RATING MIGRATIONS AS INDICATORS FOR BUSINESS CYCLES

(preliminary and incomplete)

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1 Introduction

New rules are currently introduced by the regulator [see Basle Committee (...)] to control the risk taken by the banks, in particular to define the capital required to hedge a risky credit portfolio [the so-called Credit VaR]. The implementation of a regulation requires a careful analysis of corporate default risk and of its expected evolution. As a by-product the rating agencies as Standard & Poor's, Moody's, Fitch, or some central banks, as the Banque de France, have been led to improve the quality of their proprietary rating data bases and to weaken their confidentiality constraints[1]. Typically they report regularly summary statistics of the rating histories under the form of transition matrices providing the migration probabilities between given rating classes for different years and industrial sectors.

These aggregate data on rating histories include a lot of information on the general state of the economy and in particular on business cycles. The aim of this paper is to study if the observed migration probabilities can be used as leading indicators for cycles, and be competitive with the other leading indicators based on macrovariables such as unemployment, inflation, which are usually considered.

The basic data sets are presented and discussed in section 2, where some typical evolution of migration probabilities are also reported. The migration data are usually analyzed in the literature (see e.g. Albanese, Chen (2003), Bangia et alii (2002), Gupton et alii (1997), Crouhy et alii (2000)) by means of an ordered qualitative model with (observable or unobservable) factors. This model is reviewed in section 3, and especially its dynamic properties, when some factors represent the cycles. A complete factor analysis of the migration data is performed in Section 4 to estimate the number of underlying factors and to reconstruct the factor values. Then the filtered values are compared with the evolution of the GNP.

2 The migration probabilities

The definition of the rating and the population of firms differ according to the rating agencies.

i) For instance the main rating agencies obtain information about the situation of the corporates, generally when they are issuing bonds. This explains why their data bases concern large firms, mainly US companies, even if the proportion of European and Japanese firms represented in the bases has grown rapidly at the end the eighties. The number of

[1] See e.g. Brady et alii (2003) for a description of S&P data base, Carty (1997) for the Moody's data base, Foulcher et alii (2003), Bardos (2004), for a description of the Banque de France data base.

rated firms is around 10 000, with a proportion of missing data (alternative NR : not rated) between 10 and 20 %. These data are reliable since 1985 approximately, providing 17 years of observed transitions matrices. They use a rating with ten[2] classes from the highest rating [AAA for S&P for instance] to the worst one D corresponding to default.

ii) The data collected by the French Central Bank are of a different type. They include the balance sheets of all French firms, covering about 100 000 firms. The information included in the balance sheet is transformed into a quantitative score representing the risk level by a discriminant analysis technique [see Bardos (2004) for the description of the score] ; then the score is discretized to get a qualitative rating with height alternatives, noted $0, 1, ..., 7$. The alternative "0" corresponds to default, and the alternative "7" is the analogue of the AAA or Aaa of the rating agencies. The data have been collected since 1992, which provides 11 years of transition matrices.

Examples of transition matrices reported by S&P and the Banque de France are given in Tables 1 and 2 :

[Insert Table 1 : Migration probabilities, S&P year]

[Insert Table 2 : Migration probabilities, BdF year]

Such transition matrices are the input for the cycle analysis performed in the paper. A lot of migration probabilities are rather small. The significant values are mainly around the main diagonals, or for default as arrival state.

This matrices can be summarized by considering the probabilities of a down-grade [resp. an up-grade]. For instance for a firm rated BBB by S&P at the beginning of the year, the probability of an up-grade is the probability to migrate to one of the state A, AA or AAA.

The evolution of the down-grade probability by BdF for the wholesale and retail trade sectors is displayed in Figure 1.

[Insert Figure 1 : Evolution of the down-grade probabilities BdF ; wholesale and retail trade sectors]

Similarly Figure 2 displays the evolution of the up-grade probabilities by BdF for the wholesale and retail trade sectors.

[Insert Figure 2 : Evolution of the up-grade probabilities BdF :

[2] The rating classes for S&P are :
AAA, AA, A, BBB, BB, B, CCC, CC, C, D.
Sometimes the classes CCC, CC, C are aggregated providing a rating with eight buckets.

wholesale and retail trade sectors]

3 The ordered qualitative model with factor

The ordered qualitative model with factor is usually considered in the literature as a convenient specification for stochastic transition matrices. Indeed it allows to reduce the number of factors, which are driving the different migration probabilities, while keeping the model tractable with a reasonable number of parameters.

