equilibria. A crucial step in our approach is the definition of two sequences of contracts, that are showed to converge to a separating equilibrium whenever one does exist. Using this tool, we prove that separating equilibria may take three different forms, two of which are non standard (in the sense that they do not appear in a pure adverse selection model with single crossing). Also, we study how the introduction of adverse selection may influence the equilibrium effort levels; we show that effort may be either discouraged or stimulated, depending on the parameters. The existence of an equilibrium is discussed in Section 5. Finally, we extend our results to the case of continuous effort levels in Section 6, while section 7 is devoted to a brief summary of our results, in relation with the conclusions of the standard setting.

2. The model

2.1. The basic framework

We consider an elementary model of insurance, in which agents with identical risk aversion and initial wealth W may, with a given probability, incur a loss of a given amount D. The accident probability is related to an effort that each agent privately chooses in the set $\{0, 1\}$. There are two types of agents, L and H, with respective VNM utilities :

$$U_L(x, e) = u(x) - c_L.e_L, \quad U_H(x, e) = u(x) - c_H.e_H$$

(where u is increasing and strictly concave). In particular, we assume that effort is separable with respect to consumption - a feature that is restrictive, but by now standard in the literature.

The accident probability of agent i $(i = L, H)$ is p_i if $e_i = 1$, and P_i if $e_i = 0$. Note that agents are allowed to differ in both their respective risks and their respective effort cost. The basic intuition of moral hazard in an insurance framework relies upon the existence of some 'risk reduction technology', according to which an agent can, at some cost, influence her accident probability. The additional, adverse selection ingredient is that this technology differs across individuals, and that the differences are not observable by the insurer. This, in turn, has two consequences. One is similar to standard model of adverse selection a la Rothschild-Stiglitz -

namely, different individuals present different levels of risk. Note, however, that risk is now endogenous to the contract (since different contracts will induce different effort levels). A second consequence is that different agents face *different incentive problem*, since both the cost of increased effort and the corresponding benefits are specific. This aspect is original, and will be carefully described in the next subsection.

Finally, risk-neutral insurers propose contracts of the form $x_i = (\alpha_i, \beta_i)$, where $i = L, H$. Here, β denotes the premium, and α the (net) reimbursement; so that the wealth of agent i is $W - D + \alpha_i$ if an accident occurs, and $W - \beta_i$ otherwise. Such a contract can be represented by a point in the (α, β) plane. In what follows, $S \subset \mathbb{R}^2$ denotes the set of possible contracts.

In general, the respective values of p_i, P_i and c_i are independent. In particular, we might have, say, that

$$p_L < p_H \qquad \text{and} \qquad P_L > P_H$$

meaning that L agents are less risky when both agents choose the maximum effort level, but more risky when no agent does. In what follows, we choose to somewhat specialize the model by assuming that one agent, say L, is a "low risk", in the sense that, when both agents choose some *identical* effort level (either 0 or 1), the accident probability of L is always lower. Formally, we thus assume the following :

Assumption 1 :

$$p_L < p_H \quad ; \quad P_L < P_H \quad ; \quad P_L \neq p_H \tag{2.1}$$

As discussed in the introduction, the purpose of this restriction is to keep our model as close as possible to the initial Rothschild-Stiglitz setting, where one agent is a 'low risk' (at least as compared to the other). While this assumption simplifies the interpretation of our results, and especially the comparison between our findings and those of the standard, pure adverse selection framework, it must be emphasized that there is no loss of generality entailed by this choice. In particular, *all results below would remain valid in the more general case sketched above.* In fact, restricting oneself to preferences that satisfy (2.1) does not reduce the forms of the possible equilibria, the existence conditions, or any other substantial qualitative conclusions of the model. Or, to put it differently : all deviations from the standard setting that can be observed in the more general case do obtain here. The key remark is that, even under (2.1), p_H may still be lower than P_L; i.e., the "risky" agent may sometimes be actually less risky than the other, provided that he chooses a high effort level while the other does not. As it will become clear later, allowing for this case enable to recover the full richness of the model. However, the case $P_L = p_H$ would lead to very peculiar situations, and is anyway non-generic; so we rule it out in what follows.

2.2. Incentive constraints

We first consider the moral hazard problem facing agent i. A given contract (α_i, β_i) will induce the choice of the high effort level if :

$$(1-p_i)\,u(W-\beta_i)+p_i\,u(W-D+\alpha_i)-c_i \geq (1-P_i)\,u(W-\beta_i)+P_i\,u(W-D+\alpha_i)$$

which writes down as :

$$u(W-\beta_i)-u(W-D+\alpha_i) \geq \frac{c_i}{P_i-p_i} \overset{def}{=} \varphi_i$$

When this condition is fulfilled, the contract is said to be *incentive-compatible.* Let IC_i denote the set of incentive-compatible contracts, and ε_i its frontier. The equation of ε_i is :

$$u(W-\beta_i)-u(W-D+\alpha_i) = \varphi_i$$

The constant φ_i can be interpreted as describing the "technology" that underlies the moral hazard effect. It characterizes the agents' respective "performances" - how much it costs to them, in utility terms, to reduce the accident probability by a given amount. It should be emphasized that the values of φ_H and φ_L - hence the respective locations of ε_L and ε_H - cannot be deduced from the sole hypothesis that H agents are more risky. Lower risk agents L may well turn out to be *more difficult to incite* than higher-risk ones. Indeed, while riskiness is related to the *absolute values* of accident probabilities, incentives depend on the *difference* $P-p$ - i.e., on the shift in probability resulting from a change in the effort level. Risky agents may still be more "productive" in that sense; this is when effort results in a large reduction of the agent's (large) accident probability[8].

In what follows, we assume that without insurance, the agent will always choose $e=1$. Then IC_i is non-empty; its properties are summarized in the following Lemma:

[8]Another reason is that, while agents are allowed to differ in the cost of effort, we make no assumption about respective costs. But this is by no means needed to get the results.

Lemma 2.1. *In the* (α, β) *plane :*

- ε_i *is decreasing, with a slope always smaller than -1*
- ε_L *and* ε_H *do not intersect unless* $\varphi_H = \varphi_L$ *(in which case they coincide); moreover,* $IC_L \subset IC_H$ *if and only if* $\varphi_H < \varphi_L$

Proof : See Appendix ■

As a consequence, one may note that the line of zero-profit contracts for each type of risk will exhibit a *discontinuity* when crossing the corresponding incentive frontier ε_i . This is because the accident probability changes in a discontinuous way when agents change their effort level (note, however, that utilities change continuously, as it will become clear below). As it is by now standard, we suppose that whenever an agent is indifferent between the two effort levels, he will always choose effort 1.

An illustration is given in Figure 1 (for the case $\varphi_H < \varphi_L$.).

Include here Figure 1

2.3. Indifference curves and revelation constraints

For any given pair of contracts $x_L = (\alpha_L, \beta_L)$ and $x_H = (\alpha_H, \beta_H)$, the *revelation constraints* write down :

$$\widetilde{u}_L(x_L) \geq \widetilde{u}_L(x_H) \quad \text{and} \quad \widetilde{u}_H(x_H) \geq \widetilde{u}_H(x_L)$$

where

$$\widetilde{u}_i(x_k) = (1 - \pi_i)\, u\,(W - \beta_k) + \pi_i\ u(W - D + \alpha_k) - c_i e_i$$

is agent i's expected utility when choosing the contract x_k $(i, k = L, H)$, and where π_i denotes the accident probability corresponding to the effort level e_i induced by the contract; i.e., $\pi_i = p_i$ and $e_i = 1$ if $x_k \in IC_i$, $\pi_i = P_i$ and $e_i = 0$ otherwise.

