

# Equilibrium Risk Shifting and Interest Rate in an Opaque Financial System\*

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## Abstract

We analyse the risk-taking behaviour of heterogeneous intermediaries that are protected by limited liability and choose both their amount of leverage and the risk exposure of their portfolio. Due to the opacity of the financial sector, outside providers of funds cannot distinguish “prudent” intermediaries from those “imprudent” ones that voluntarily hold high-risk portfolios and expose themselves to the risk of bankruptcy. We show how the number of imprudent intermediaries is determined in equilibrium jointly with the interest rate, and how both ultimately depend on the cross-sectional distribution of intermediaries’ capital. One implication of our analysis is that an exogenous increase in the supply of funds to the intermediary sector (following, e.g., capital inflows) lowers interest rates and raises the number of imprudent intermediaries (the *risk-taking channel* of low interest rates). Another one is that easy financing may lead an increasing number of intermediaries to gamble for resurrection following a bad shock to the sector’s capital, again raising economywide systemic risk (the *gambling-for-resurrection channel* of falling equity).

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# 1 Introduction

The 2007-2010 financial crisis has rejuvenated the interest in systemic risk in the financial system, its dramatic spill over to the real economy and whether and how it should be addressed by public policies. We contribute to this debate with an analysis of the risk taking behaviour of financial intermediaries that have limited liabilities and may deliberately choose a level of risk in excess of the social optimum. We show how the level of economywide risk taking depends on the distribution of equity among intermediaries and the level of interest rate in the economy.

Our key assumption is that outside providers of funds cannot tell apart “prudent” and well diversified banks from “imprudent” ones overly exposed to one particular asset, because the balance sheets of individual intermediaries is imperfectly observable, or *opaque*. This assumption is consistent with the view of several commentators of the crisis including Brunnermeier (2008), Acharya and Richardson (2009), and Dubecq et al. (2009)<sup>1</sup>. In the decade prior to the crisis, risk transfer instruments, which have reached a very large scale in the U.S., have increased the opacity of banks’ leverage and risk-taking incentives (Acharya and Schnabl, 2009). First, regulatory loopholes allowed banks evade capital requirements by securitising assets and providing (unregulated) liquidity support to “shadow” (i.e., off-balance-sheet) entities (Acharya and Richardson, 2009). Second, the financial sector as a whole effectively repurchased much of the senior tranches of structured products, whose payoff distributions was particularly difficult to assess (see, e.g., Coval et al., 2009). Third, some banks actively relied on “window dressing” to manipulate leverage figures –by selling asset before the books releases to repurchase them at a later date (see, e.g., The Financial Crisis Inquiry Commission, 2011). Last but not least, this opacity may have taken the form of shadow subsidiaries that were used to absorb poorly performing assets, as was revealed by the investigation on Lehman’s bankruptcy.<sup>2</sup> All these trends arguably made it

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<sup>1</sup>And the references therein.

<sup>2</sup>On April 13, 2010, the New York Time reported that “Lehman Brothers operated a side business that allowed the defunct brokerage to transfer risky investments off its books in the years leading up to its collapse, according to a report published Tuesday 13 April 2010. The firm, called Hudson Castle, appeared to be an independent company, but played an important "behind-the-scenes role" at Lehman, .... Hudson is part of a "vast financial system" that operates largely beyond the reach of banking regulators. But banks can use such entities to raise cash by trading investments and, at times, make their finances look artificially strong. The report said Lehman conducted several transactions greater than \$1 billion with Hudson vehicles, but

increasingly difficult for outside providers of funds to accurately assess the true net worth levels of individual banks.

When intermediaries' balance sheet is opaque, those with relatively low levels of capital may be tempted to hold high-risk asset portfolios, or even to gamble for resurrection in the face of worsening economic conditions. In our model, intermediaries' limited liability creates an incentives to correlate asset portfolios and raise leverage, thereby allowing intermediaries to raise their return on equity in case of success while transferring much of their losses to their creditors in case of failure. This tendency, however, is alleviated by intermediaries' shareholders' initial equity stake, which disciplines risk-taking, thereby limiting leverage and favouring diversification. We show that this tradeoff gives rise to an endogenous sorting of intermediaries along the equity dimension, with well capitalised intermediaries holding diversified portfolios and keeping a limited level of leverage (that is, behaving prudently), and poorly capitalised ones heavily resorting to leverage to invest in correlated assets (i.e., behaving imprudently). Opacity implies that the former are not readily distinguishable from the latter, so that risk-prone behaviour may prosper without being immediatly sanctioned by higher borrowing rates.

