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ANALYSES OF EVOLUTION OF THE SCHOOL SYSTEM STRUCTURE

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ANALYSES OF EVOLUTION OF THE SCHOOL SYSTEM STRUCTURE

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Analyses of evolution of the school system structure

Abstracts

The aim of this paper is to analyse a change of the French education system using the Markovian model and the method imbued with the adaptive control.

The retrospective analysis of the flux of pupils in education and their departure into active life is carried out first. We examined also the possibilities of adapting the education system to the needs of the productive system. This last step tests the applicability of the algorithm proposed to optimization of the socio-economic model based on the theory of Markov chains.

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Problematic

We will study the System of initial education called the "Educational System". It will be considered in terms of flux at different levels.

The educational system is divided into six academic levels : primary, 1st period cycle secondary, short second cycle period, long second period cycle, 1st undergraduate period cycle of tertiary education, 2nd and 3rd periods cycles (postgraduate) of tertiary education and technical colleges. The entries are known through demographic hypotheses. They are therefore considered as exogenous variables.

The transition from one educational level to another are marked by a transition matrix as in the works of R. STONE (1971). This matrix also determines the output flux towards professional life. Given a particular educational structure, all the transitions are not possible ; thus only some coefficients are not zero. The value of the coefficients which are not zero can be estimated with a varying degree of precision according to the coefficients and starting with available statistics.

Following this, we will try to render dynamic the educational model, that is, to give the transition matrix a changeing value per period of a few years or per year. The available statistics to evaluate repetition, transition, withdrawal and output are of varying reliability and often lacking. Thus the transition coefficients cannot be directly estimated. To the contrary, they can be evaluated from observable factors : (the flux of pupils).

The simulation model is reduced to two equations :

(1) $X_t = p \cdot X_{t-1} + E_t$

where X_t - the activity vector of the school system apart from the pre-elementary level (6 levels : primary, 1st period secondary, short second period, long second period, 1st undergraduate period, 2nd and 3rd postgraduate periods)

p - matrix of the rate of transit from one level to another

E_t - Vector of entries of the school apparatus which includes as a first component the children of 6 year old age.

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(2) $Y_t = s\ X_t$

Y_t - vector of graduates per formation level (6 following levels : VI, V A, V, IV, III and II + I) [1]

s - matrix of the output rate of the school apparatus at the end of school attendance per formation level.

As the matrices p and s are complementary, the sum of the addition of their coefficients of the line is equal to one. In fact, all children entering the school system must one day leave it (the mortality phenomenon, very weak at this age, has been neglected).

(3) $$\sum_{j=1}^{6} p_{ij} + \sum_{j=1}^{6} s_{ij} = 1 \qquad i = 1,\ldots,6$$

The overall matrix shows the properties of the transitional probability matrix of the markovian kind :

(4) $$p_{ij} \geq 0 \qquad s_{ij} \geq 0$$

The transitional matrices p_{ij} and s_{ij} are not constant ; they are therefore anually estimated by a method mentioned above.
Usualy the adaptive process takes place in the following way. At first the model is in an initial state t_0 (the value of its parameters are known). The forecast is made with this model (in which the economic agent choses its behaviour). The next step t_1 is to compare the calculated results and the observed values of the series at t_1 (where the result of the choice of the agent is analysed, that is : whether it is compatible with the actualised state of the environment). The error of the forecast acts in retrospect on the input of the system and is used in conformity with the logic of the model so as to allow the transit to a state which corresponds best to the dynamics of the observed series (or, more generally, to the environment). The adjustments are carried out by repetition until they are sufficient to minimize error.

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(1) See nomenclature in Appendix.

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In accordance with this principle let us assume from the outset that for each new period, the structural coefficients of the school system are unknown (rate of repetition, transit, withdrawal and output). They will then be taken as equal to those of the preceding period. According to the observed fluxes and those of the coefficients for the period calculated with this hypothesis, it is possible to adjust their values. Thus an iterative proceedings gives a new matrix of the coefficients for each period.

The aim of this repetitive method is to minimize the divergence of the observed fluxes and those calculated with the model.