As it is well known, an important issue, in adverse selection models, is whether the Spence-Mirrlees single crossing condition holds true; that is, taking an arbitrary indifference curve for each type, can these cross more than once ?

To answer, let us now consider the indifference curves of agent i in the set of possible contracts. These are continuous curves, but, because of the moral hazard component, they are no longer concave. Specifically, they exhibit a kink when crossing ε_i, because the MRS is then proportional to $\frac{p_i}{1-p_i}$ on one side of the frontier, and to $\frac{P_i}{1-P_i}$ on the other side.

An immediate consequence is that the indifference curves of the two types of agents may cross more than once. Specifically :

Lemma 2.2. *The indifference curves of H and L cross only once if and only if any of the following two conditions is fulfilled :*

- $p_H \geq P_L$
- $\varphi_H \geq \varphi_L$

Proof : Multiple crossing requires that, for some contracts in the (α, β) plane, the MRS of L is greater than that of H. This can only be the case if $p_H \geq P_L$. Moreover, the contract must be such that H makes an effort while L does not, which requires that $IC_L \subset IC_H$. ■

Conversely, assume that $p_H < P_L$ (high-risk agents who make an effort are less risky than low-risk agents who don't), and $IC_L \subset IC_H$ (there exist contracts that induce maximum effort for high-risk agents but not for low-risk agents). Then some indifference curves will cross twice (and may even cross up to three times). This will be referred to in what follows as the "multiple crossing" case, as illustrated in Figure 2.

Include here Figure 2

2.4. Equilibrium

The most usual definition of an equilibrium was introduced by Rothschild and Stiglitz. We may briefly recall it as follows :

Definition 2.3. *A pair of contracts $x_L = (\alpha_L, \beta_L)$ and $x_H = (\alpha_H, \beta_H)$ is an equilibrium a la Rothschild and Stiglitz (a RS equilibrium from now on) if the following two conditions are fulfilled :*

- *no contract in the equilibrium pair makes negative (expected) profits*
- *no new contract can be offered and make positive profits*

The intuition underlying the definition is clear : given the set of existing equilibrium contracts, it must not be possible to some new entrant to offer a contract that makes a positive profit. It should be noted, however, that in this definition a new entrant can only offer *one* contract - not a menu. In terms of game theory, a RS equilibrium can be seen as a Nash equilibrium of a two-stage game, where insurers first propose contracts, then agents choose among the set of available contracts their most preferred one. However, each agent's strategy space consists of contracts, not of menus of contracts. Also, note that we do not impose that $x_L \neq x_H$; i.e., we allow for pooling contracts. It is well known, however, that, in the standard framework, such contracts cannot be equilibria. We shall see below that this intuition is preserved in the ASMH case; i.e., equilibria a la Rothschild and Stiglitz, when they exist, must be separating.

Various extensions of this concept have been proposed in the literature. For instance, Hellwig (1987) introduces a third stage, in which insurers can either accept the clients or leave the market; he then considers the outcome of the game, and shows in particular that stable equilibria à la Kohlberg and Mertens may be pooling. More related to our approach is the concept formalized by Hahn (1978). Equilibria a la Hahn are defined in exactly the same way as RS equilibria, except for the strategy spaces: in Hahn's version, insurers are allowed to offer several contracts simultaneously. Formally:

Definition 2.4. *A pair of contracts $x_L = (\alpha_L, \beta_L)$ and $x_H = (\alpha_H, \beta_H)$ is an equilibrium a la Hahn if the following two conditions are fulfilled :*

- *the equilibrium pair makes non-negative total (expected) profits*
- *no menu of new contracts can be offered and make positive profits*

In the original Rothschild-Stiglitz framework, there is a close link between the set of equilibrium allocations a la Hahn and a la RS. The set of Hahn equilibria is always included within the set of RS equilibria; conversely, any RS equilibrium is an equilibrium a la Hahn if and only if it is efficient. This property is essentially preserved in our context, though in a somewhat different manner. We first have the following result :

Proposition 2.5. *Under the assumptions above :*

- *At any equilibrium a la RS, each contract makes zero profit.*
- *At any equilibrium a la Hahn, each contract makes zero profit.*
- *Any equilibrium a la Hahn is an equilibrium a la RS.*

Proof : see Appendix ■

In particular, though Hahn equilibria do not preclude cross-subsidies across contracts, these will never occur at equilibrium, just like in the standard framework.

2.5. The Pure Moral Hazard (PMH) case

In what follows, we shall concentrate upon the deviations due to adverse selection. These deviations must be defined with respect to some benchmark. The benchmark we shall be interested in is the equilibrium that would obtain in the absence of adverse selection, i.e., if agents' type was publicly observable. Obviously, this does not correspond to the first best allocation, because public observation of agents' type would not eliminate the moral hazard problem. Hence, our reference will be what we call the "Pure Moral Hazard" (PMH) case.

We use the following notations : for $i = H, L,$ let $\bar{x}_i = (\bar{\alpha}_i, \bar{\beta}_i)$ and $\bar{e}_i$ denote the equilibrium PMH policies and effort level, while $x_i^* = (\alpha_i^*, \beta_i^*)$ and e_i^* refer to the (general) case of adverse selection plus moral hazard (from now on ASMH).

From standard moral hazard theory, we know the following :

Lemma 2.6. *Under PMH and competition, there are two different contracts (one for each type of agent). Each contract may take one of the following two forms :*

- *either $\bar{e}_i = 0$, then $\bar{\alpha}_i + \bar{\beta}_i = D$; the policy is located at the intersection of the zero-profit line and the full insurance line (point A (resp. A') in Fig.1).*
- *or $\bar{e}_i = 1$, then the incentive constraint is binding; the policy is located at the intersection of the zero-profit line and the incentive frontier ε_i (point B (resp. B') in Fig.1).*

From the previous Lemma, two cases are possible; in both cases, the insurance company makes zero profit, so that the corresponding contract is located on the zero profit line. It may be the case, on the one hand, that inciting the agent to make an effort is just too costly. Then the equilibrium contract will entail zero effort; as a consequence, the agent will receive full coverage. If, on the other hand, equilibrium requires an effort to be made, the incentive constraint will be exactly binding; the intuition being that increasing the deductible beyond this minimum level would reduce agents' welfare without any gain in terms of incentives. These two contracts, being the two possible candidates for PMH equilibrium, will be called in the remainder "*PMH locally optimal*". We assume that they are not equivalent from the agent's viewpoint, an assumption that is generically fulfilled. Note that we implicitly assume insurance policies are exclusive. This assumption is natural in this context; moreover, it avoids the complexities described in Arnott and Stiglitz (1993) or Bisin and Guaitoli (1993).

3. The tools

Given the simplicity of our setting, a direct resolution, using only the specificities of the framework at stake, would probably be possible. But, of course, the robustness of the conclusions would then be doubtful. Our goal, here, is instead to introduce, within this specific context, some tools that can be used in a very general way. In particular, while the various properties of these constructs are established only for the model at stake, their scope is much more general (see Chassagnon (1996) for a general presentation).

3.1. The basic correspondence

In all what follows, our basic tool will be the correspondence Φ, defined as follows. Take any couple of contracts (x_H, x_L). Starting with x_H, consider the set of contracts y_L that fulfill three properties :

- they make non-negative profits (on L agents)
- they do not attract H agents out of x_H (i.e., they are not preferred to x_H by H agents)
- they are preferred by L agents among all contracts satisfying the two previous conditions.