One property of our model is that the proportion of imprudent intermediaries and, therefore, the level of systemic risk in the financial system, crucially depend on both the cross-sectional distribution of capital and the prevailing interest rate. The endogenous determination of the number of imprudent intermediaries jointly with the (equilibrium) interest rate is our key contribution. Equiped with this joint equilibrium outcome, we analyse the impact on the interest rate and the number of imprudent intermediaries of two exogenous shocks: a *lending boom* that shifts the loan supply curve rightwards; and a *equity squeeze* that shifts the distribution of banks' capital leftwards. As we show, the downward pressure on the equilibrium interest rate that follows the lending boom raises the number of imprudent intermediaries and hence the level of economywide risk shifting (the *risk-taking channel* of low interest rates). An "equity squeeze", that is, a reduction in the equity value of intermediaries' shareholders after a negative aggregate shock, has the the same effect provided that the

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added that it is unclear how much money was involved since 2001. Critics charge that this type of creative financing allowed Lehman and other major banks to temporarily transfer risky investments in subprime mortgages and commercial real estate, the report said. While most of the deals done through operations such as Hudson are legal, the report points out that bank examiners have recently raised questions about other dubious accounting practices at Lehman."

supply of funds is sufficiently elastic (the *gambling-for-resurrection channel* of falling equity). The evidence strongly suggests that both shocks occurred in the run-up to the current crisis. In the first half of the 2000 decade, both capital inflows from China and oil-exporting countries into the U.S. and the accommodative monetary policy of the Fed contributed to keep the yield curve very low; according to our model, this would have favoured imprudent behaviour by an increasing number of banks –those at the lower end of the capital distribution– thereby raising their risk exposure and the amount of systemic risk in the economy.<sup>3</sup> Second, the tightening of U.S. monetary policy in 2004 and the rise in delinquency rates on subprime mortgages from 2006 onwards may have deteriorated the equity position of exposed intermediaries, and hence favored gambling-for-resurrection strategies. Landier et al. (2010) provide direct evidence of this behaviour for New Century Financial Corporation, a major subprime originator prior to its bankruptcy in 2007.<sup>4</sup>

**Related literature.** In our model, systemic risk in the financial sector arises from the interaction between i. intermediaries’ limited liability and option to default (the *risk shifting problem*); ii. their incentive to correlate their risk exposure (the *endogenous correlation problem*); and iii. the difficulty for outside lenders to discriminate individual institutions on the basis of their true net worth level (the *opacity problem*). While our model is the first to explicitly connect these three dimensions, we build on many contributions that have studied each of them separately.

Our modelling of the risk shifting problem closely follows Allen and Gale (2000) and Acharya (2009), who show that limited liability leads financial institutions to overweight risky assets in their portfolio, relative to the first best.<sup>5</sup> There are two main differences between earlier models of risk shifting and ours. In Allen and Gale, market segmentation

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<sup>3</sup>See Jimenez et al. (2010) for direct evidence that falling short-term interest rates favour risk-taking by banks that are at the lower end of the capital distribution.

<sup>4</sup>According to Landier et al. (2010), rising interest rate were conducive to more risk shifting, while many authors (e.g. Diamond and Rajan, 2009) suggests that low interest rates favor leverage and excess risk-taking. However, to effects of interest rates on intermediaries’ risk taking must be distinguished. First, holding the equity stake of bankowners fixed, low interest rates may indeed favor high leverage and risk taking. However, rising interest rates lower asset values, which in turn deplete bankowners’ net equity positions ex post and may trigger the gambling-for-resurrection logic. Our model is consistent both views.