(5) $$J_t\ (\phi,\ \alpha) = |F_t^r - F_t| < \varepsilon$$

with F_t^r observation of the fluxes : X_t^r, Y_t^r

F_t fluxes calculated with the model : X_t, Y_t

$J_t\ (\phi,\ \alpha)$ function minimizing the ϕ factors and the α structural coefficients of the model.

The algorithm used is derived from the works of TOVSTOUKHA (1974). The linear dynamic model with the equations (1) and (2) for the elements of the vector F_t can be summed up :

(6) $$F_{it} = \sum_{j=1}^{j=N} \alpha_{ij}^{\ t}\ \phi_{ij}^{\ t} + u_{it}$$

where α_{ij} matrix of coefficients : p_{ij} and s_{ij}

ϕ_{ij} matrix of factors : demographic vectors E_t, effectives X_t, graduates (diplomas and withdrawals) Y_t

u_{it} vector of errors , variables with normal distribution, mean zero and finite dispersion.

The iterative algorithm of adjustment for the vector line α_i of the matrix is as follows :

(7) $$\alpha_i^{\ t}[n] = \alpha_i^{\ t}[n-1]+A_i[n]\ \nabla_{\alpha_i} Q_i(\phi_i^{\ t}[n],\ \alpha_i^{\ t}[n-1])$$

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where

(8) $$Q_i(\phi_i{}^t[n],\ \alpha_i{}^t[n-1]) = F_{it}{}^r - \sum_{j=1}^{N} \alpha_{ij}{}^t[n-1]\,\phi_{ij}{}^t[n]$$

$i,j = 1,\ldots,N$

$n = 1,2,\ldots$ the repetitions

t period

$\nabla_\alpha Q$ gradient of the function Q.

For the initial values $\alpha_i{}^1[0]$ of the first period, the estimated coefficients for the basic year are taken. For a period $t > 1$, $\alpha_i{}^t[0]$ are taken as initial values — the coefficients $\overline{\alpha}_i{}^{t-1}$ obtained for the preceding period.

As the matrix A possesses the following elements (9) assures the convergence of the algorithm and satisfies the conditions previously described :

(9) $$a_{ij}\,[n] = \frac{1}{\sqrt{n}(\phi_{ij}{}^t[n])}$$

And the algorithm is then as follows

(10) $$\alpha_{ij}{}^t[n] = \alpha_{ij}{}^t[n-1] + c_{ij}\, b_{ij}$$

where :

(11) $$b_{ij} = \left(\frac{(F_{it}{}^r - \sum_{j=1}^{N} \alpha_{ij}{}^t[n-1]\phi_{ij}{}^t[n])}{\sqrt{n}\ (\phi_{ij}{}^t[n])} \right)$$

Given that the matrix α is markovian and the sum of its coefficients equal to 1 in each line, the adjustment of their value during the repetition must keep this restriction. A way of introducing it into the formulas is to give a specific value to the coefficients c_{ij} so that the sum of the adjustment divergence of the coefficients of a line is null. Thus :

(12) $$\begin{cases} c_{ij} = \dfrac{1}{\sum_j b_{ij}} & \text{when the adjustment } b_{ij} \text{ is positive} \\[2ex] c_{ij} = \dfrac{-1}{\sum_j b_{ij}} & \text{when the adjustment } b_{ij} \text{ is negative.} \end{cases}$$

The adjustment algorithm in terms of the school model is to be found in the annexe.

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Some results of the structural analysis of the school system

Retrospective analysis of the period 1972-1978

a) The degree of precision of the estimates

The equations and the adjustment algorithm of the model used are to be found in the Annexe.

We are looking for the values of 22 coefficients of the matrices p and s which the analysis of the school system presumes not zero. Firstly, the coefficients for the whole period were estimated through various statistical surveys of varying fullness and precision (1). Secondly, the adaptation was carried out. The adaptation concerns the 15 coefficients (2) $\alpha_{ij} \neq 0$ (p_{ij} and s_{ij}). They are adjusted at each stage of the operation of the product $c_{ij}b_{ij}$ (equations (11) and (12)) :

b_{ij} is dependant on the difference between the observed values and those resulting from the old values of the parameters (b_{ij}^1 for the effectives and b_{ij}^2 for the output)

c_{ij} is an adjustment factor so that the matrices keep their markovian quality (respectively c_{ij}^1 for the effectives and c_{ij}^2 for the output).

