MANAGEMENT OF INVENTORIES

Christian Courieroux and Irina Peaucelle[1]
CEPREMAP, 142 rue du Chevaleret,
75013 Paris, France
Tel: 33.1.40.77.84.09

Abstract

The model analyzes the consequences of firms' behavior concerning the share of inventories they introduce on the market, for example, deciding to reduce the supply of products in order to increase future prices.

The model is based on dynamic optimizing behaviour of the private agents. It is an equilibrium model in which the inventories do not appear as disequilibrium indicators but as means of intertemporal transfers and pressure.

The comparison of variables' dynamics (by simulating the model) in competitive and monopolistic cases, under the same economic policy, shows that:

- the share of product put on the market is comparatively bigger in competitive case;
- the monopolistic behaviour is more compatible with investment increasing;
- the monopolistic behaviour has an undesirable influence on employment volume.

1 Author for correspondence : peaucelle@pse.ens.fr

MANAGEMENT OF INVENTORIES

Introduction

The transformation of a planned economy into a market one is accompanied by transfers to firms of some government prerogative. At the beginning of the process the firms behave as monopoly and competitive aspect is introduced progressively. We will study this phenomenon and some means of control that the government has to contain the monopolistic behaviour of firms. The kernel of our model concerns the firm behaviours. To influence the prices they can decide to introduce more or less of their inventories on the market. For example they can create artificially under-supply of goods and influence by that practice the future increasing of prices.

Model is composed of two parts: in the first one, firms manage optimally their inventories without market power on prices formation. In the second part the firms consider the consumers' and governments' reactions, and "play" with the volume of inventories. Simulations are presented in the third paragraph and, in particular, it is shown how the government can restrain the monopolistic behaviour by settling some suitable taxes.

The model is an equilibrium one. The inventories are not indicators of disequilibrium; they are mostly a mean for intertemporal redistribution of revenues and a device of pressure.

1. Competitive situation

The manager decides, in an optimal way at each moment, the production volume, the level of inventory and the quantities he introduces on the market, considering prices, taxes and actual and previous capital as exogenous. The dynamic is introduced consequently to these choices concerning the evolutions of the inventories and of the capital.

Production use labour L_t and capital K_{t-1}. The supply of goods

(1) $Q_t^s = f(K_{t-1}, L_t)$

is obtained from a production function, which is assumed increasing and concave in labour at fixed capital. The firm disposes at the end of t-1 period of S_{t-1} inventory of output, and if the rate of depreciation of this inventory is ? at the beginning of period t the inventory is equal to (1-?) S_{t-1}. The firm can then interject on the market the quantity:

(2) $Q_t = ß_t [Q_t^s + (1-?) S_{t-1}]$

where $o \leq ß_t \leq 1$.

This volume of goods is sold, and the level of inventories at the end of the period became:

(3) $S_t = (1-ß_t) [Q_t^s + (1-?) S_{t-1}]$

At each moment the manager can fix the quantity of production Q_t^s by fixing the volume of labour L_t and the proportion of goods $ß_t$ he decides to interject on the market. We can suppose that this choice is realised on the basis of firms' results which include immediately available profits used for investment, and inventories.

The profit is determined from

(4) $p_t^* = p_t Q_t - w_t L_t$

where p_t, w_t - denote the price of good and the wage at t. We introduce a tax on profit; therefore the available profit is equal to:

(5) $p_t = (1-t_t) p_t^*$,

where t_t - is the tax rate. This profit is used for investment I_t. If the price of a unit of capital is equal to one, we have :

(6) $I_t = p_t = K_t - (1-d) K_{t-1}$

where d - is the rate of capital depreciation.

Besides, manager disposes through the inventories S_t, of a dotation, not available immediately, but which could be used for engender profits and through them investments in the future. For simplicity we assume that $?_t$ - is an unity-price of inventory evaluated by the firm at t. Then the evaluation of inventory is equal to $?_t S_t$. The price of inventory $?_t$ must take into account different aspects such as : the depreciation of quantity of inventory, expectation of future prices and the expectation of demand.