Also, y_H can be defined from x_L in a similar way. Then Φ is the correspondence that, to any (x_H, x_L), associates the contracts (y_H, y_L) thus defined. Formally :

Definition 3.1. *Φ is the correspondence from $S \times S$ to itself that associates, to any $(x_H, x_L) \in S \times S$, the set of couples (y_H, y_L) such that, for $i = H, L$:*

$$y_i \in \arg \max_{\hat{x}_i = (\hat{\alpha}_i, \hat{\beta}_i)} \widetilde{u}_i \ (\hat{x}_i)$$

$$\begin{aligned} (1 - \pi_i)\,\hat{\beta}_i - \pi_i \ \hat{\alpha}_i \geq 0 \\ \widetilde{u}_j(\hat{x}_i) \leq \widetilde{u}_j(x_j) \end{aligned} \tag{3.1}$$

where, as above,

$$\widetilde{u}_i(x_k) = (1 - \pi_i)\, u\,(W - \beta_k) + \pi_i \ u(W - D + \alpha_k) - c_i e_i$$

is agent i's expected utility when choosing the contract x_k $(i, k = L, H)$, and where π_i denotes the accident probability corresponding to the effort level e_i induced by the contract; i.e., $\pi_i = p_i$ and $e_i = 1$ if $x_k \in IC_i$, $\pi_i = P_i$ and $e_i = 0$ otherwise.

Note that $\Phi(x_H, x_L)$ may consist in several contracts. However, if (y_H, y_L) and (y'_H, y'_L) both belong to $\Phi(x_H, x_L)$, then it must be the case that

$$\widetilde{u}_j(y_j) = \widetilde{u}_j(y'_j)$$

for $j = H, L$. In particular, whenever the single-crossing property is fulfilled, then Φ is in fact a mapping.

3.2. RS equilibria : a necessary condition

What the above definition is aimed at capturing is the idea that competition will provide each agent with the best contract available, subject to two restrictions : non negative profits and the revelation constraint. Its scope will become clear from the following Proposition :

Proposition 3.2. *A pair of contracts (x^*_H, x^*_L) is a RS equilibrium if and only if :*

1. *it is a fixed point of* Φ

2. *it is not strictly Pareto-dominated by a pooling contract that makes non-negative profit (on the whole population)*

Proof : see Appendix ■

In words : a RS equilibrium is a fixed point of Φ such that all contracts preferred by all agents make negative profits. This property would of course be true (and in a sense trivial) in the standard setting. In our general framework, it reveals useful for two reasons

- it proposes a direct characterization of each equilibrium contract that only depends on the utility level reached by the other agent. In particular, this characterization relies upon two independent computations, each of them being only parametrized by one utility level.

- the set of fixed points of Φ can be determined using traditional tools of equilibrium analysis (as will become clear below). In particular, this set does not depend on the respective proportions of high and low risk agents in the population - while, of course, condition 2 may (but need not) depend on that.

Also, note that conditions 1 and 2 characterizes RS equilibria; since equilibria a la Hahn form a subset, the conditions are still necessary for Hahn equilibria. But they may not be sufficient. Indeed, a RS equilibrium might be Pareto dominated by a menu of separating contracts (with or without cross-subsidies), in which case it will not be a Hahn equilibrium, as we shall see below.

It is important to note that Φ *is not a contraction in general.* A consequence is that the uniqueness of the fixed point can by no means be guaranteed. In fact, we shall see later on that, in some cases, several fixed points do coexist. However, in case of multiplicity, we have the following result :

Proposition 3.3. *Assume* Φ *has several fixed points. Then the corresponding contracts are Pareto-ranked. In other words, if* (y_H, y_L) *and* (y'_H, y'_L) *are two fixed points of* Φ *and if*

$$\widetilde{u}_L(y_L) > \widetilde{u}_L(y'_L)$$

then necessarily

$$\widetilde{u}_H(y_H) > \widetilde{u}_H(y'_H)$$

Proof :

Assume

$$\widetilde{u}_L(y_L) > \widetilde{u}_L(y'_L)$$

From the revelation constraints,

$$\widetilde{u}_L(y'_L) \geq \widetilde{u}_L(y'_H)$$

This means that, taking y_L as given, y'_H makes positive profits and satisfies

$$\widetilde{u}_L(y_L) \geq \widetilde{u}_L(y'_H)$$

From the definition of y_H, it follows that

$$\widetilde{u}_H(y_H) \geq \widetilde{u}_H(y'_H)$$

Finally, assume that the previous relationship holds with equality. Then y_L and y'_L are solutions of the same program. This implies that $\widetilde{u}_L(y_L) = \widetilde{u}_L(y'_H)$, a contradiction. ■

This result has an immediate consequence :

Proposition 3.4. *Assume there exists at least one RS equilibrium, say (X^*_H, X^*_L). Assume there exist a fixed point of Φ, say (Y_H, Y_L), that Pareto dominates (X^*_H, X^*_L). Then (Y_H, Y_L) is a RS equilibrium.*

Proof : Just note that (Y_H, Y_L) cannot be Pareto dominated by a pooling contract (since the latter would also Pareto-dominate (X^*_H, X^*_L), a contradiction), and apply Proposition 3.2. ■

3.3. The basic sequences

It is clear, from the results above, that one should pay particular attention to fixed points of the correspondence Φ, since the latter constitute natural candidates for an equilibrium. Since Φ is not a contraction, the most natural way to get such a fixed point is by iterating Φ. This leads us to considering the following two sequences of contracts :

Definition 3.5. *The sequences $S_H = (x^k_H)$ and $S_L = (x^k_L)$ (for $k \in R$) are defined by:*

- $x_H^0 = \bar{x}_H$ *and* $x_L^0 = \bar{x}_L$
- *for* $k \geq 1$, $\left(x_H^k, x_L^k\right) \in \Phi\left(x_H^{k-1}, x_L^{k-1}\right)$

Now, we know that whenever such a sequence does converge, it must be to a fixed point of Φ. But can we expect the sequences to converge at all? The answer is positive, as stated by the following lemma :

Lemma 3.6. *The sequences* (x_H^k) *and* (x_L^k) *always converge to a fixed point of* Φ*. If, in particular, a RS equilibrium exists, then the sequences converge to a RS equilibrium. Moreover, the latter Pareto-dominates all RS equilibria.*

Proof : see Appendix

This result is easy to interpret. Start from the PMH contracts $\bar{x}_H$ and $\bar{x}_L$. In most cases, these cannot constitute an equilibrium, because one revelation constraint (at least) is violated[9]. The idea is then to modify the contracts proposed to both agents, so as to eliminate this violation; specifically, each agent will receive the best contract available among those that make positive profits and satisfy the *previous* revelation constraint. This leaves us with two new contracts, x_H^1 and x_L^1. But, of course, the revelation constraints may still be violated, because the contracts were moved independently. If this is the case, then we just define a new couple of contracts exactly as before, and so on. It remains to check that the sequences do converge. The key point, here, is that the expected utility of each type of agent *decreases along the sequence*. This is because we maximize the same utility functions under increasingly restrictive constraints, as proved by a simple induction argument (if the kth iteration of Φ decreases $\tilde{u}_i$, the revelation constraint of agent j for the $(k+1)$th iteration will be more stringent). Since both sequences are bounded below (say, by the expected utility without insurance), they must converge by a standard Lyapunov argument; and the limit will naturally be a fixed point of Φ. The corresponding contracts are good candidates to constitute an equilibrium - provided, of course, that an equilibrium exists. Finally, since the sequences are starting from the PMH contracts, they will necessarily Pareto-dominate any equilibrium. This means, in particular, that when several equilibria a la Rothschild and Stiglitz coexist, the sequences can only converge to one of them - namely, the Pareto superior one.

