Tournaments

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There are two kinds of jobs:

A Good Job (Supervisor's position)	1
A Bad Job (Worker's position)	2

With wages of $W_1 > W_2$.

There are two individuals, *j* and *k*.

The firm wants workers to expend effort, μ, as this increases profit. This part of output is under the worker's control, and will depend on their investment in human capital and their effort.

However, it is not possible to write a contract conditional on μ as the firm does not observe it directly, but only receives a noisy signal of effort, *q*. The noise or luck component of output is denoted ε:

$$q_i = \mu_i + \varepsilon_i$$

The winner of the tournament, the one who is promoted to the good job, is the worker with the highest value of *q*. Worker *j* will win the tournament if the observed value of their output is higher than that of worker *k*:

$q_j > q_k$, which implies $\quad \mu_j + \varepsilon_j > \mu_k + \varepsilon_k$.

The random luck term is normally distributed: $\varepsilon \sim N(0, \sigma^2)$.

Increasing effort thus increases the chance of winning the tournament (but can never guarantee success, due to the random elements, ε_j and ε_k). However, effort is costly for the individual, with a cost of effort function (in terms of dollars) of $C(\mu)$, which is the same for both workers. Cost is convex: C', $C'' > 0$.

Solve the model by first considering how workers determine effort as a function of the two wages, and then consider how the firm sets wages to maximise profit.

Workers

Workers choose effort to maximise their expected utility; they are risk-neutral. If P(.) is the probability of being promoted then they will choose their effort level μ in order to maximise:

$W_1P + W_2(1-P) - C(\mu)$

Lazear equation (3.2).

i.e. to maximise the expected wage minus the cost of effort. The first-order condition is:

(W_1-W_2)dellaP/dellaμ - $C'(\mu) = 0$.

Marginal benefit equals marginal cost.

Lazear equation (3.3).

[There is also the second-order condition that (W1-W2)della^{2}P/dellaμ^2 – $C''(\mu) < 0$]

The probability that j wins = $P(q_j > q_k) = P(\mu_j + \varepsilon_j > \mu_k + \varepsilon_k) = P(\varepsilon_k - \varepsilon_j < \mu_j - \mu_k) = P^*$.

The term $\varepsilon_k - \varepsilon_j$ is the difference between two random normally-distributed luck terms, and is itself thus normally distributed [in fact, $\varepsilon_k - \varepsilon_j \sim N(0, 2\sigma^2)$]. Denote by G(.) the cumulative distribution function of $\varepsilon_k - \varepsilon_j$, with an associated probability distribution function of g(.).

We therefore have the probability that j wins is equal to $G(\mu_j - \mu_k)$.

The PDF, g(.) is normal: bell-shaped and symmetric around 0.

The CDF, is thus sine-shaped with a horizontal asymptote at one. Read P* off from the vertical axis as corresponding to $\mu_j - \mu_k$ on the horizontal axis.

How does the probability of j winning change with j's effort (i.e., what is dellaP_j/dellaμ_j in equation (3.3) above)?

dellaP/dellaμ = della $G(\mu_j - \mu_k)$/dellaμ_j = $g(\mu_j - \mu_k)$.

Note that *j* and *k* are ex ante identical, so they will make the same choices:

$\mu_j = \mu_k$.

Individual *j*'s reaction function therefore becomes:

$(W_1-W_2)g(0) = C'(\mu)$.

Lazear equation (3.4).

This is illustrated in Figure 3.1 The marginal cost of effort curve is positive convex. Read μ^* off on the horizontal axis from $(W_1-W_2)g(0)$ on the vertical axis.

As the wage gap between good and bad jobs grows, worker effort rises (to μ'). Perhaps less obviously, as g(0) falls the lower is the level of effort. As g(0) falls, then the distribution of the relative error term $\varepsilon_k - \varepsilon_j$ is more spread out, so that luck becomes more important in determining the winner. Thus the return to effort is lower. If the tails become fatter, so that g(0) falls to g-tilda(0), then effort falls to μ-tilda. In the limit, if luck equally distributed from plus to minus infinity, then g(0) is plus epsilon, and effort is correspondingly almost zero.

Firm

The firm wants to maximise expected profit, or expected profit per worker (as the number of workers, here two, is exogenous to the problem). The expected value of output, q, is just μ, so the firm maximises

$\mu - (W_1+W_2)/2$

Subject to

$(W_1+W_2)/2=C(\mu)$.

The expected wage must be enough to compensate for the cost of effort, otherwise workers won't apply to the job.

Substitute the latter into the former, then the firm wants to maximise

μ - C(μ).

With the first-order conditions

(1-C′(μ))dellaμ/dellaW_i = 0.

Which implies that the solution requires C′(μ)=1. The RHS is the social reward of effort (price normalised to one, so that this is the value of output). The LHS is the social cost of effort. Thus tournaments are efficient.

Substitute C′(μ)=1 into (3.4) above, then the optimal wage spread is:

$(W_1-W_2) = 1/g(0)$.

Lazear equation (3.9).

As luck becomes more important (fatter tails), then g(0) falls (which would lead to lower effort, ceteris paribus). The firm responds by raising the wage spread until (3.9) holds again.

An increase in risk/luck leads to a higher wage spread. Not to compensate for the risk as such, but to ensure that incitations remain at their optimal level.

Notes

1. W_1 has nothing directly to do with μ. (wages do not equal VMPL at a point in time).
2. Don't consider W_1 in isolation, but rather the whole wage profile.
3. Risk is key.
4. Stylised model. Will tournaments, by explicitly introducing competition between workers, also lead to a breakdown of co-operation?
5. In terms of wage differences, no reason why the optimal wage spread should be the same everywhere in the economy (role of risk different between sectors: example of call-centre workers, whose productivity is directly measured, also truckers, whose every move is monitored).