In the estimations, of course, the zero coefficients stay the same. This is obtained by annulling the value of the differences (b_{ij}) calculated for these coefficients. This is also one of the specificities of the adaptation used : the multiplicity of restrictions on the coefficients (zero values, total of constant lines) limits the variations which result from the adjustment at each repetition.

To judge the quality of the method, we shall compare the results of the simulations with the adjustment of the structural matrix of the system (after 5 repetitions within each period (3)) and a simulation carried out on a fixed matrix each time relating to the observed values.

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(1) Estimations carried out for the AGORA model (1978).

(2) The seven others result from a direct solution of the equation system, that is, of the restrictions. This is done to eliminate the existing redundancy in the equations (1)-(4).

(3) Beyond 5 iterations the convergence of the adjustment algorithm slows down.

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Table 1

Total absolute error (average per year) and
average relative error of the simulations

(13) $EA = \frac{1}{m} \sum_{t=1}^{m} \sum_{j=1}^{12} |P^t_j - A^t_j|$ where m = 5 periods (from 1972 to 1977)

(14) $ER = \frac{EA}{\frac{1}{m} \sum_{t=1}^{m} \sum_{j} A^t_j}$

P^t_j - value of effectives and calculated output

A^t_j - value of effectives and observed output

j=1,... 6 - formation levels

j=7,...12 - output levels

		Simulations with	
		the fixed matrix	adjustments of coefficients
EA in thousands of persons	Effectives	84,7	16,4
	Graduates	63,6	29,3
ER as a %	Effectives	0,89	0,17
	Graduates	10,02	4,61
	Totals	1,47	0,45

We notice the advantage of the adaptive method with regard to the diminution of errors of the estimations compared with the simulation with the fixed matrix.

In absolute value the greatest error reaches the effectives in the estimations with the fixed matrix but the graduates in the simulations with adaptation. The importance of these absolute errors, in thousands of persons, differs according to the value of the fluxes on which they bear.

In relative value the main errors in both cases relate to the output estimations for they accumulate, on the one hand, inaccuracies of the structure coefficients (rate of withdrawal and diplomad output) and, on the other hand, the unadjusted errors of the effectives of the preceding period of which they are functions (see equation (2)).

Furthermore, the values of the <u>global errors</u> of simulation are presented <u>year by year</u> in diagram 1.

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For the simulation with adjustment, two periods seem particularly "difficult" to reach the aim of the adjustment which is the maximum reduction of error between the calculated and the observed values. These are the years 1973 and 1976. The estimations for the year 1973 are strongly marked by the coefficients of the basic year, 1972, with all the uncertainty of its coefficients. After 5 iterations for 1973 the absolute error is still great. For the simulation with the stable coefficients it is to the contrary the smallest for the structure of the system has in fact changed the least since the preceding year and the stable coefficients give relatively good forecasts.

The decline in the quality of approximation in 1976 will be discussed in the following paragraph where the situation will be further examined. Depending on the level of studies and the finish diploma levels, the quality of the results differs.

The variations are small in the report on effectives, except for the overestimated 37 thousand in 1976 in the long second cycle , which however represents a mere 3,8 % of the concerned effectives. The other variations are less than 1 % in terms of relative variation. The uncertainty is greater with respect to the fluxes of graduates. Error, dépending on the year and the level, is as much as 14 % of the fluxes (in 1976 for theVbis finish diploma level). This relatively small approximation is because the restrictions interfere with the efficacy of the adjustment formula. This formula calculates the variations for all the coefficients and would tend to change them all but is only allowed to change some of them.

Within each period all the non zero coefficients converge asymptotically to their optimum values when n increases (n = iteration number).