[9] Note, however, that this *needs not* be the case, as we shall see below - a conclusion in sharp contrast with pure adverse selection.

In what follow, let u_H^k (resp. u_L^k) denote the utility level reached at each step of the sequences defined below :

$$u_i^k = \tilde{u}\left(x_i^k\right) \quad , i = H, L$$

Obviously, the sequences $U_H = \left(u_H^k\right)$ and $U_L = \left(u_L^k\right)$ converge.

4. RS equilibria : general results

4.1. The general form(s) of RS equilibria

With the help of the previous results, we may first characterize the form of RS equilibria. This is done in the following Theorem :

Theorem 4.1. *RS equilibria, when they exist, must be separating. Moreover, they are generically of either of the three following forms :*

- **Type 1** *("no adverse selection") : each agent gets his PMH optimal contract (as characterized in Lemma 2.5); no revelation constraint is binding.*
- **Type 2** *("weak adverse selection") :*
  - *one agent (at least) receives a PMH locally optimal contract*
  - *one revelation constraint (at most) is binding*
- **Type 3** *("strong adverse selection") :*
  - *no agent receives a PMH locally optimal contract*
  - *both revelation constraints are binding*

Though type 2 equilibria are reminiscent of the pure adverse selection case, some innovations with respect to the standard framework should be emphasized. For instance, no agent may get his PMH contract (while bad risk always get their first-best contract in the standard setting). Also, it can be the case that no revelation constraint is binding. More important is the fact that the agent with a PMH contract and a binding revelation constraint can be *any* of the two - not necessarily the high-risk one. In fact, a further classification of the type 2 case is the following :

Proposition 4.2. *Type 2 equilibria generically belong to one of the following subtypes :*

- **Type 2a :** *H receives a PMH locally optimal contract, L does not; the revelation constraint of H is binding.*
- **Type 2b :** *L receives a PMH locally optimal contract, H does not; the revelation constraint of L is binding.*
- **Type 2c :** *both agents receive a PMH locally optimal contract; no revelation constraint is binding.*

A complete proof is given in the Appendix. Note that non-generical pathologies are disregarded. The three possible cases are illustrated in figure 3.

Include here Figure 3

We may briefly comment these equilibrium forms, some of which drastically differ from the traditional Rothschild-Stiglitz conclusions. In type 1 equilibria, adverse selection does not change the PMH situation - which means that the corresponding contracts do in fact fulfill the revelation constraints. The intuition is straightforward. In a pure adverse selection framework, revelation is obtained through the introduction of deductible. Since first-best contracts are characterized by full insurance, this implies a welfare loss for the lower risks. In our case, however, the benchmark (i.e., PMH contracts) is a second-best outcome. It may already entail partial coverage, because of the incentive constraints due to the moral hazard component. It may be the case that the corresponding deductibles, in addition to their incentive properties, do screen the agents in an adequate way. In this case, no agent looses from the fact that his true nature is not observable.

Let us now consider equilibria ot type 2. Type 2a equilibria are closest to standard Rothschild-Stiglitz equilibria. Note, however, that the contract received by H is one of the two PMH locally optimal, but may fail to be *the* PMH one; it may be the case that *even* H looses from the introduction of adverse selection, because of a switch from his PMH contract to the alternative local optimum. Such a switch must be due to a change in the effort level; that is, the presence of adverse selection may discourage effort, the agent switching from $\bar{e}_H = 1$ at the PMH equilibrium to $e_H^* = 0$ - a point that we consider later. The lower-risk agent, on the other hand, does suffer from adverse selection, essentially because he is further away from full insurance than in the PMH situation. Type 2b equilibria are of the same kind, but with a permutation of types. The idea is that, in the case of multiple crossing, there exist areas in the plane where H agents are in fact better risks than L agents; the initial intuition a la Rothschild-Stiglitz may then apply up to a switch of types. Again, the contract received by L is PMH locally optimal, but not necessarily the PMH equilibrium. Finally, equilibria 2c are even more specific. Here, both agents receive a PMH local optimum, but for one of them (at least) it is not the PMH optimum (which is the difference with equlibria of Type 1); however, no revelation constraint is binding. In other words, there is a cost associated to the presence of adverse selection, but this cost only comes from a switch between locally optimum PMH contracts (i.e., between effort levels). The intuition can be seen on the following example. Take a standard Rothschild-Stiglitz situation, and assume that the PMH contract of L entails no effort. Such a contract is out of reach under adverse selection, because the revelation constraint of H would always be violated. Assume that, under adverse selection, L takes the maximum effort. But then it may be the case that, just like in Type 1 equilibria, the deductible needed for incentive purposes is sufficient to achieve full revelation - in which case no revelation constraint is binding. Interestingly enough, the converse situation (with H replacing L) is also possible in that case.

In all Type 2 equilibria, however, adverse selection is said to be weak because one agent gets either his PMH level of expected utility, or at least a PMH locally optimal contract. The final situation is even more interesting. Here, adverse selection always makes *both* agents worse off, even with respect to PMH local optima. To grasp the intuition, note two points. First, this situation can only occur in the case of multiple crossing. Second, the equilibrium contracts, x_H^* and x_L^* , have a particular property in that case : they are located on the same indifference curve for *both* H and L. How is this possible ? The idea is that x_L^* is located in an area where both agents make the same effort (which may be 0 or 1). By assumption, L is a better risk in that case. The revelation constraint of H is binding like in the pure adverse selection case; note, in particular, that attracting H agents to the L contract would make the latter unprofitable. But, at the same time, x_H^* is located in an area where *only* H is incited. Remember that, in this case, H must be a better risk (otherwise, multiple crossing would not obtain). This means that attracting L agents to the H contract would loose money; and the revelation constraint of L is then binding.

Also, it should be emphasized that the situation depicted in Figure 3e is by no means pathological, and cannot be ruled out by a genericity argument. In the case of multiple crossing, there will *always* exist a pair of contracts that make zero profits and are located on the same indifference curves for both agents (though this pair may not be an equilibrium). To see why, start from point K on figure 3e, and consider the two indifference curves going through K. Since H is less risky, his curve is flatter, and intersects H's zero-profit line above L's curve. But in the neighborhood of the no-insurance point O, the converse is true : L is less risky, and her curve intersects H's zero-profit line above H's curve. If one continuously moves the initial point between O and K, there exist a point such that both intersections coincide. As a consequence, in the multiple crossing case, the pattern described in Figure 3e will typically exist, and the corresponding contracts constitute a fixed point of the mapping Φ - although it may not be an equilibrium.

4.2. Coexistence of several RS equilibria

A consequence of the previous analysis is that under multiple crossing, several RS equilibria may coexist. An illustration is given in Figure 4. Here, both equilibria are of type 3. The first, Pareto-inferior equilibrium, (y_H, y_L), is such that y_L is located in an area where both agents choose effort 1, while at y_H only H makes an effort. For the second equilibrium, (x_H^*, x_L^*), at x_H^* only H makes an effort while at x_L^* no one does. Now, note that, in the neighborhood of (y_H, y_L), any new contract preferred by H agents will attract L agents as well, hence make losses. Also, though x_L^* is obviously preferred to y_L, unilateral introduction of x_L^* in a market where only (y_H, y_L) exist will attract all agents, hence make negative profits; and the same argument applies, mutatis mutandis, to x_H^*. This explains why (y_H, y_L) may be a RS equilibrium. Of course, (y_H, y_L) cannot be an equilibrium a la Hahn, because the introduction of the *pair* (x_H^*, x_L^*) would attract all consumers. The tricky part is to construct an example where (y_H, y_L) is indeed a RS equilibrium - i.e., is not dominated by some pooling contract. This is left to the reader.