<sup>5</sup>see also Rochet (1992) for an early analysis of bank risk taking under limited liability.

and limited-liability debt contracts twist intermediaries' risk-taking incentives and leads to an overvaluation (or "bubble") in the price of the risky asset (a feature that is also in Challe and Ragot and Dubecq et al.). In Acharya, risky assets are in flexible supply so that their quantity (rather than price) adjust to clear the market. All these models share the property that intermediaries' excessive risk-taking is ubiquitous: risky assets are always have excessive space in intermediaries portfolios, leading all of them to be exposed to bankruptcy risk. We see this property as somewhat extreme, which leads us to emphasise the disciplining role of initial shareholders' equity stake and to endogenise each intermediary's (discrete) choice of adopting or not a bankruptcy-prone behaviour based on the expected costs and benefits of doing so. The second difference with earlier concerns the way we model excessive risk taking: while earlier models rely on intermediaries' overexposure to a risky asset relative to a safe one, in ours excessive risk taking exclusively takes the form of insufficient portfolio diversification in equilibrium.

This asset correlation problem has been the focus on several recent contributions, both empirical and theoretical. Acharya and Richardson (2009) notably document the overexposure of the U.S. banking sector to securitised mortgages prior to the current crisis, with the risk associated with those securities being effectively kept within the sector (via the use of unregulated liquidity enhancements or the repurchase of CDO tranches) rather than transferred to other investors and disseminated throughout the economy. Greenlaw et al. (2008) had reached similar conclusions. The dominant explanation for this excessive correlation, apparently at odds with standard finance theory, is that it is natural consequence of the time-inconsistency of ex post bail-out or interest rate policies; namely, it is optimal to save banks ex post when a large number of them fails, which precisely occurs when they have chosen correlated portfolios in the first place –see Acharya and Yorulmazer (2007) and Fahri and Tirole (2010). Our model differs from those in the source of moral hazard that leads to excess portfolio correlation, i.e., limited liability rather than time-inconsistent policies. In Acharya (2009), the economywide correlation of risks arises from systemic failure externalities amongst intermediaries. The main difference between Acharya's endogenous correlation mechanism and ours is that in his framework banks are assumed to hold undiversified portfolio (because they are industry-specific lenders), and the puzzle to be explained is why correlation occurs *across* banks (i.e., why they tend to lend to the same industries). By contrast, in our model banks are unspecialised and choose the correlation of their portfolio

at the individual level; but since those who opt for highly correlated portfolios favour the stochastically dominated asset, the very same asset is overinvested in at the aggregate level, hence more risk-taking at the individual level directly translates into greater systemic risk.

Finally, a number of authors have discussed the adverse consequences of the opacity of the financial sector for financial stability. The difficulty for (unsophisticated) outside lenders of perfectly observing bank assets is a traditional argument for why banks need to be supervised (e.g., Dewatripont and Tirole, 1994). More recently, Biais et al. (2010) have argued that financial innovations create asymmetric information problems that worsen the opacity of the financial sector. Our model focuses on one specific (and yet unexplored) implication of opacity: the fact that outside providers of funds may find it difficult to accurately measure bank shareholders' true stake and hence to adequately assess their risk-taking incentives.

The remainder of the paper is organised as follows. Section 2 introduces the model and characterises the optimal behaviour of intermediaries. Section 3 derives the equilibrium level of interest rate and systemic risk in the opaque economy, and carries out some comparative statics experiments. Section 4 concludes.

## 2 The model

### 2.1 Timing, states and assets

There are two dates,  $\mathbf{t} = \{1, 2\}$ , two possible states at date 2,  $\mathbf{s} = \{s_1, s_2\}$ , and two (supply-elastic) real assets available for purchase at date 1,  $\mathbf{a} = \{a_1, a_2\}$ . At date 1, loan contracts are signed and investments in the real assets take place; at date 2, the state is revealed, asset payoffs are collected and financial contracts are resolved –possibly via one party's default. Any unit of investment in  $a_1$  pays  $R_1 = R_1^h > 0$  if  $s = s_1$  and 0 otherwise, while any unit of investment in  $a_2$  pays  $R_2 = R_2^h > 0$  if  $s = s_2$  and  $R_2^l > 0$  otherwise.<sup>6</sup> State  $s_1$  ( $s_2$ ) occurs with probability  $p(s_1) \equiv p = 0.5 - \epsilon$ ,  $\epsilon > 0$  ( $p(s_2) = 1 - p$ .) Finally the two assets are

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<sup>6</sup>Assuming that asset  $a_1$  has no liquidation value in state  $s_2$  greatly simplify the analysis of Section 4, where we need to sum up the repayments of heterogenous intermediaries to the lenders across states. Alternatively, one may assume that the asset has some liquidation value but that the latter cannot be recovered in case of default (due, e.g., to bankruptcy costs).