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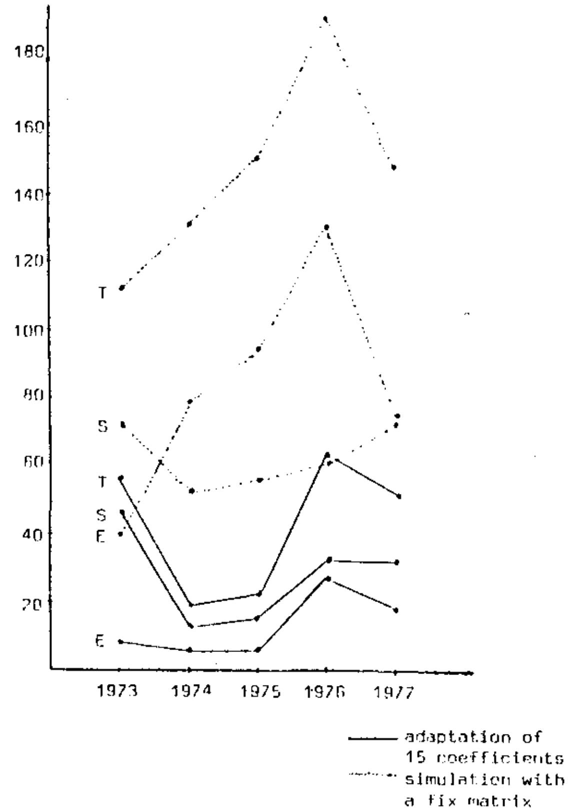
Diagram 1

Absolute error of the simulations
in thousands persons
(effectives, graduates and summes)
(E) (S) (T)



b) Retrospective analysis of the structure of the school system

The degree of stability of the structure over the retrospective period can be estimated from the variance, the standard deviation and the mean of each coefficient of the matrix (Annexe 2 gives the table of results).

The diagonal coefficients

The diagonal coefficients of the transition matrix (p_{ij}, $i = j$) represents the proportion of effectives which stay at the same level during the following period. Their value is proportional to the length of the education cycle, to the input rate and to the rate of repetition and inversely proportional to the withdrawal rate.

Amongst these factors the length of the cycle is the major one. This is why the highest proportion is in the second and third cycles of tertiary education and in primary education where the average duration is equal (primary) or superior to 5 years. In the perfect system, without repetition and withdrawal and a constant input flux, the diagonal coefficients would be equal to the theoretical rate. ./.

Table 2

	Primary	Secondary			Tertiary	
		1st cycle	2nd cycle short	2nd cycle long	1st cycle	2nd and 3rd cycles
	(P_{11})	(P_{22})	(P_{33})	(P_{44})	(P_{55})	(P_{66})
Theoretical rate	0,8	0,75	0,5-0,67	0,75	0,5	0,75-0,9
Average rate for the period	0,811	0,725	0,59	0,65	0,48	0,866

We notice that in the first cycle of secondary education (p_{22}) and the long second cycle (p_{44}) the average rate is greatly inferior to the theoretical rate. This means that the average duration of studies is inferior to the theoretical duration and that withdrawals before completion of scolarity in these cycles are numerous.

In primary education, to the contrary (where withdrawals are absent because of compulsory scolarity for children of elementary school age) the average rate is superior to the theoretical rate due to fairly frequent repetitions.

The variance of coefficients of the diagonal (except p_{11}) is relatively high compared with other coefficients of the matrix. It is particularly high (p_{55}) in the first cycle of tertiary education, as the behaviour of students with respect to the prolonging of their studies varies from one period to another.

The coefficients of transit from one level to another $(p_{ij}, i \neq j)$

The other coefficients of the transit matrix, measuring the proportions of transition to another state, different from the preceding period, reflects the students' possibilities to chose and the capacity of reception of schools. Furthermore, the value of the rates is a function of the average duration of the cycle they have completed and of the proportion of them entering into professional life.

The rate of transit in the first cycle of secondary education has a value complementary to the rate of effectives remaining in the primary level. The sum of values is equal to one.

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The transits in the long cycle after the first short cycle of the secondary level are more frequent than the transits in the short cycle (the average coefficients are respectively equal to 11,1 % and 9,8 % of the effectives).

The variance of the transit coefficients is weak. The exception gives the variance of the transit coefficient between the cycles at the tertiary level which, just as the coefficient of the rate of continuation in the first cycle of the tertiary level, reflects the behaviour of students in liaison with the conjuncture of diplomas in the workfield.

The rate of graduates of the system, with or without diplomas

Apart from the coefficients concerning higher education (withdrawals from the first cycle and output from the second and third) the others do not vary greatly.