At each date the manager is assumed to maximize an objective function depending of the two kinds of results :

(7) $\underset{ß_t, L_t}{\text{Max}} V [p_t, ?_t S_t]$

It is by intermediary of this function V that the sharing between actual profits p_t and future profits is performed (future profit is represented by $?_t S_t$).

If V_1 and V_2 are respectively the derivatives of V with respect to each component, evaluated in optimum, we obtain the first order conditions:

$$(8) \qquad \frac{\partial}{\partial \mathrm{L_t}} = 0 \Leftrightarrow \mathrm{V_1}(1-t_t)\left[\mathrm{p_t}\, b_t \frac{\partial \mathrm{f_t}}{\partial \mathrm{L_t}} - \mathrm{w_t}\right] + \mathrm{V_2}\, l_t (1-b_t)\frac{\partial \mathrm{f_t}}{\partial \mathrm{L_t}} = 0$$

$$\frac{\partial}{\partial b_t} = 0 \Leftrightarrow \mathrm{V_1}(1-t_t)\mathrm{p_t}\,\mathrm{Q_t^s} - \mathrm{V_2}\, l_t \mathrm{Q_t^s} = 0$$

Taking into account the second equation the first one can be written as:

$$(9) \qquad \mathrm{w_t} = \mathrm{p_t}\frac{\partial \mathrm{f_t}}{\partial \mathrm{L_t}}(\mathrm{K_{t-1}}, \mathrm{L_t})$$

The demand for labour is easy to calculate by equating the marginal productivity and the ratio of the wage to the price of output, which is the usual condition. The optimal quantities may be described by:

(10) $L_t = l_t^D [p_t, w_t, t_t, ?_t, K_{t-1}]$,

$ß_t = ß_t^S [p_t, w_t, t_t, ?_t, K_{t-1}]$.

Previous solutions can be written differently in order to perform a dynamic study. For such a purpose we have to specify the evolution of inventory unit price:

(11) $?_t = ?\,(\underline{p}_t, \underline{w}_t, \underline{t}_t)$,

where $\underline{p}_t$ is introduced for time series p_t, p_{t-1}... We obtain the system of equations in L_t, $ß_t$, $?_t$, K_t, S_t, which, when the behaviour of producer is optimal, allows to express the variables (and all other variables linked to them p_t^*, p_t, Q_t^s, Q_t) as functions of current and passed values of prices and tax, only. Then, it is possible to write:

$L_t = l^d\,(\underline{p}_t, \underline{w}_t, \underline{t}_t)$

$Q_t = q\,(\underline{p}_t, \underline{w}_t, \underline{t}_t)$,...

We also introduce a demand for produced output:

(12) $C_t = C_t\,(p_t)$.

For simplicity we take into account only its dependence on price p_t, the dependence on others factors is summarized by index t. The equilibrium condition on goods' market is:

(13) $C_t = Q_t$,

From this condition we derive the value of the equilibrium price.

2. Market power situation

The monopolistic behaviour of the firm is described by analogy with the competitive one.

The producer knows that the quantity of goods he will introduce on the market, in particular managing of the inventories, may modify the price. He may try to use for his best such a market power. For that he has to take into account the consumer reaction, that is summarized under the form of an inverse demand function :

$$(14) \qquad p_t = -\frac{c_t}{b_0} + \frac{b_1}{b_0} = -\frac{b_t}{b_0}[Q_t^s + (1-g)S_{t-1}] + \frac{b_1}{b_0}$$

The producers' objective function will be :

V [p_t, ?$_t$ S_t]

=p_t + v(?$_t$ S_t)

= (1-t_t) [p_t ß$_t$ (Qs,t + (1-?) S_{t-1}) - w_t L_t]

+ v {(c_o + c_1 p_t) (1-ß$_t$) (Q_t^s + (1-?) S_{t-1})}

In this expression the price p_t intervenes at the same time in the components of utility associated with the present and the future.