Include here Figure 4

4.3. When do the various types obtain ?

4.3.1. A characterization using the sequences

We shall now characterize the situations in which each type of contract may occur. A first characterization relies upon the sequences constructed above; for the sake of simplicity, they are expressed in utility terms.

Proposition 4.3. *Consider the sequences $U_H = \left(u_H^k\right)$ and $U_L = \left(u_L^k\right)$ constructed in the previous section. Then :*

- *either the sequences are constant from the beginning. Then equilibrium is of type 1*
- *or the sequences converge in a finite number of step. Then equilibrium is of type 2.*
- *or both sequences converge in an infinite number of steps; equilibrium is then of type 3.*

Proof : See Appendix

4.3.2. A characterization using the values of the parameters

A natural question, at this point, is whether the existence of some types of equilibria is restricted to certain configurations of the initial parameters. This turns out to be the case. A first, very general result is the following :

Theorem 4.4. *Under single crossing, H agents receive their PMH contracts.*

The proof is immediate. Under single-crossing, any contract that makes nonnegative profits for H agents will also make nonnegative profits for L agents. Under adverse selection, H agents will always be proposed their PMH contract, because it attracts H agents away from any other contract making nonnegative profits, and that it cannot make negative profits even if it attracts L agents as well. This intuition is a direct generalization of Rothschild and Stiglitz's initial argument. It must however be emphasized that it does not hold with multiple crossing, essentially because, now, *H's PMH contract might loose money if L agents were attracted.*

An immediate application to the type of equilibrium that may obtain is the following :

Proposition 4.5. *Assume there is single crossing. Then equilibria must be of type 1, 2a or 2c. Specifically, if $IC_H \subset IC_L$, the equilibrium must be of type 2a; if $IC_L \subset IC_H$ and $P_L \leq p_H$, the equilibrium may be either of type 1, 2a or 2c.*

In particular, equilibria of type 1 are not linked to the presence of multiple crossing, but rather to the fact that the reference situation (PMH contracts) already is second-best (instead of first-best) one. However, equilibria of type 2b or 3 are specific to an adverse selection model where the Spence-Mirrlees condition does not hold.

4.4. The influence of adverse selection upon the choice of effort

We finally consider the way in which adverse selection may influence the second best effort level. Assume, for instance, that under PMH one agent chooses $\bar{e} = 1$ at the equilibrium. The introduction of adverse selection might, in this context, alter the incentive properties of the equilibrium contract, and eventually result in zero effort. Conversely, we may wonder whether, as a consequence of hidden information, more incentive could obtain. Answers to these questions are given in the following result.

Proposition 4.6. *Assume that the PMH contracts entail zero effort for high-risk agents ($\bar{e}_H = 0$). Then the same is true under ASMH (i.e., $e^*_H = 0$). Conversely, it may be the case that $\bar{e}_H = 1$ and $e^*_H = 0$.*

Also, the PMH effort level for low-risk agents may be changed at the ASMH equilibrium.

> *Proof :* If $\bar{e}_H = 0$, then the high-risk agent's utility under PMH is maximum for zero effort. Obviously, under ASMH, H's utility will not decrease, because an insurer can always propose the PMH contract and make zero profit. This implies that $e^*_H = 0$.
>
> Counter examples for the three other cases are given in Figure 5 below. In Figure 5a, $\bar{e}_H = 1$ and $e^*_H = 0$. In 5b, $\bar{e}_L = 1$ and $e^*_L = 0$. Finally, in 5c, $\bar{e}_L = 0$ and $e^*_L = 1$.

Include here Figure 5

So adverse selection may either weaken or strengthen the incentive properties of equilibrium contract, at least for lower risk agents. Though this conclusion is not unexpected (it sounds like a classical second best result), it may have surprising consequences. Assume, for instance, that the agents' choices of effort have external effects that are not taken into account by the competitive equilibrium. It may be the case that, under PMH, competition leads to $\bar{e}_L = 0$, while $e_L = 1$ would lead to socially superior outcomes. Since the introduction of adverse selection may change incentives in such a way that $e_L^* = 1$, we may end up with a situation where the introduction of adverse selection turns out to be welfare increasing[10].

5. Existence of an equilibrium

Finally, we may consider the question of existence of an equilibrium. The result, here, is quite different from the standard case. The answer may, as in Rothschild-Stiglitz, depend on the proportions of agents of each type. But, in addition, it also depends on the structure of the model, and more precisely of the type of the candidate equilibrium (as defined by Theorem 3.3). Specifically, consider the sequences $S_H = (x_H^k)$ and $S_L = (x_L^k)$ defined in section 3, and let x_H^∞ and x_L^∞ denote their (respective) limits. If a separating equilibrium does exist, then the pair $X^\infty = (x_H^\infty, x_L^\infty)$ is a separating equilibrium. Now, existence is related to the structure of X^∞ as follows :

Theorem 5.1. *Let $\lambda \in [0, 1]$ denote the proportion of H agents in the population. Then :*

- *Assume that, at X^∞, both agents receive their PMH contract. Then X^∞ is always a (type 1) equilibrium.*
- *Assume that, at X^∞, agents H only receive their PMH contract. Then there exists a value $\underline{\lambda} > 0$ such that X^∞ is an equilibrium if and only if $\lambda \geq \underline{\lambda}$.*
- *Assume that, at X^∞, agents L only receive their PMH contract. Then there exists a value $\bar{\lambda} < 1$ such that X^∞ is an equilibrium if and only if $\lambda \leq \bar{\lambda}$.*

[10] It is well known that less information can lead to socially better outcomes, when ignorance remains symmetric. The innovation, here, is that *asymmetric* information is needed to achieve the pareo improvement !

- *Assume that, at X^∞, neither agents H nor agents L receive their PMH contract. Then there exists two values $\underline{\lambda}$ and $\bar{\lambda}$ such that X^∞ is an equilibrium if and only if $\underline{\lambda} \leq \lambda \leq \bar{\lambda}$. If, in particular, $\underline{\lambda} > \bar{\lambda}$, there is no RS equilibrium whatever λ.*

  Proof : See Appendix.

The interpretation goes as follows. Take, first, a type 1 equilibrium where both agents get their PMH contracts. Obviously, no pooling contract can be preferred by L agents, so that an equilibrium always exists. The next two cases are standard, except possibly for a permutation of types. Finally, consider a situation where no agent gets his PMH contract; this is the case in Type 3 equilibria, but also in some Type 2 cases. Assume the proportion of agents of type X is 'very small'. Then a pooling contract will be close to X's PMH contract, hence preferred by X agents. But, in addition, if X agents do not get their PMH contract at equilibrium, it must be because this would violate the revelation constraint of the agents of the other type - say, type Y. Hence Y agents prefer X's PMH contract to their own equilibrium contract; by continuity, they will also prefer a pooling contract located close enough to X's PMH contract. This means that both agents prefer the pooling contract; it follows that no equilibrium can exist.

It can be noted that this conclusion is in sharp contrast with the standard setting. For instance, equilibria may exist whatever the proportions of agents of various types. Conversely, they may fail to exist, whatever these proportions may be. The intuition that equilibria are jeopardized when good risks are too numerous is not robust to the introduction of moral hazard - and, as a matter of fact, of violations of the Spence-Mirrlees property.

6. Extension : the case of a continuous effort

Though most of the results above are general, some are linked with the particular setting at stake, and especially with the assumption that effort can only take two values. In this section, we investigate a first generalization by assuming that effort is continuous. The basic conclusions - in particular the properties of the sequences and the characterization of the various types of equilibria - are preserved. However, some new features appear. We show, in particular, that pooling equilibria may exist; however, they are not robust to small perturbations of the parameters.