assumed to have identical expected payoffs, i.e.,

$$pR_1^h = pR_2^l + (1 - p) R_2^h. \quad (1)$$

Our assumed joint payoff distribution has the following properties: when considered in isolation,  $a_1$  is more risky (in the sense of mean-preserving spread) than  $a_2$ ; however, the strict negative correlation between the two assets implies that one of them may be used as a hedge against the portfolio risk generated by the other. In particular, a suitably diversified portfolio pays the certain gross return  $pR_1^h$ —thereby entirely eliminating bankruptcy risk for a leveraged investor. This simple payoff structure allows us to focus on the joint choice of leverage and portfolio correlation as the ultimate source of endogenous aggregate risk in the economy.

## 2.2 Agents and market structure

There are two types of agents in the economy: “lenders” and “intermediaries”, both risk-neutral and in mass one. Our market structure (and implied decisions) is similar to that in Allen and Gale (2000) and Acharya (2009). In particular, markets are segmented, in the sense that intermediaries have exclusive access to the menu of assets  $\mathbf{a}$  (due, for example, to asymmetric information, difference in asset management abilities, regulation etc.). Intermediaries may borrow from the lenders to achieve their desired level of asset investment, and are protected by limited liability debt contracts. Once lending has taken place, the portfolio chosen by the intermediaries is out of the control of the lenders.

We modify this basic framework in two directions. First, we assume that an intermediary’s funding partly comes out of *inside equity*, which will serve both to buffer the intermediary’s balance-sheets against adverse shocks and to discipline its shareholders’ risk-taking attitude.<sup>7</sup> Second, we study the equilibrium of an economy populated by a large number of intermediaries with heterogenous equity levels that are imperfectly observed by outside providers of funds.

**Intermediaries.** Intermediaries’ shareholders maximise value, given their (exogenously given) initial equity stake  $e > 0$ . Denoting by  $(x_i)_{i=1,2} \geq 0$  the portfolio of an intermediary,

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<sup>7</sup>This paper focuses on agency problems between the intermediary’s owner-manager and its creditors, and hence abstracts from incorporating inobservability and conflict of interest between the owners and the managers. See Acharya et al. (2010) for a model of risk-shifting that explicitly incorporates both dimensions.

its balance sheet constraint may be written as:

$$\sum x_i \leq e + b, \quad (2)$$

with  $i = 1, 2$  and where  $b$  is the intermediary's debt. Intermediaries face a convex investment cost, which is assumed to take the form:<sup>8</sup>

$$c(\sum x_i) = (2\theta)^{-1} (\sum x_i)^2, \quad \theta > 0. \quad (3)$$

Given its initial equity stake  $e$  and a contracted gross interest rate  $r$  on borrowed funds, an intermediary chooses  $(x_i)$  and, by implication,  $b$  –i.e., it chooses both the size and the structure of the balance sheet. Limited liability implies that an intermediary's payoff net of debt repayment is bounded below by zero, so the ex post net payoff generated by the portfolio  $(x_i)$  is:

$$\max [\sum x_i R_i - rb - c(\sum x_i), 0]$$

Substituting (2) (with equality) into the latter expression, we find the date 1 value of an intermediary with initial equity  $e$  to be:

$$V(e) = \max_{x_i \geq 0} \sum_s p(s) (\max [re + \sum x_i (R_i - r) - c(\sum x_i), 0]), \quad (4)$$

with  $s = 1, 2$ .

In solving (4), intermediaries differ in the amount of the inside equity stake of its shareholders,  $e$ . The cross-sectional equity distribution is assumed to be characterised by a continuous density function  $f(e; \epsilon)$  with support  $[0, e_{\max}]$  and c.d.f.  $F(e; \epsilon) = \int_0^e f(i; \epsilon) di$ . Since the number of intermediaries is normalised to one we have  $F(e_{\max}; \epsilon) = 1$ , while  $E \equiv \int_0^{e_{\max}} e f(e; \epsilon) di$  is the total capital of the intermediary sector. The parameter  $\epsilon$  indexes the location of the density function, with an increase in  $\epsilon$  being associated with a rightward shifts in the distribution of equity level (i.e.,  $F_\epsilon(e; \epsilon) < 0$ .)