(*) The first approach would be to consider the reaction of consumer on these two levels. For our model we arrive to a system of equations which may be solved only numerically.

(**) The second approach consists to appreciate the consumer reaction only on the level of current utility. To realise it we retain ?$_{t-1}$ in the place of ?$_t$ as expected unit price of inventory. In this condition the equality between the marginal productivity and the ratio of the wage to price no more holds. We have combined previous conditions and obtained the following expression :

$$\frac{\partial f_t}{\partial L_t} = \frac{w_t}{p_t - \frac{Q_t}{b_0}}$$

which replaces equation (9) introduced for the competitive framework.

Finally the equilibrium model with market power of firm, allowing only current utility, corresponds to the equations (1) (2) (3) (5) (6) plus the equations for prices :

$$p_t = -\frac{c_t}{b_0} + \frac{b_1}{b_0}$$

and $?_t = c_0 + c_1 p_t$

3. Dynamic analysis of non-linearities

i) Specification

In order to realise a simulation of prices and quantities in the two situations corresponding to monopolistic and competitive behaviour of a firm, we have to explicit the production and utility functions of the firm, and to fix the values of the parameters.

Consider a Cobb-Douglas type production function :

(15) $$f_0(k) = k^{a_0}$$

in which we do not introduce the multiplicative constant because it can be easily normalised by an adequate choice of measure units.

The utility function is of a quadratic form :

$$v(y) = a_0 y - a_1 \frac{y^2}{2},$$

If so, we have :

$$\frac{dv(y)}{dy} = a_0 - a_1 y,$$

and its inverse function is :

$$v_0(y) = a_0/a_1 - y/a_1$$

Finally, we have to precise the expectation function. Consider the case, where $t_t = t_\infty$, for $\forall t$ and let us denote p_∞ the corresponding value of stationary equilibrium. Suppose, that the producer think that prices satisfy approximately an autoregression of first order :

$$p_t = p_\infty + \mu (p_{t-1} - p_\infty) + u_t$$

His price expectation is :

$$\hat{p}_{t+h} = p_\infty + \mu^h (p_t - p_\infty)$$

If ? - is a coefficient of preference for the present, the expected price corrected for this effect is :

$$\tilde{p}_{t+h} = ?^h \ \hat{p}_{t+h} = ?^h \ [p_\infty + \mu^h (p_t - p_\infty)]$$

If the producer supposes, that his inventory S_t can be realised in the future markets only in some proportion ?, it is reasonable to consider, taking into account the inventory depreciation, that :

in t+1 $?(1-?)\ S_t$ is introduced on the market,

in t+2 $?(1-?)\ [(1-?)\ (1-?)]\ S_t$ is introduced on the market,

in t+h $?(1-?)^{h-1}\ (1-?)^h\ S_t \equiv \tilde{S}_{t+h}$ is introduced on the market.

So, the expected value of inventory is equal to :

$$\sum_{h=1}^{\infty} \tilde{p}_{t+h} \ \tilde{S}_{t+h}$$

$$= \sum_{h-1}^{\infty} ?\ (1-?)^{h-1}\ (1-?)^h\ S_t\ ?^h [p_\infty + \mu^h (p_t - p_\infty)]$$

$$= c_0 S_t + c_1 p_t S_t,$$

where c_0 and c_1 depend on parameters ?, ?, ?, p_∞.

ii) Competitive equilibrium

According to the chosen forms of the production, utility and expectation functions, the equation defining the equilibrium price is :

$$K_{t-1}^{a_0}\left(\frac{w_t}{a\ p_t K_{t-1}^{a_0}}\right)^{\frac{1}{a-1}} + (1-g)\,S_{t-1} - \frac{a_0}{a_1}\frac{1}{c_0 + c_1 P_t} + \frac{(1-t_t)p_t}{a_1(c_0 + c_1 p_t)^2} = -b_0 p_t + b_1$$