6.1. The framework : moral hazard with continuous effort

The previous model is extended by the assumption that the effort level e is continuous, and belongs to $[0,+\infty)$. Utility of an agent of type i becomes :

$$U_i(x,e) = u(x) - c_i\,(e)$$

where c_i is twice continuously differentiable, $c_i'(0)=0$, $c_i'(e)>0$ for $e>0$, and $c_i''(e)>0$. In words : the marginal disutility of effort is positive and increasing..

In the same way, the accident probability is of the form $P_i(e)$, where P_i is twice continuously differentiable, $P_i'(e)<0$ and $P_i''(e)>0$: effort decreases accident probability, but with decreasing returns. As in the previous model, we assume that H agents are bad risks, in the sense that $P_L(e)<P_H(e)$ for all e.

A first remark is that, in this setting, the first-order approach can be used, as stated in the following lemma :

Lemma 6.1. *Assume agent i is faced with some insurance contract $x_i=(\alpha_i,\beta_i)$ that does not entail over insurance. The effort level he will choose is of the form :*

$$e_i = \delta\ \left[u(W-\beta_i) - u(W-D+\alpha_i)\right]$$

where δ is continuously differentiable, $\delta(0)=0$, and $\delta'>0$ over R_+. In particular:

$$\frac{\partial\, e_i}{\partial\, \alpha_i} = -\delta'\,.\,u'\,(W-D+\alpha_i)<0$$
$$\frac{\partial\, e_i}{\partial\, \beta_i} = -\delta'\,.\,u'\,(W-\beta_i)<0$$

Proof : consider the program :

$$\max_e\ H(e) = \ [1-P_i(e)]\,u(W-\beta_i) + P_i(e)\,u(W-D+\alpha_i) - c_i(e)$$

Note that H is concave for any contract that does not entail over insurance, so first order conditions are necessary and sufficient to characterize a local optimum. These are given by :

$$g(e) = -\frac{c'(e)}{P'(e)} = u(W-\beta_i) - u(W-D+\alpha_i) = \Delta\,u$$

Here, g is continuously differentiable and increasing, with $g(0) = 0$; and δ is defined as g^{-1}. We only have to check that a corner solution $e = 0$ cannot obtain for $\Delta u > 0$. But then H is strictly increasing at $e = 0$, which terminates the proof.

An important consequences is the following. Define E_e^i as the set of contracts for which agent i chooses the effort level e. :

$$E_e^i = \{x = (\alpha, \beta) \ / \ \delta \ [u(W - \beta) - u(W - D + \alpha)] = e\}$$

Then we have the following result :

Lemma 6.2. *In the (α, β) plane :*

- *E_e^i is a differentiable, decreasing curve with a slope $s(x) = -\frac{u'(W-D+\alpha)}{u'(W-\beta)} < -1$ (where $x = (\alpha, \beta)$)*
- *There exist some $\bar{e} > 0$ such that, for any $e \leq \bar{e}$, the sets of E_e^i curves for $i = L$ and $i = H$ coincide :*

$$\begin{aligned} \forall e &\leq \bar{e}, \exists e' \ s.t. \ E_e^H = E_{e'}^L \\ \forall e' &\leq \bar{e}, \exists e \ s.t. \ E_e^H = E_{e'}^L \end{aligned}$$

This result is the counterpart, in the continuous setting, of Lemma 2.1 in section 2. The incentive frontier is now replaced by a foliation of the set of contracts by iso-effort curves, with similar forms. From Lemma 6.1, the equation of such a curve is of the form $\Delta u = K$, where K is a constant; in particular, the *set* of iso-effort curves does not depend on the agent's type (though, of course, the particular effort level associated with each curve does). Also, it can be seen that, as before, 'low risk' agents L may well turn out to be *more difficult to incite* than high risk ones. Indeed, while riskiness is related to the *absolute values* of accident probabilities, incentives depend on the *derivatives* P_i' - i.e., on the shift in probability resulting from a change in the effort level.

In what follows, we let $\pi_i(x)$ denote the accident probability of a type i agent facing the contract $x = (\alpha, \beta)$ - taking into account the effort level induced by the contract. Formally :

$$\pi_i(x) = P_i \ \{\delta \ [u(W - \beta) - u(W - D + \alpha)]\}$$

Similarly, let $\gamma_i(x)$ denote the effort cost of a type i agent facing the contract $x = (\alpha, \beta)$:

$$\gamma_i(x) = c_i \left\{\delta \ \left[u(W-\beta) - \ u(W-D+\alpha)\right]\right\}$$

Finally, let Z_i denote the zero-profit curve of agent i, defined as the set of contracts providing zero profit for agent i :

$$Z_i = \left\{x = (\alpha, \beta) \ / \ \left[1 - \pi_i(x)\right] . \beta - \pi_i(x) \, \alpha = 0\right\}$$

In particular, for any $x \in Z_i$, we have that

$$\frac{\beta}{\alpha} = \frac{\pi_i \ (x)}{1 - \pi_i \ (x)}$$

- in words, that the straight line Ox (where O is the origin) has a slope equal to $\frac{\pi_i \ (x)}{1-\pi_i \ (x)}$. Now, the slope of the tangent to Z_i at x can also be characterized :

Lemma 6.3. *The zero-profit curves Z_i of agent i is differentiable almost everywhere. Moreover, at any point $x = (\alpha, \beta) \neq (0,0)$, the slope $\zeta_i(x)$ of Z_i is such that :*

- *either* $\zeta_i(x) > \frac{\pi_i \ (x)}{1-\pi_i \ (x)} > 0$
- *or* $\zeta_i(x) < -\frac{u' \, (W-D+\alpha)}{u' \, (W-\beta)} < -1$

Proof. From

$$\beta = \frac{\pi(\alpha, \beta)}{1 - \pi(\alpha, \beta)} \alpha$$

it follows that

$$\zeta_i(x) = \frac{\pi(\alpha, \beta)}{1 - \pi(\alpha, \beta)} - \frac{\alpha}{(1-\pi)^2} P_i' \delta' \left[u' \left(W - D + \alpha\right) + u' \left(W - \beta\right) \zeta_i(x)\right] \quad (6.1)$$

or

$$\zeta_i(x) \left[1 + \frac{\alpha}{(1-\pi)^2} P_i' \delta' u' \left(W - \beta\right)\right] = \frac{\pi}{1-\pi} - \frac{\alpha}{(1-\pi)^2} P_i' \delta' u' \left(W - D + \alpha\right)$$

Each term on the rhs is positive, whereas the sign of the lhs is ambiguous. If the term between brackets is positive, then $\zeta_i(x) > 0$, and the first property follows immediatly from (6.1). If not, then :

$$\begin{aligned}\zeta_i(x) &= \frac{\pi(1-\pi) - \alpha P_i' \delta' u'(W - D + \alpha)}{(1-\pi)^2 + \alpha P_i' \delta' u'(W-\beta)} \\ &= -\frac{u'(W - D + \alpha) - \frac{\pi(1-\pi)}{\alpha P_i' \delta'}}{u'(W-\beta) + \frac{(1-\pi)^2}{\alpha P_i' \delta'}}\end{aligned}$$

and the second property follows from the fact that $P_i' < 0$. ■

Note, in particular, that the zero-profit curve can be *downward slopping*. The intuition is that, starting from any point x, increasing β may in fact decrease the profit, because the agent will respond by a reduction of his effort, resulting in higher accident probability. When this is the case, a decrease in α will be needed to compensate this effect. However, the slope, when it is negative, is always steeper than that of iso-effort curves. Conversely, when the slope is positive, it is always steeper than that of the Ox line. An illustration is provided by Figure 6.