**Lenders.** Funds are supplied by households (the “lenders”), who lend their funds to the intermediary sector at date 1 in order to collect repayments at date 2. Each lender enjoys labour income  $w > 0$  at date 1 and maximises the  $u(c_1) + c_{2s}$ , where  $c_1$  is date 1 consumption,  $c_{2s}$  consumption at date 2 in state  $s$ , and  $u(\cdot)$  a twice continuously differentiable, strictly

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<sup>8</sup>We model this cost as a standard pecuniary cost, as in, e.g., Rochet (1992), and unlike Allen and Gale (2000) or Acharya (2009). Our results are insensitive as to which specification is adopted.

increasing and strictly concave function. Let  $\rho_s$  denote lenders' ex post date 2 return in state  $s$  from lending to the intermediary sector, and  $\rho \equiv \sum_s p(s) \rho_s$  the corresponding ex ante return (Note that both in general differ from the face lending rate  $r$  due to the possibility of intermediaries' default.) Lenders choose their loan supply  $B^s$ , where  $B^s = \arg \max u(c_1) + \sum_s p(s) c_{2s}$ , subject to  $c_1 = w - B$  and  $c_{2s} = B \rho_s$ . The implied loan supply curve is:

$$B^s(\rho; w) = w - u'^{-1}(\rho), \quad (5)$$

which is continuous and strictly increasing in both arguments. In short, risk neutrality implies that lenders value the expected return on loans,  $\rho$ , with the implied loan return curve being shifted by date 1 income,  $w$ . We impose specific parameter restrictions later on ensuring that  $B^s(\rho; w) > 0$  in equilibrium.

### 2.3 First best outcome

The key contractual friction in this economy is that an intermediary maximises the expected terminal payoff to its (risk-neutral) shareholders who are protected by limited liability – and hence transfer losses to the debtors in case of default. Before further analysing the implied portfolio distortions, let us first derive the first-best outcome wherein lenders can force intermediaries to fully exploit the diversification opportunities offered by the menu of available assets.

**Efficient portfolio.** Since  $a_1$  and  $a_2$  have identical expected payoff and are perfectly negatively correlated, a fully diversified portfolio that entirely eliminates payoff risk is always efficient (at least weakly). A portfolio  $(x_i)_{i=1,2}$  pays the certain payoff  $(\sum x_i) p R_1^h$  if it pays identical payoffs across states, that is, if it satisfies

$$x_1 R_1^h + x_2 R_2^l = x_2 R_2^h, \quad (6)$$

where the left and right hand sides are the portfolio payoff in states  $s_1$  and  $s_2$ , respectively. This riskless portfolio eliminates the possibility of bankruptcy and thus makes the limited-liability clause inoperative. The value of an intermediary thus becomes:

$$\hat{V}(e) = \max_{x_i \geq 0} r e + (\sum x_i) (p R_1^h - r) - c(\sum x_i), \quad (7)$$

where we have used the fact that  $\sum_s p(s) \sum x_i (R_i - r) = (\sum x_i) (pR_1^h - r)$  under full diversification (and given the payoff distribution (1)). From (7), the first-best efficient portfolio satisfies  $\sum \hat{x}_i = \theta (pR_1^h - r)$ . Combined with (1) and (6), we find it to be:

$$(\hat{x}_1, \hat{x}_2) = (\theta p (pR_1^h - r) (1 - R_2^l/R_2^h), \theta p (pR_1^h - r) R_1^h/R_2^h). \quad (8)$$

Portfolio (8) is only weakly dominant because, due to agents' risk neutrality, a continuum of portfolios in fact achieve the same welfare outcome as (8). Indeed, for lenders to enjoy the face return  $r$  in both states, it is enough that intermediaries be *solvent* in both states, which is possible (to some extent) with an imperfectly diversified portfolio thanks to the buffering role of intermediaries' capital. The solvency conditions impose that a given portfolio  $(x_i)_{i=1,2}$  never generates a negative net payoff ex post, i.e.,