This equation is non-linear in price p_t. In order to avoid solving such an equation at every stage of simulation of the model, we analyze the model as if the producer took his decision on the basis of price, wage and expected price of the previous period. This enables us to have the whole model in a recursive form:

(m.1) $$p_t = -\frac{1}{b_0}\{ K_{t-1}^{a_0}[\frac{w_{t-1}}{a\, p_{t-1} K_{t-1}^{a_0}}]^{\frac{a}{a-1}} + (1-g) S_{t-1}$$

$$- \frac{a_0}{a_1}\frac{1}{c_0 + c_1 p_{t-1}} + \frac{(1-t_{t-1}) p_{t-1}}{a_1 (c_0 + c_1 p_{t-1})^2}\} + \frac{b_1}{b_0}.$$

(m.2) $$L_t = [\frac{w_{t-1}}{a\, p_{t-1} K_{t-1}^{a_0}}]^{\frac{1}{a-1}}$$

(m.3) $$l_t = c_0 + c_1 p_t$$

(m.4) $$Q_t^s = K_{t-1}^{a_0} L_t^a$$

(m.5) $$b_t = [-\frac{a_0}{a_1} + \frac{(1-t_{t-1}) p_{t-1}}{a_1 l_{t-1}}] \frac{1}{l_{t-1}[Q_t^s + (1-l) S_{t-1}]}$$

(m.6) $$S_t = (1-b_t)[Q_t^s + (1-g) S_{t-1}]$$

(m.7) $$I_t = (1-t_t)\left[p_t b_t [Q_t^s + (1-g) S_{t-1}] - w_t L_t\right]$$

(m.8) $$K_t = (1-d) K_{t-1} + I_t$$

This system of equations contains a non-linear dynamic, in particular in the equation giving price, and except in some particular cases, this dynamic can be analyzed only numerically.

Simulated paths

We have performed different simulations of variables evolution to study, in particular, the influence of taxation t in function of the value of parameters: ? - level of inventory depreciation and d - level of capital depreciation, parameters that impel the intertemporal transfers.

Other parameters are fixed at the following values:

$a_0 = 3$, $a_1 = 1$, $b_0 = 10$, $b_1 = 12$, $a = a_0 = 0.5$, $c_0 = 0$, $c_1 = 1$.

The initial values of variables are:

$p_0 = ?_0 = 1.2$, $L_0 = 3$, $Q_0^s = 6$, $ß_0 = 0.5$, $S_0 = 4$, $I_0 = 2$, $K_0 = 10$.

In addition we fix the wage at each moment equal to one ($w_t = 1$, $\forall$ t), so that evolutions are appreciated in wage equivalent; and the level of tax is supposed constant $t_t = t$.

The revealed evolutions depend sensibly on the choice of three other parameters ?,d, t. The increasing of taxes' rate display the decreasing of capital (it is the consequence of the lack in the model of subsidies or governmental help, capable to compensate a tax effect), and at the same time as stabilising the development. We realised different simulations. One of scenario is presented on figure 1, indicating the alternation of periods of adjustment and stabilisation in firms' development. In such studies it is to highlight that some variables are less flexible than others for non-linearity of phenomenon. We may stress that simulations are performed for sufficiently large number of iterations (between 50 and 150), therefore some links or relations reflect long-run characteristics of the model.

iii) Monopolistic equilibrium case

Fixing, as in competitive case, the wage evolution $w_t=1$, $\forall$ t we obtain equilibrium model, composed on following equations:

(m*1) $L_t = \left[a_0 b_0 l_{t-1} K_{t-1}^{a_0} + 0.5 K_{t-1}^{a_0} a_1 b_0 b_1 l_{t-1}^2 - a_1 b_0 K_{t-1}^{a_0} l_{t-1}^2 (1-g) S_{t-1}\right]^2$