As usual the specification is based on a latent variable $S_{i,t}$, say, which can be interpreted as an underlying quantitative score, directly related to an expected probability of default at some horizon. Typically the horizon is 3 years for the BdF score (and larger than 1 year for the S&P or Moody's methodologies). Thus $S_{i,t}$ is the value of the score for corporate i at date t. It is assumed that the conditional distribution of variable $S_{i,t}$ given the information available at the beginning of period t depends on some factor Z_t and on the most recent rating $Y_{i,t-1}$. It is such that :

Latent model for the score

$$\begin{aligned} S_{i,t} = \quad & \alpha_k + \beta_k \, Z_t + \sigma_k \, u_{i,t}, \\ & \text{if } Y_{i,t-1} = k, \quad k = 0, ..., K-1, \end{aligned} \tag{1}$$

where α_k, β_k, σ_k are scalar parameters and $u_{i,t}$ are independent error terms, identically distributed with cdf G. Thus three parameters are introduced for each rating class : α_k measures a level effect, β_k is the risk sensitivity with respect to the factor[3], whereas σ_k is the idiosyncratic (firm specific) standard error.

Latent model for the factor

The factor has to be assumed non observable, to allow for migration correlation [see the discussion in Gagliardini, Gouriéroux (2003)]. Loosely speaking, if the different sensitivity coefficient are nonnegative, a positive movement on Z_t will imply a joint increase of the individual risks whatever the rating class of the companies. Thus we will get a positive migration up-grade correlation, if the future of the factor is not a priori known. It will be assumed that the evolution of the factor is independent of the values of the shocks $u_{i,t}$, that they depend on the past by means of the most recent factor value (which is the Markov assumption), and that the transition factor density can be parameterized. This density will be denoted by :

[3] We have implicitely consider a one factor model for expository purpose.

$$\frac{1}{dz} P\left[Z_t \epsilon (z_t, z_t + dz) | Z_{t-1} = z_{t-1}\right] \simeq f(z_t | z_{t-1}\ \theta), \quad (2)$$

for small dz.

Link between latent and observable variables

Finally the rating of period t is defined by discretizing the quantitative score :

$$Y_{i,t} = \ell, \text{ if and only if } a_\ell < S_{it} < a_{\ell+1}, \quad (3)$$

where : $a_0 = -\infty < a_1 < ... < a_{K-1} < a_K = +\infty$ are fixed (unknown) thresholds.

Under the specification above it is easy to derive the migration probabilities given the factor value. Since the factor is indexed by time only, these quantities are closed to the observed transition matrices, computed per year. We get :

$$\begin{aligned} p_{k,\ell,t} &= P\left[Y_{i,t} = \ell | Y_{i,t-1} = k, Z_t\right] \\ &= P\left[a_\ell < S_{i,t} < a_{\ell+1} | Y_{i,t-1} = k,\ Z_t\right] \\ &= P\left[a_\ell < \alpha_k + \beta_k\ Z_t + \sigma_k\ u_{i,t} < a_{\ell+1} | Z_t\right] \\ &= P\left[\tfrac{a_\ell - \alpha_k - \beta_k\ Z_t}{\sigma_k} < u_{i,t} < \tfrac{a_{\ell-1} - \alpha_k - \beta_k\ Z_t}{\sigma_k} | Z_t\right] \\ &= G\left(\tfrac{a_{\ell+1} - \alpha_k - \beta_k\ Z_t}{\sigma_k}\right) - G\left(\tfrac{a_\ell - \alpha_k - \beta_k\ Z_t}{\sigma_k}\right). \end{aligned}$$

The model reduces to an ordered probit model with factor, if the common distribution G corresponds to the standard gaussian distribution (as in Credit Matrics), and to a Cox model with stochastic intensity if exp u_{it} follows an exponential distribution [see e.g. Lando (1998)].

Different specifications of the factor dynamics can be retained in practice. Feng et alii (2003), Gagliardini-Gouriéroux (2003) assume a gaussian vector autoregressive factor. However the business cycle in often specified by means of au hidden Markov chain. Thus it seems naturel to consider a factor with discrete state space $\{0, 1\}$, where "0" represents "recession" and "1" represents "expansion". The factor dynamics in summarized by a (2,2) transitions matrix $P = \begin{pmatrix} p_{oo} & p_{01} \\ p_{10} & p_{11} \end{pmatrix}$, say. For this model the joint process $(Y_{1t}, ..., Y_{nt}, Z_t)$ defines a Markov chain, with some recursivity in the joint transition matrix. The Markov property is lost when the factor is not observed and only the past ratings are known.