On an average, 6,6 % of the effectives of the first cycle of the secondary level leave school each year, 3,8 % without any qualification. The greater part of the graduates of the short second cycle leave with a diploma corresponding to the end of their studies (level V) and eventually they leave with the level Vbis. However, the percentage of pupil living school without qualification is very weak (16,6 %, 7,2 % and 1 % respectively). The diplomad output of the long second cycle is twice that of the withdrawals (respectively 9,1 % and 4,1 % of the cycle's effectives). The diptych of the graduates of the first-tertiary cycle is analagous (diplomad output represents 16,6 % and without diploma ($a_{5,IV}$) 9,3 %).

The dynamics of the structure

The results of the estimation show a stability of the school system's structure for the period.

A slight tendancy to prolong scolarity can be noted in elementary education and the first cycle of secondary school. In the latter case this is due to the proportional decrease of pupils definitively leaving the system but also to the arrival in the cycle of children born between 1964-65. However the rate of withdrawal is high for this cycle.

Beyond the first cycle of secondary school, scolarity is no longer obligatory and pupils are free to chose between continuing their studies or a professional outlet. The distortion of the transition coefficients after the second cycle bring to mind the behavioural changes of youth facing the 1975 crisis.

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The strong downward curve in the rate of students remaining in the short second cycle between 1974 and 1975 must be explained by the increased rate of repetitions. The length of study in this cycle is not well-defined for it groups together two and three year formations. Between 1973 and 1977 the percentage of 2 year formations increased from 33 % to 39 % of the total effectives (subordinate to the Ministry of Education) which corresponds to a decrease in the average institutional duration (without repetition) and which could have caused the acceleration of "rotation". But this did not happen.

The proportion of pupils who leave the school before the end of studies is clearly decreasing, especially at the level Vbis ($a_{3,Vbis}$), which corresponds to the remarks made by J. AFFICHARD (1981, p. 11). The dynamics of the output in level V, with the ruptures in 1974 and 1976 does not show any perceptible tendancy.

The percentage of pupils who stay in the long second cycle (p_{44}) underwent two opposite evolutionary tendancies between 1973 and 1978. There was a flagging tendancy until 1975 (-2,4 % per year) and a recovery (of +2,2 % per year) since then.

This phenomenon leads one to believe that pupils were withdrawing ($a_{4,V}$) or leaving ($s_{4,IV}$) the school system without fear for their professional future. But as from 1975, "threatened by unemployment", they preferred to stay at school and even to repeat. Furthermore, the withdrawal rate and especially the rate of graduates decreases less than the proportion of transit to higher education, which remains very high ; more than 21 % of pupils annually undergo this transit.

In higher education, the coefficients' evolution shows the voluntary prolonging of post-obligatory scolarity. The average duration of studies (expressed by p_{55} and p_{66}) is increasing and the output rate decreasing. This tendancy has not been continuous ; the system of higher education was destabilised between 1975 and 76 (the least sound precision of the simulations for 1976 also shows this). The behaviour of pupils and of youth in general, with their baccalauréat and/or the equivalent diploma + 2 years, has suddenly changed.

A disturbance in the adaptation of the model with its many restrictions results, without giving a simple explanation for the immediate decrease in the rate of students in the first cycle of higher education and the increase of 10 % (also immediate) in the level of transit in the long tertiary cycle.

c) The Problem of finding an optimal structure in the school system

This problem can be formulated in the context of centralized education planning. But it is examined here solely as a way of testing the pertinence of adaptive method in the solving of long term optimisation problems.

The optimal fluxes at precise periods are obtained and the system is adapted so that this goal can be obtained.

For the so-called "optimal" fluxes in this analysis, we took the estimations of the demand for work given to youth entering professional life in the AGORA model (1978). These estimations of the work demand are derived from an "optimistic" scenario aiming both to improve employment and financial balance.

Adaptive method allows the transit from the state corresponding to the year 1978, to another state, the nearest possible to that hoped for in 1984. It is as though one were trying to minimize youth unemployment by restructuring the school system. The "optimistic" scenario forecasts the following work demand of youth :

Table 3

Demand per formation level to enter into professional life (in thousands) in 1984

VI	Vbis	V	IV	III	II+I
121	133	244	108	47	46

Table 4

Flux of graduates observed in 1978 (in thousands)

VI	Vbis	V	IV	III	II+I
131	121	285	109	77	71

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The problem to be solved consists in finding a balanced trajectory corresponding to the objective function : minimization of the variation of the calculated variables of the flux of graduates around the values of the desired fluxes. We are looking for the structural coefficients and the values of the variables (effectives and graduates) corresponding to them that would comply with the restrictions of the model and the objective function (equations 5-12).