$\left[a_1 b_0 l_{t-1}^2 K_{t-1}^{2a_0} + b_0 2(1-t_t) + a_1 b_0^2 l_{t-1}^2\right]^{-2} \cdot ,$

(m*2) $b_t = \left[- b_0 l_{t-1}(a_0 - a_1 l_{t-1}(K_{t-1}^{a_0} L_t^{0.5} + (1-g) S_{t-1})) + b_1(1-t_t)\right]$

$\left[2(1-t_t)(K_{t-1}^{a_0} L_t^{0.5} + (1-g) S_{t-1}) + a_1 b_o l_{t-1}^2 (K_{t-1}^{a_0} L_t^{0.5} + (1-g) S_{t-1}\right]^{-1},$

(m*3) $Q_t = b_t \left[K_{t-1}^{a_0} L_t^{0.5} + (1-g) S_{t-1}\right],$

(m*4) $p_t = -\frac{Q_t}{b_0} + \frac{b_1}{b_0}$

(m*5) $l_t = c_0 + c_1 p_t,$

(m*6) $S_t = (1- b_t)\left[K_{t-1}^{a_0} L_t^{0.5} + (1-g) S_{t-1}\right],$

(m*7) $I_t = (1-t_t)[p_t Q_t - L_t],$

(m*8) $K_t = I_t + (1-d) K_{t-1}$.

This system (m*1) - (m*8) contains also non-linear dynamics. The idea of its proprieties of stability may be reach regarding the price evolution in a particular case when ?=1, $t_t=1$, d=1, a=0.5, $w_t=1$, $c_0=1$ $c_1=1$. We see easily that the prices are such that:

$$p_t = \frac{a_0}{a_1 p_{t-1}(K_0^{2a_0} + b_0)} + \frac{b_1}{b_0(K_0^{2a_0} + b_0)}(0.5\, K_0^{2a_0} + b_0).$$

Once more it is an equation of non-linear recurrence of hyperbolic type with the properties of stability of Cobweb type (as in competitive case).

Fixing the same values of parameters (as in competitive case) $a_0=3$, $a_1=1$, $b_0=10$, $b_1=12$, $a= a_0=0.5$, $c_0=0$, $c_1=1$, we proceed to different simulations for different values of ?, d, t. We can observe that monopolistic behaviour favours in a sense "smoothing" of firms' development, because we do not fine the alternation of periods, as in competitive case.

4. Influence of tax rate on firm behaviour

Even when the evolution curves appear analogous in competitive and monopolistic cases, we ought to see how long-run values of different variables depend on levels of ?, d,t. To illustrate the effect of rate tax, we retain the values ? = 0.1, d = 0.1, because the two systems (m) and (m^*) are asymptotically stable for a large diapason of tax rate values. In such conditions we may fulfil the comparative static and see how the equilibrium values of variables depend on tax rate and compere the estimated dependencies in competitive and monopolistic cases. On the graph (figure 2) solid line corresponds to competitive model dynamic and broken line corresponds to monopolistic model. One can see that in general the part of goods purvey on the market ß is, for the given rate of tax, bigger in the first case than in second, or saying differently in order to obtain in each situation the same fraction of goods interjected on the market, ß, one must increase tax if firm behaves as monopoly. Analogous remarks may be done for other variables. Monopolistic behaviour yields, what is classical, to increase prices and decrease output. We see that it has a positive effect on investment (in our model equivalent to profit) and negative on employment. Those influences are more or less important accordingly to level of tax rate and in our example certain reversals appear since the rate attains the values of 50-60%.