Include here Figure 6

6.2. Indifference curves and revelation constraints

We now turn to indifference curves S_i. These are defined by the following equation :

$$[1 - \pi_i(x)]\, u(W-\beta) + \pi_i(x)\, u(W - D + \alpha) - \gamma_i(x) = K$$

where $x = (\alpha, \beta)$, and where K is an arbitrary constant. These curves can be described as follows:

Lemma 6.4. *The indifference curves S_i are increasing, and their slope $\sigma_i(x)$ at any point x satisfies*:

$$\sigma_i(x) = \frac{\pi_i\ (x)}{1-\pi_i\ (x)} \frac{u'(W - D + \alpha)}{u'(W-\beta)} \geq \frac{\pi_i\ (x)}{1-\pi_i\ (x)} > 0$$

This property is exactly preserved from the discrete case; that the introduction of a continuous effort does not change the result is in fact an immediate consequence of the envelope theorem. It should be noted, however, that (as in the discrete case) these curves are not necessarily concave, as already noted by Arnott and Stiglitz (1993). To see why, take any contract x, and move slightly along the indifference curve going through x, in the direction of increased insurance (i.e., towards north-east). Two effects are at stake. One is risk aversion; as in the standard model, this will tend to decrease the slope of the indifference curve. But, at the same time, getting nearer to full insurance implies a reduction in the effort level, hence an increased accident probability π_i - which tends to increase the slope. The final result depends on the respective magnitude of these two effects.

As before, the case of multiple crossing deserves special attention. A simple and strong characterization is given by the following :

Lemma 6.5. *The following four statements are equivalent :*

- *any two indifference curves of H and L cross only once*
- *an indifference curve of H can never be tangent to an indifference curve of L*
- *$\pi_H\ (x) > \pi_L\ (x)$ for all x.*
- *the zero-profit curves Z_H and Z_L do not intersect.*

  Proof : Assume that two indifference curves S_H and S_L cross more than once. Then there must be contracts x such that $\pi_H\ (x) < \pi_L\ (x)$. Take the iso-effort curve going through x, and let y be its intersection with Z_H. At y, the profit for agent L must be negative, so that Z_L lies above Z_H. But on the full insurance line, Z_H lies above; since Z_H and Z_L are continuous, they must cross in-between.

  Conversely, let x be a point where Z_H and Z_L intersect. Then $\pi_H\ (x) = \pi_L\ (x)$. This implies that, at any point on the iso-effort curve going through x, the corresponding indifference curves S_H and S_L are tangent. One can then choose an indifference curve S'_H 'close enough' to S_H, such that S'_H and S_L intersect twice.

An interesting outcome of the proof is that whenever the zero-profit curves intersect, then at any point on the iso-effort curve going through the intersection, the indifference curves of the two types of agents are tangent. Conversely, if the indifference curves of the two types of agents are tangent at some point x, they are also tangent at any point located on the same iso-effort curve; moreover, zero-profit curves must also intersect on this iso-effort.

6.3. The case of pure moral hazard (PMH)

Assume, first, that types are public information. What will the optimal contracts look like ? A consequence of the assumptions made is that the optimal contract will never entail zero effort.

Lemma 6.6. *Under pure moral hazard, the optimal contract cannot provide full insurance. As a consequence, effort is always positive.*

> *Proof* : Let X_i denote the zero-profit, full insurance contract. Then X_i cannot be optimal unless the respective slopes of the zero-profit curve and of the indifference curve satisfy :
>
> $$\sigma_i(X_i) > \zeta_i(X_i) > 0$$
>
> But since $\sigma_i(X_i) = \frac{\pi_i\ (X_i)}{1-\pi_i\ (X_i)}$, this would contradict Lemma 5.3

Hence, the optimal contract will be such that he indifference curve is tangent to the zero-profit curve. Note, however, that tangency can occur at any point of the zero-profit curve. This remark is particularly interesting if these curves intersect. Also, though tangency is a necessary condition for optimality, it is by no means sufficient. Remember, indeed, that neither zero-profit nor indifference curves exhibit concavity properties of any kind, so that local optima need not be global optima. As before, tangency points will be said to be 'PMH locally optimal'; we know that the (global) equilibrium must belong to the set of PMH locally optimal contracts.

6.4. A first characterization of separating equilibria

We now address the (general) case of moral hazard and adverse selection. A first, rather pleasant result is that the simple characterization given in Proposition 3.2 is still valid:

Proposition 6.7. *Assume a separating equilibrium* (x_H^*, x_L^*) *exists. Then* x_i^* *must be a solution of the program :*

$$\max_{x_i} \ \widetilde{u}_i \ (x_i)$$

$$\begin{aligned} [1 - \pi_i(x_k)] \ \beta_i - \pi_i(x_k) \ \alpha_i &\geq 0 \\ \widetilde{u}_j(x_j^*) &\geq \widetilde{u}_j(x_i) \end{aligned}$$

where

$$\widetilde{u}_i(x_k) = [1 - \pi_i(x_k)] \ u\,(W - \beta_k) + \pi_i(x_k) \ u(W - D + \alpha_k) - \gamma_i(x_k)$$

is agent i*'s expected utility when choosing the contract* x_k

In particular, the zero-profit condition still applies :

Corollary 6.8. *In the case of continuous effort, and under the assumptions above, profit must be zero at the equilibrium*

> *Proof :* Let (x_H^*, x_L^*) denote the separating equilibrium (when it does exist). Then x_i^* is i's preferred contract in the area A of the (α, β) plane lying above i's zero-profit curve and north-west of j's indifference curve (see fig. 6). For a positive profit to obtain, it must be the case that i's best choice in A is on j's indifference curve, away from i's zero-profit curve. This is possible only if, at x_i^* , i's and j's indifference curves are tangent. Now, take the iso-effort curve at x_i^*, and let X be its intersection with i's zero-profit curve. From Lemma 6.5, i's and j's zero-profit curves intersect at X; hence, *both* agents make positive profits at x_i^* . But then, in the neighborhood of x_i^* , there must exist either a contract for i or a contract for j that makes positive profits, satisfy the revelation constraint and is preferred by the agent (the shaded area in Fig. 7), a contradiction with Proposition 5.7.

Include here Figure 7

This zero-profit property may seem rather natural, within a framework where exclusivity is assumed (in contrast with, for instance, Arnott and Stiglitz (1993), who find that profits may be positive at equilibrium because of non exclusivity). In fact, it is somewhat specific, for the following reason. From Proposition 3.2, we know that L agents will receive the best contract available, among those that provide non-negative profit to the insurance company and fulfill the revelation constraint. The essence of the zero-profit result with discrete effort is that this maximization problem will have a corner solution; i.e., both the non negative profit and revelation constraints are exactly binding. Why is this the case ? Why can't the solution be located on the revelation frontier, but away from the intersection with the zero profit line ? Well, remember the revelation frontier is in fact an indifference curve for H agents. So an interior location of the maximum would require a tangency between some indifference curve of L agents and some indifference curve of H - a feature that is impossible in both the initial RS setting and in the discrete effort framework. Now, with a continuous effort, we know that such a tangency is no longer impossible; so an interior solution is more difficult to rule out. It turns out to be excluded, for reasons linked in particular with the linearity of expected utility with respect to probabilities and with the assumption of identical preferences. Not surprisingly, if one of these assumptions is modified, positive profits become possible in a RS equilibrium (see for instance Villeneuve (1996)).