The resolution of this kind of optimisation problem gives :

1) the trajectory of the evolution of the model's variables (in this case the fluxes of effectives), and
2) the trajectory of change of the matrix's coefficients.

We impose a progressive change from the trajectory formed during the retrospective period to the optimal trajectory under the hypothesis of transformational regularity of the output flux structure. This change took place during the 6 year period from 1978 to 1984.

Table 5

Absolute difference between the fluxes of graduates sought for and the fluxes calculated by adaptive control

Years Periods	1978 1	1979 2	1980 3	1981 4	1982 5	1983 6	1984 7	In the case of estimations without adjustment in 1984
Levels of graduates								
VI	8	4	3	3	1	2	0	6
Vbis	10	6	2	1	0	12	2	8
V	0	7	5	8	10	31	10	37
IV	16	15	18	10	10	11	9	25
III	0	1	0	0	0	0	0	30
II+I	0	0	0	0	0	0	0	53
Sum of absolute differences	34	33	28	22	21	56	21	159

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The variations presented in Table 5 show the degree of approximation of the estimations on the optimal trajectory. It can be seen that the adjustment algorithm is sufficiently effective so as to react to radical changes imposed on the system.
The sum of the variations gradually decreases although the imposed organization becomes more restrictive (this sum of variations represents 2 to 3 % of the sum of the variables). An exception can be noticed in the 6th period : the disturbance is due to the decrease of school age children in the school system in 1983.

The end column (Table 5) shows the variations in output estimations in the scenario of the extension of the functioning of the school system according to tendancies observed in the past, always with reference to the "desired" fluxes.

How is the structure of the system modified ?
In the context of the demographic input flux without a decrease (up till 1983) and under the restriction of the total decrease in the fluxes of graduates, the coefficients of the diagonal (t_{ii}) increase and those of graduates and withdrawal decrease (diagram 2).

The greatest modifications in the distribution of effectives occurs in higher education where restrictions on the purcentage of students leaving the system are greatest. The change in coefficients in the primary and secondary levels seem smaller. But one must keep in mind that the sensitivity of the variables reaction to the changes of the coefficients is greater for these channels than for higher education because their effectives are far greater.

Table 6

Average annual rate of growth of effectives
and graduates in the optimisation scenario
of the flux of graduates per formation level (as a %)

	Primary	Secondary			Tertiary	
		1st cycle	2nd cycle short	2nd cycle long	1st cycle	2nd and 3rd cycles
Effectives	-0,6	0,38	0,85	0,62	2,8	5,9
	VI	Vbis	V	IV	III	II+I
Graduates	-1,3	+1,6	-2,5	-0,15	-7,9	-7,0

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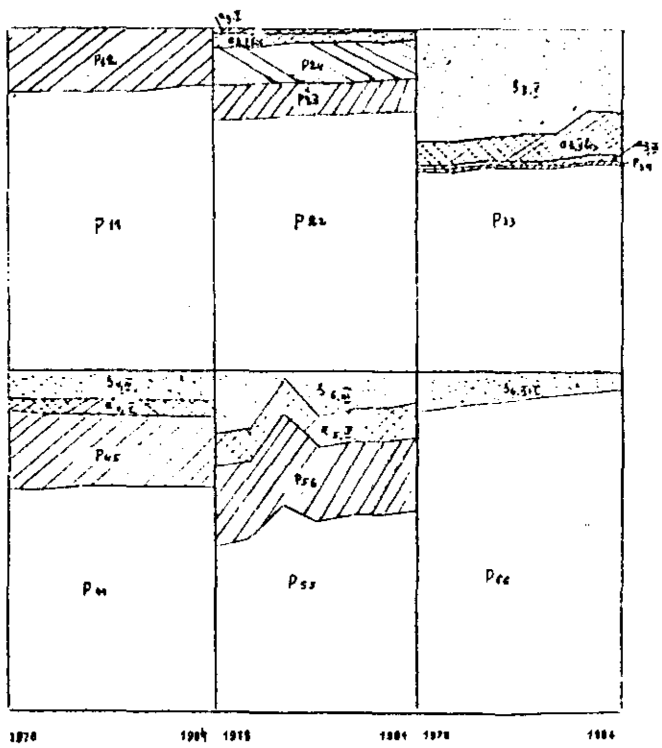
In 1984, to match the desired output structure (Table 3) with the exogenous structure of entry forecasting the progressive transition to the average annual rate of growth (figuring in table 6), the variables (schooled effectives) must follow the trajectory of evolution indicated in Table 6, and the coefficients of the matrix must follow the trajectories described in diagram 2.