4 Credit migration and business cycle

The relationship between corporate default and business cycle has been advocated since a long time in the economic literature. During a recession period, the firms have more difficulty to sell their productions, which can deteriorate their balance sheet, increase their probability of failure, and the credit rate which is offered by their lender. Conversely, if the probability of failure of a firm increases, it has more difficulty to find credit at a small rate, which can increase the amount of regular reimbursement and reacts negatively on its situation. This creates an accelerated movements towards default, with the usual consequences on employment, growth, deflation .... The new existing data bases on credit histories allow for a better understanding of the relationship between the business cycle and the factors influencing credit risk and credit migration. A few number of studies have already been done on this topic [see Nickell, Perraudin, Varotto (2000), Bangia et alii (2002)], but with proxies of the business cycle such as the NBER indicator and not the GNP growth itself. Moreover they essentially concern the US business cycle at a general level, and do not take into account the effect specific of a country or of an industrial sector.

The evaluation of migration probabilities given in Figures 1 and 2 show clearly some common feature, with at most two underlying patterns (note that the evolutions of up-grade are in the inverse direction of the evolutions of down-grade). For instance the evolutions corresponding to the high ratings are almost parallel, with sometimes a lag, or a lead on the location of the peaks. This pattern can be compared withe the French growth rate given in Figure 3.

[Insert Figure 3 : French growth rate]

The comparison shows the strong link between migration probabilities and growth rate. At this level it is interesting to study if the somewhat parallel curves present some lag, that is to say if the evolutions of migration probabilities can be considered as an advanced indicator for economic growth, or is simultaneous.

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Migration probabilities, S& P year 1997

	1	2	3	4	5	6	7	8
1	95.92	4.08	0.00	0.00	0.00	0.00	0.00	0.00
2	0.88	94.87	3.01	0.88	0.00	0.35	0.00	0.00
3	0.00	1.72	93.75	3.89	0.18	0.45	0.00	0.00
4	0.00	0.37	3.89	91.60	2.89	0.75	0.13	0.37
5	0.00	0.00	0.20	9.60	84.79	5.20	0.00	0.20
6	0.00	0.00	0.71	0.47	8.02	84.19	2.84	3.77
7	0.00	0.00	0.00	0.00	0.00	18.19	68.18	13.63

Table 2 Migration probabilities, whole sale sector, year 2001

	7	6	5	4	3	2	1	0
7	0.8392	0.1148	0.0282	0.0133	0.0036	0.0003	0.0003	0.0003
6	0.1396	0.6804	0.1174	0.0366	0.0213	0.0025	0.0009	0.0013
5	0.0194	0.2925	0.4867	0.1316	0.0549	0.0090	0.0029	0.0030
4	0.0101	0.0713	0.2991	0.4177	0.1452	0.0352	0.0130	0.0084
3	0.0014	0.0514	0.1152	0.3053	0.3656	0.0940	0.0422	0.0249
2	0.0000	0.0154	0.0686	0.2024	0.3373	0.2213	0.1136	0.0414
1	0.0000	0.0116	0.0486	0.0833	0.2917	0.1875	0.2778	0.0995
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Figure 1 : Evolution of the downgrade probabilities