Finally, the impossibility of pooling equilibria still obtains, but only generically :

Proposition 6.9. *Pooling equilibria cannot exist, but may be for a zero-measure set of particular values λ_P of λ.*

Proof. In four steps :

- Assume that $\bar{x}$ is a pooling equilibria, then the indifference curves of H and L must be tangent at $\bar{x}$ (otherwise, the standard RS argument would apply). Note that this is possible in this setting
- Assume that $\bar{x}$ is a pooling equilibria, then it must make zero-profit (otherwise, other pooling contracts would make non negative profits and attract all agents). Hence it must be located on the 'pooling zero-profit curve' (i.e;, the set of contracts that make zero profit when attracting all agents).

- Assume that $\bar{x}$ is a pooling equilibria, then the indifference curves of H and L must be tangent to the pooling zero-profit curve at $\bar{x}$ (otherwise, other pooling contracts would make non negative profits and attract all agents).
- In this setting, we may have a tangency point of indifference curves of H and L located on the pooling zero-profit curve (in general, the locus of tangency points may intersect the pooling zero-profit curve). But, generically, the indifference curves will not be tangent to this curve (a fact that can be established using transversality arguments).

■

In particular, it is possible to construct pooling RS equilibria, but such examples cannot be robust. An illustration is provided by Figure 8.

Include here Figure 8

6.5. Separating equilibria : general form

First, consider the sequences $S_H = (x_H^k)$ and $S_L = (x_L^k)$ defined in subsection 3.1. Note, first, that the definition given does not require specific assumptions upon the nature of effort; it is still fully relevant in our context. In addition, the main property still holds true :

Proposition 6.10. *Assume a separating equilibrium exists. Consider the sequences $S_H = (x_H^k)$ and $S_L = (x_L^k)$ defined in subsection 3.1. These sequences converge to the separating equilibrium.*

Then the form of separating equilibria can be characterized in exactly the same way as before, as stated in the following results :

Theorem 6.11. *Separating equilibria, if they exist, can be of either of the three following forms :*

- **Type 1** *("no adverse selection") : each agent gets his PMH contract , no revelation constraint is binding.*

- **Type 2** *(”weak adverse selection”) :*
  - *one agent (at least) receives a PMH locally optimal contract*
  - *one revelation constraint (at most) is binding*
- **Type 3** *(”strong adverse selection”) :*
  - *no agent receives a PMH locally optimal contract*
  - *both revelation constraints are binding*

Proposition 6.12. *Type 2 equilibria generically belong to one of the following subtypes :*

- **Type 2a :** *H receives a PMH locally optimal contract, L does not; the revelation constraint of H is binding.*
- **Type 2b :** *L receives a PMH locally optimal contract, H does not; the revelation constraint of L is binding.*
- **Type 2c :** *both agents receive a PMH locally optimal contract; no revelation constraint is binding.*

A complete proof is given in the Appendix. Note that non-generical pathologies are disregarded.

7. Conclusion

In this paper, we introduce of moral hazard within the standard adverse selection model of pure competition, and check which results of the initial framework are preserved. A summary is given in Table 1

Insert here Table 1

As it turns out, some of the initial insights of the Rothschild-Stiglitz model are preserved. For instance, agents will be offered a menu of contracts, i.e., of premium-deductible schemes, a higher premium being always associated to better coverage. Profits are zero; also, it is still true that, whatever the equilibrium, agents with lower deductible are more likely to have an accident - a fact that is important in view of empirical application, since it provides a testable prediction of the model (see Chiappori and Salanié (1997) for an empirical test along these lines).

However, many of the initial results no longer hold in our context. In the Rothschild-Stiglitz model, only risky agents do not suffer from adverse selection. Here, it may be the case that both agents loose - or, conversely, that all agents are exactly as well off as if characteristics were fully observable. Another conclusion of the standard approach is that an equilibrium exists if and only if there are 'enough' bad risks. Again, this is not robust. Equilibria may exist whatever the proportions of various types. They may also fail to exist if there are two many bad risks, good risks, or both.

These results are by no means specific to the case of moral hazard plus adverse selection. In fact, two main ingredients drive our results. One is that, in our framework, the Spence-Mirrlees condition may not hold (indifference curves of different agents may cross more than once); the other, that the benchmark situation (the 'PMH' case in the paper) does not necessarily entail full coverage for the bad risks. One may think of various insurance models where this may be the case. Our conjecture is that, in most of these frameworks, equilibria of the types described in the paper will also be present.

Also, it should be stressed that some of the initial conclusions that appear to be robust here may not hold in different contexts. Take the fact that profit are zero at the equilibrium. Whenever indifference curves may be tangent, this property may be jeopardized, because interior solutions may appear. Though it is not the case in our setting, it is fairly clear that positive profits might appear in other context. The case where agent differ not only by their risk but also by their risk aversion provides a typical example.

Finally, a natural extension of our model is to consider more than two different types of agents. This is a very difficult task, if only because without single crossing, no monotonicity condition can be expected to hold, so that revelation constraints may have to be tested for all possible pairs of agents. This is the topic of ongoing research.

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- equilibria must be separating : **True (generically)**
- higher deductibles associated with lower premia : **True**
- higher deductibles associated with lower (ex post) risk : **True**
- one type of equilibrium : **Not in general**
  - 'high risks' do not loose from adverse selection; **Not in general**
  - 'low risks' do loose from averse selection : **Not in general**
  - only 'high risks' RC binding : **Not in general**
- equilibrium (if any) always unique : **Not in general**
- no equilibrium if 'too many' good risks : **Not in general**
- equilibrium always exist if 'enough' bad risks : **Not in general**
- profits are zero if competitive equilibrium : **True here, not robust**

Table 1 : are the main original conclusions still true in our setting ?

For any contract (α, β) on the frontier ε_i, if the following inequality is true

$$(1 - p_i)\,\beta - p_i\;\alpha > c$$

for some real number c, then there exists a neighborhood of the contract on which the same inequality remains true. Since $P_i > p_i$, it follows that :

$$(1 - P_i)\,\beta - P_i\;\alpha > c$$

which is sufficient to insure that the profit function is upper-hemicontinuous, even at the discontinuity contracts.

$$$

It follows that, although the insurer profit function is discontinuous on the frontier ε_i, it remains upper-hemicontinuous - a fact that will play a role in the properties below.

Whenever several RS equilibria coexist, they must be Pareto-ranked. This result clearly draws attention toward one specific RS equilibrium, namely the highest one (in the Pareto ranking)[11]. Actually, this equilibrium has an interesting property :

Corollary 7.1. *Let $Y = (Y_H^*, Y_L^*)$ be a RS equilibrium. Assume Y is (second-best) Pareto efficient.Then Y is an equilibrium a la Hahn. Conversely, assume that Y is strictly dominated by some RS equilibrium. Then Y is not an equilibrium a la Hahn.*

Proof : See Appendix ■

This result suggests a natural interpretation of equilibria a la Hahn. The concept is especially relevant when several RS equilibria coexist; a situation that is ruled out in the standard model, but may well appear in our context, and more generally in any adverse selection setting in which the Spence-Mirrlees single-crossing condition is not fulfilled (see Chassagnon (1996) for a detailed investigation). Then these equilibria must be Pareto-ranked, and Hahn's concept essentially selects the Pareto efficient equilibrium (if any).

[11] In principle, there could be several such 'superior' equilibria; but then one must have that, for any two of them - say, (X_H^*, X_L^*) and (Y_H^*, Y_L^*) - $\widetilde{u}_H(X_H^*) = \widetilde{u}_H(Y_H^*)$ and $\widetilde{u}_L(X_L^*) = \widetilde{u}_L(Y_L^*)$. This case can be showed to be non generic.