$$re + \sum x_i (R_i - r) - c(\sum x_i) \geq 0, \quad s = s_1, s_2, \quad (9)$$

where  $(R_1, R_2) = (R_1^h, R_2^l)$  if  $s = s_1$  and  $(0, R_2^h)$  if  $s = s_2$ . Combining (9) with the optimal balance-sheet size  $\sum \hat{x}_i = \theta (pR_1^h - r)$ , we find that a solvent portfolio must be such that  $x_1 \in [\underline{x}_1^*, \bar{x}_1^*]$ ,  $0 < \underline{x}_1^* < \bar{x}_1^* < \infty$ , where  $\underline{x}_1^*$  and  $\bar{x}_1^*$  are:

$$\underline{x}_1^* = \frac{\theta (pR_1^h - r)^2 / 2 + \theta (pR_1^h - r) (r - R_2^l) - re}{R_1^h - R_2^l},$$

$$\bar{x}_1^* = \frac{-\theta (pR_1^h - r)^2 / 2 + \theta (pR_1^h - r) (R_2^h - r) + re}{R_2^h},$$

and where  $\bar{x}_1^* < \theta (pR_1^h - r)$  whenever the intermediary is leveraged. In short, given the optimal balance-sheet size  $\theta (pR_1^h - r)$ ,  $x_1$  cannot be too low, otherwise the intermediary would default in state  $s_1$ ; it cannot be too high either (and hence  $x_2$  too low), otherwise default would occur in state  $s_2$ . Intermediaries that deviate from the riskless portfolio  $(\hat{x}_i)$  while still satisfying  $\sum \hat{x}_i = \theta (pR_1^h - r)$  and  $x_1 \in [\underline{x}_1^*, \bar{x}_1^*]$  will bear some asset risk but are indifferent to it thanks to risk neutrality. These portfolios, which we refer to as "prudent", lie along a closed subinterval of the  $x_2 = \theta (pR_1^h - r) - x_1$  line and include the riskless portfolio  $(\hat{x}_i)$  –see Figure 1(a) below.

**Equilibrium.** To complete the characterisation of the first best outcome, we must compute the equilibrium interest rate  $\hat{r}$  that results from the confrontation of the aggregate demand and supply for loanable funds. Since  $\sum \hat{x}_i = \theta (pR_1^h - r)$ , it follows that the leverage of

an intermediary with inside equity  $e$  and facing the interest rate  $r$  is given by  $\hat{b}(r, e) = \theta(pR_1^h - r) - e$ . The aggregate demand for funds is obtained by summing up the demands for debt by all intermediaries, i.e.,

$$\hat{B}^d(r; \epsilon) = \int_0^{\epsilon_{\max}} \hat{b}(r, e) dF(e; \epsilon) = \theta(pR_1^h - r) - E. \quad (10)$$

On the other hand, since intermediaries never default in the first-best equilibrium, lenders are repaid  $r$  with certainty. Hence, we may rewrite (5) as:

$$\hat{B}^s(r; w) = w - u'^{-1}(r). \quad (11)$$

$\hat{B}^d(r; \epsilon)$  is continuous and linearly decreasing in  $r$ , while  $\hat{B}^s(r; w)$  is continuous and strictly increasing in  $r$  (since  $u''(\cdot) < 0$ ). Hence the two curves cross at most once and, if they do, give a unique equilibrium interest rate  $\hat{r}$ . In the remainder of the paper, we focus on equilibria in which all intermediaries are active and leveraged. Lemma 1 provides a sufficient condition for the existence of a first-best equilibrium with this property.

**Lemma 1.** Assume that i.  $\theta pR_1^h > e_{\max}$  and ii.  $w > e_{\max} - E + u'^{-1}(pR_1^h - e_{\max}/\theta)$ . Then, the first-best equilibrium is unique and such that  $\hat{b}(r, e) > 0$  for all  $e \in [0, e_{\max}]$ .

All proofs are in the Appendix. Essentially, a unique equilibrium with all intermediaries being leveraged exists if both expected asset payoffs (i.e.,  $pR_1^h$ ) and lenders' income (i.e.,  $w$ ) are sufficiently large. This equilibrium is depicted in Figure 2(b) below.

## 3 Loanable funds equilibrium under risk-shifting

### 3.1 Intermediaries' behaviour

The presence of the limited-liability debt contracts affect investment choices by altering intermediaries' shareholders payoffs relative to the first best. Namely, value maximisation under limited liability may lead an intermediary to choose a high risk/high expected payoff strategy, thereby maximising its own payoff in case of success while transferring losses to the lenders in case default.