<u>Diagram 2</u>

Evolution of the structure of the educational system in the optimational model



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Conclusion

Adaptive method is used to estimate the probabilities of transition in the education system. This method appears to be fairly effective in treating problems at the level of general behaviour. But his possibilities are lessened in the treatment of the education system where, at this level of globality, the possibility of transition is limited to the main levels.

The series of data of the fluxes of graduates are at present very short (6 points). The reconstruction of the system by estimating the proportions of the fluxes between the school levels and the levels of graduate diplomas is however possible, under the given conditions, with the adaptive method.

The dynamic rendering of the model with optimisation is given so as to illustrate the possibility of treating the impact of the work market on the functioning of the school system (impact on the rates of transition).

This analysis allows us to advance a "numbered" reply to the question asked by planners to-day : at what school level and on what scale should one organize continuous education of youth so as to delay their exit into professional life ?

Appendix

Nomenclature of the formation levels

Level VI. - Graduates of the first cycle, second degree (6th, 5th, 4th, 3rd practical) and one year pre-professional formations (CEP : certificate of professional education, CPPN : pre-professional class, CPA : class preparatory to apprenticeship).

Level V bis . - Graduates of the 3rd level and classes of the short second cycle before final year.

Level V. - Graduates in the final year of the short professional cycles and withdrawals from scolarity in the long second cycle before the final year.

Level IV. - Graduates of terminal classes of the long second cycle and post-baccalauréat withdrawals from scolarity before level III.

Level III. - Graduates with a diploma bac level + 2 years, in some cases bac + 3 years (DUT : university technical diploma ; BTS : higher technicians certificate ; primary school teachers ; DEUG : diploma of general university studies ; healthcare schools, etc.).

Level II and I. - Graduates with a university second or third cycle degree, or degree from an institute of technology.

Annexe I

Terminology

$XEAH^{t}[n]$	vector of effectives estimated at the n-th iteration of the algorithm at t	
$XEAH^{t}$	vector of effectives observed at t	
$XEAH_{i}^{t}$	element of vector $XEAH^{t}$	$i = 1, 2, \ldots 6$
$XEAH_{k}^{t}[n]$, $XEAH_{j}^{t}[n]$	element of vector $XEAH^{t}[n]$	$k = 1, 2, \ldots 6$
$TPH^{t}[n]$	matrix of transition estimated at n-th iteration	
$TPH_{ij}^{t}[n]$	element of the matrix $TPH^{t}[n]$	
$TNH^{t}[n]$	matrix of output from the school system estimated at the n-th iteration	
$TNH_{ij}^{t}[n]$	element of the matrix $TNH^{t}[n]$	
$ENTH^{t}$	vector of entries observed	
$ENTH_{i}^{t}$	element of vector $ENTH^{t}$	$i = 1, 2, \ldots 6$
$XEOH^{t}$	vector of output from the school system observed	
$XEOH^{t}[n]$	vector of output estimated at n-th iteration	
$XEOH_{i}^{t}[n]$	element of vector $XEOH^{t}[n]$	$i = 1, 2, \ldots 6$
b_{ij}^{1}, b_{ij}^{2}	elements of correction of the coefficients	
c_{ij}^{1}, c_{ij}^{2}	coefficients of normalisation in order to respect the equality to 1 the sum of the coefficients of the model.	