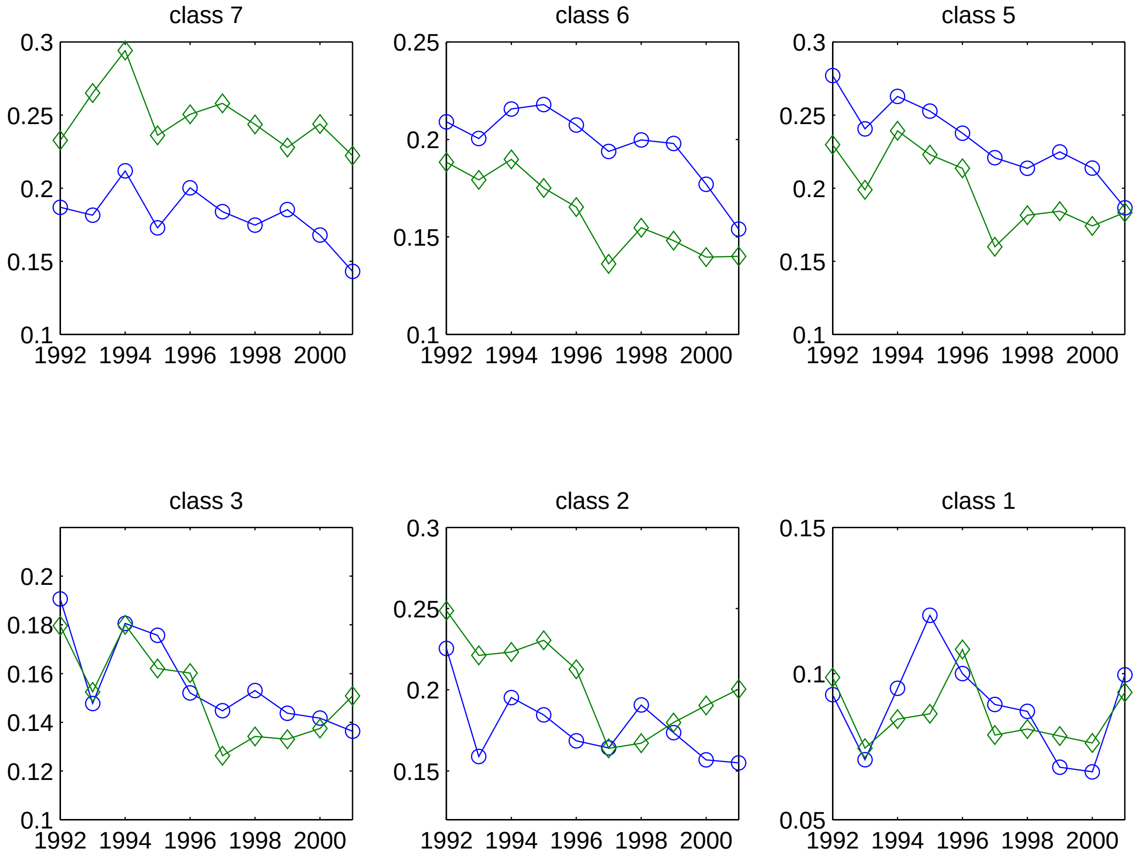

Figure 2 : Evolution of the up-grade probabilities

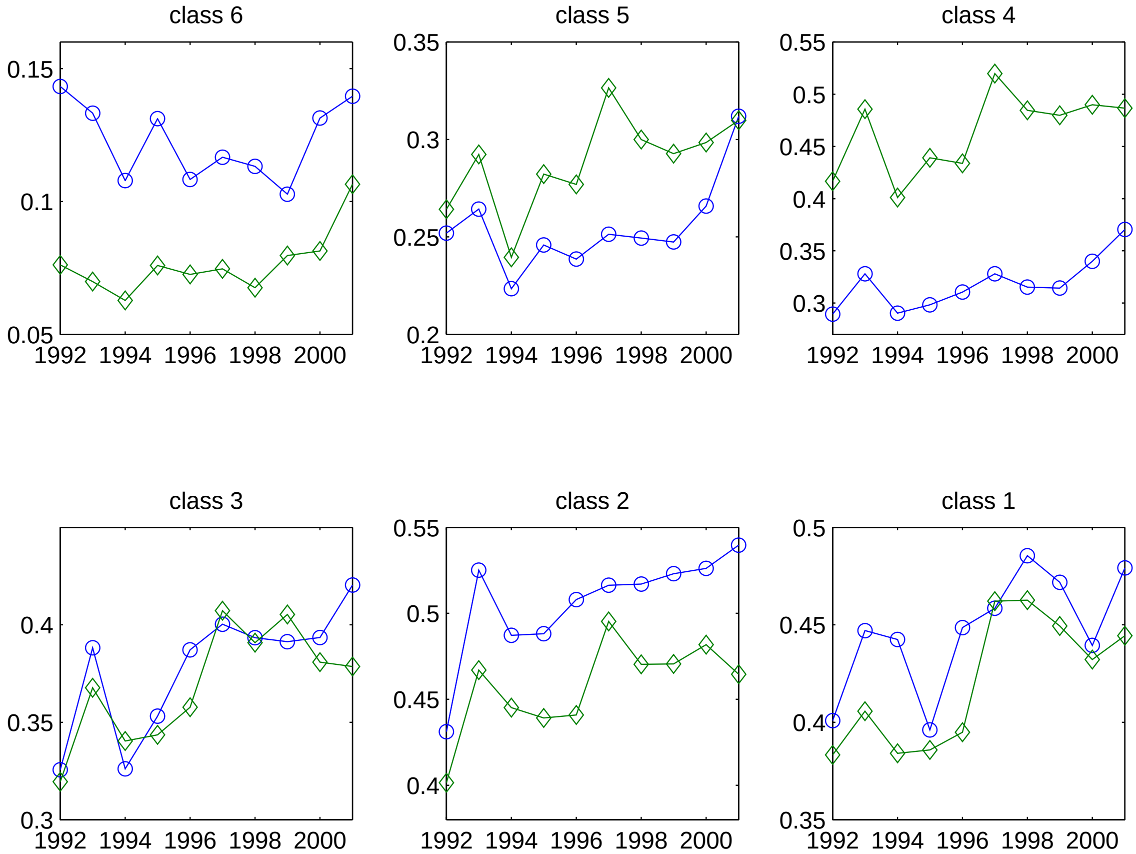

Figure 3 : GDP increment