We work the problem of an intermediary (i.e., equation (4)) backwards. Let us refer to as “prudent” an intermediary whose asset portfolio satisfies both solvency constraints in (9), and denote its value as  $V^*(e)$ . Similarly, let us call “imprudent” an intermediary whose

portfolio violates one of the two inequalities in (9) –thereby triggering default in one of the two states–, and denote its value by  $V^{**}(e)$ . The intermediary chooses the best option, giving a value to the initial equity holders of  $V(e) = \max[V^*(e), V^{**}(e)]$ .

**Prudent intermediaries.** Trivially, the absence of the option to default implies that the portfolio choice of a prudent intermediary is the same as in the first best:

$$\sum x_i^* = \theta (pR_1^h - r), \quad \underline{x}_1^* < x_1^* < \bar{x}_1^*, \quad (12)$$

$$b^*(r, e) = \theta (pR_1^h - r) - e. \quad (13)$$

Substituting (12)–(12) into (7), we find the value of a prudent intermediary to be:

$$V^*(e) = re + (\theta/2) (pR_1^h - r)^2 (= \hat{V}(e)). \quad (14)$$

**Imprudent intermediaries.** Imprudent intermediaries, unlike prudent ones, correlate their portfolio and consequently default in one of the two states. Consider first the the optimal portfolio choice of an intermediary having chosen to overweight asset  $a_1$  in its portfolio, and thus defaults at date 2 if state  $s_2$  occurs. Ex ante, this intermediary earns zero with probability  $1 - p$ , so the objective (4) becomes:

$$\max_{x_i \geq 0} p (re + x_1 (R_1^h - r) + x_2 (R_2^l - r)) - c(\sum x_i) \quad (15)$$

Since  $x_1$  and  $x_2$  enter symmetrically in the cost function while  $R_1^h > R_2^l$ , the intermediary must entirely disregard  $a_2$ , leading to the optimal portfolio:

$$(x_1^{**}, x_2^{**}) = (\theta p (R_1^h - r), 0), \quad (16)$$

$$b^{**}(r, e) = \theta p (R_1^h - r) - e. \quad (17)$$

An alternative investment strategy for an imprudent intermediary would be to overweight  $a_2$ , and hence to default if  $s_1$  occurs ex post. However, it is straitforward to show that it is never optimal to do so under our distributional assumptions. Indeed, imprudent behaviour implies that the intermediary earns zero if the wrong state occurs, and accordingly only values the state corresponding to the asset being invested in. Since the univariate distribution of  $a_1$  is a mean-preserving spread of that of  $a_2$ ,  $a_1$  has more value to the imprudent inter-

mediary than  $a_2$ .<sup>9</sup> Substituting (16) into (15), we find the optimised value of an imprudent intermediary to be:

$$V^{**}(e) = pre + (\theta/2) (p(R_1^h - r))^2. \quad (18)$$

To summarise, imprudent intermediaries have two distinguishing characteristics, relative to prudent intermediaries. First, they perfectly correlated their asset portfolio (since  $x_2^{**} = 0$ ), thereby maximising both their payoff in case of success and lenders' losses in case of default. Second, they endogenously choose a larger balance sheet size (since  $x_1^{**} > \sum x_i^*$ ), which in turns implies more leverage for any given level of equity  $e$  (i.e.,  $b^{**}(r, e) > b^*(r, e)$ ). This latter property is a direct implication of the fact that imprudent intermediaries avoid repayment with probability  $1 - p$ . This effectively lowers the cost of debt ex ante for any given face interest rate  $r$ , relative to the cost faced by prudent intermediaries (who repay in both states). In the  $(x_1, x_2)$  plane, the imprudent portfolio lies on the  $x_1$  axis and the the left of the  $x_2 = \theta(pR_1^h - r) - x_1$  line –see Figure 1(a).