Figure 1: Competitive case

Scenario: Alternation of periods of adjustment and stabilisation

? = 0.2, d = 0.065, t = 0.2

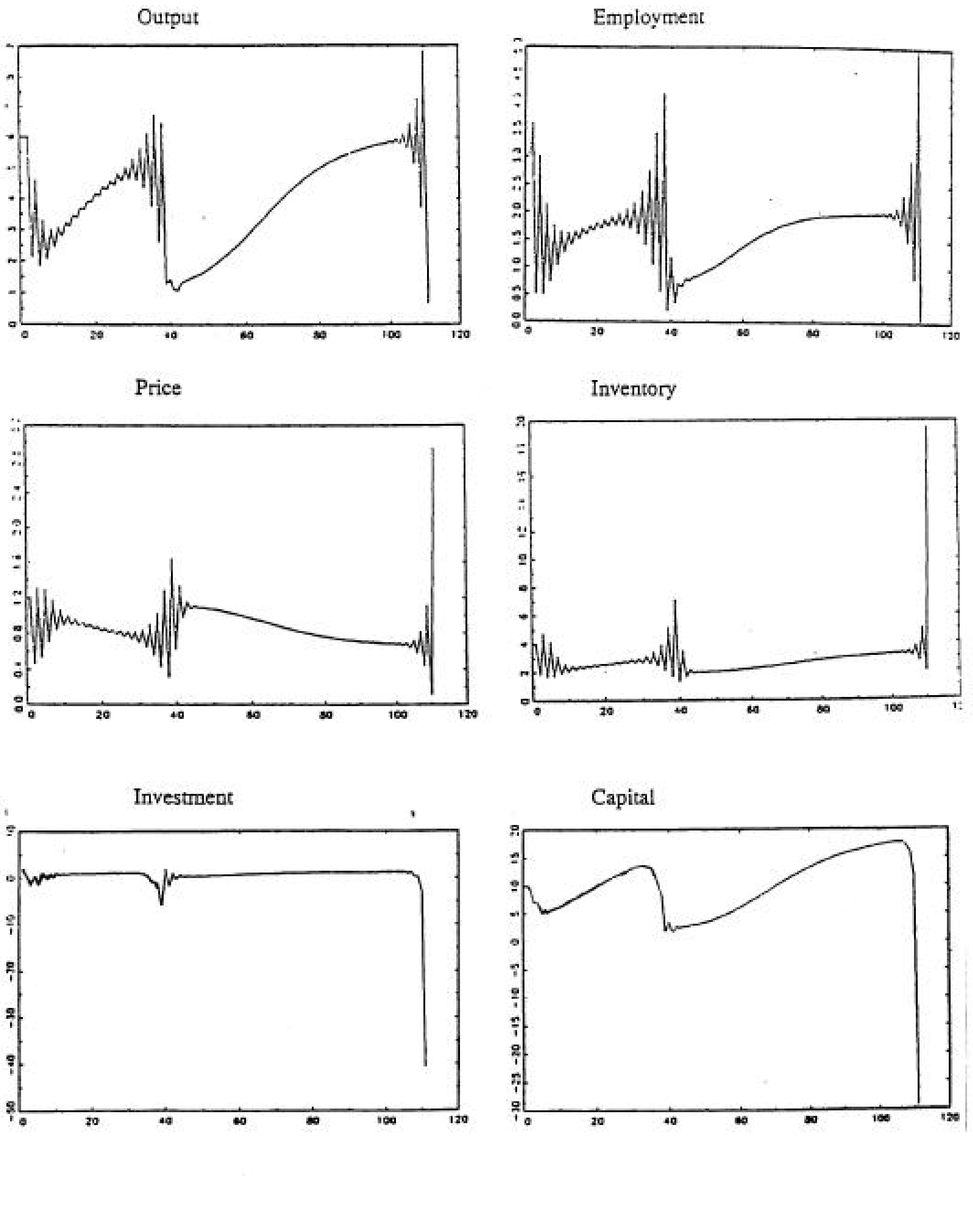

Figure 2: Comparative static

? = 0.1, d = 0.1