The model of the n-th iteration

$$XEAH^{t}[n] = TPH^{t}[n-1]\ XEAH^{t}[n-1] + ENTH^{t}$$

$$XEOH^{t}[n] = TNH^{t}[n-1]\ XEAH^{t}[n-1]$$

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Adaptation

$$TPH_{ij}^{t}[n] = TPH_{ij}^{t}[n-1] - b_{ij}^{1t}[n]c_{ij}^{1t}[n]$$

$$b_{ij}^{1t}[n] = \frac{(XEAH_i^{t} - \sum_k TPH_{ik}^{t}[n-1]XEAH_k[n] - ENTH_i)}{\sqrt{n}\ XEAH_j^{t}[n]}$$

$$TNH_{ij}^{t}[n] = TNH_{ij}^{t}[n-1] - b_{ij}^{2t}[n]c_{ij}^{2t}[n]$$

$$b_{ij}^{2t}[n] = \frac{(XEOH_i^{t} - \sum_k TNH_{ik}^{t}[n-1]XEAH_k[n])}{\sqrt{n}\ XEAH_j^{t}[n]}$$

$$b_i^{1t}+[n] = \sum_{j/b_{ij}^{1}>0} b_{ij}^{1t}[n] \qquad b_i^{1t}-[n] = \sum_{j/b_{ij}^{1}<0} b_{ij}^{1t}[n]$$

$$b_i^{2t}+[n] = \sum_{j/b_{ij}^{2}>0} b_{ij}^{2t}[n] \qquad b_i^{2t}-[n] = \sum_{j/b_{ij}^{2}<0} b_{ij}^{2t}[n]$$

$$c_{ij}^{1t}[n] = \frac{1}{b_i^{1t}+[n]+b_i^{2t}+[n]} \qquad \text{if } b_{ij}^{1t}[n]>0$$

$$c_{ij}^{1t}[n] = \frac{-1}{b_i^{1t}-[n]+b_i^{2t}-[n]} \qquad \text{if } b_{ij}^{1t}[n]<0$$

$$c_{ij}^{2t}[n] = \frac{1}{b_i^{1t}+[n]+b_i^{2t}+[n]} \qquad \text{if } b_{ij}^{2t}[n]>0$$

$$c_{ij}^{2t}[n] = \frac{-1}{b_i^{1t}-[n]+b_i^{2t}-[n]} \qquad \text{if } b_{ij}^{2t}[n]<0$$

$$\left.\begin{array}{l} \text{if } \ b_i^{1t}+[n]+b_i^{2t}+[n] < \varepsilon \\ \text{or } \ b_i^{1t}-[n]+b_i^{2t}-[n] < \varepsilon \end{array}\right\} \quad \text{end.}$$

Annexe II

Mean, variance and standard deviation
of coefficients of the matrix of transition

	Coefficients of "rotation" in the system					
	p_{11}	p_{22}	p_{33}	p_{44}	p_{55}	p_{66}
Mean	0.811	0.725	0.59	0.649	0.477	0.866
Variance	0.00004	0.00009	0.00016	0.00021	0.00064	0.00014
Standard deviation	0.00736	0.0104	0.0141	0.016	0.0277	0.01314

	Coefficients of transition to the next level					
	p_{12}	p_{23}	p_{24}	p_{34}	p_{45}	p_{56}
Mean	0.189	0.098	0.111	0.0115	0.219	0.263
Variance	0.000049	0.0000086	0.0000088	0.0000186	0.0000322	0.001114
Standard deviation	0.0077	0.0032	0.00325	0.0047	0.0062	0.036

	Coefficients of withdrawals from the school system ; $a_{i,n}$ n = VI, V, Vbis, IV, III, II+I = graduate levels; i = 1,... 6 = school levels					
	$a_{2,VI}$	$a_{2,Vbis}$	$a_{3,VI}$	$a_{3,Vbis}$	$a_{4,V}$	$a_{5,IV}$
Mean	0.0381	0.0275	0.0097	0.072	0.041	0.093
Variance	0.0000063	0.0000023	0.000005	0.000097	0.0000201	0.00014
Standard deviation	0.00275	0.00164	0.00246	0.0108	0.0049	0.013

	Coefficients of graduates ; $s_{i,n}$			
	$s_{3,V}$	$s_{4,IV}$	$s_{5,III}$	$s_{6,II+I}$
Mean	0.315	0.0915	0.166	0.133
Variance	0.0000397	0.0000889	0.000037	0.00014
Standard deviation	0.0069	0.01033	0.0066	0.0129

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