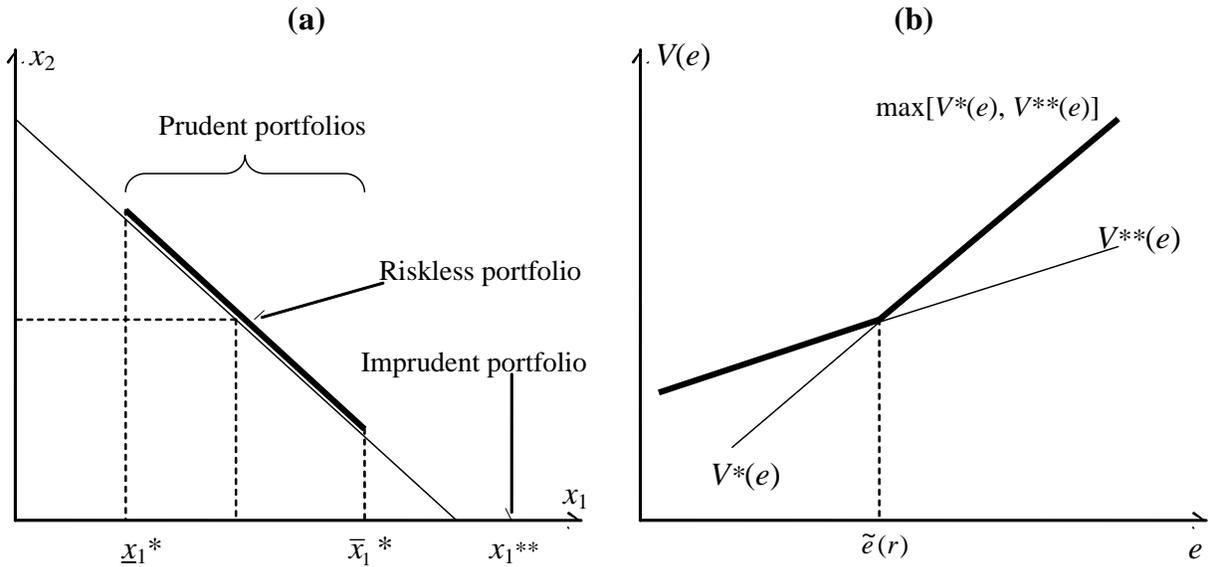


Figure 1: Intermediaries' optimal portfolios (a) and value (b).

<sup>9</sup>An intermediary choosing the default in state  $s_1$  does not value payoffs in that state and hence maximises  $(1 - p)(x_1(0 - r) + x_2(R_2^h - r) + re) - c(x_1 + x_2)$ , leading to the optimal portfolio  $(\hat{x}_1^{**}, \hat{x}_2^{**}) = (0, \theta(1 - p)(R_2^h - r))$ . Computing and comparing the ex ante utility levels associated with  $(x_1^*, x_2^*)$  and  $(\hat{x}_1^{**}, \hat{x}_2^{**})$  leads the former to be preferred, provided that  $\epsilon$  is not too large.

**Value of an intermediary.** Expressions (14) and (18) reflect the joint roles of equity and the borrowing rate in affecting the intermediary's value and thus incentives to behave prudently or imprudently. For a given level of equity and borrowing rate, imprudent intermediaries buy larger portfolios, consequently earn large payoffs in case of success, which goes towards raising value (see the second term in the right hand side of both expressions); however, they also risk losing their equity (with probability  $1 - p$ ), which tends to reduce value for any given initial equity stake (the first term). Comparing (14) and (18) and assuming that indifferent intermediaries behave prudently, we find that an intermediary engages in imprudent behaviour whenever its equity state is sufficiently low, that is, if and only if:

$$e < \tilde{e}(r) \equiv \theta \left( pR_1^h - \left( \frac{1+p}{2} \right) r \right) \quad (19)$$

Equation (19) implies that a poorly capitalised intermediary, i.e., one with low equity stake and hence relatively little to lose in case of default, will engage in imprudent behaviour, while an intermediary with high shareholders' equity stake, and hence much to lose in case of default, will behave prudently. The implied value of an intermediary as a function of  $e$ , i.e.,  $V(e) = \max [V^*(e), V^{**}(e)]$  is depicted in Figure 1(b).

A key implication of (19) is that lower borrowing rates raise the cut-off equity level below which the intermediary chooses to behave imprudently. To further understand why this is the case, compare the impact of a marginal rise in  $r$  on  $V^*(e)$  and  $V^{**}(e)$  –that is, for each strategy, the loss in the intermediary's value associated with a rise in the face financing cost. Taking derivatives and using (14) and (18), we find these falls to be:

$$\begin{aligned} V_r^*(e) &= e - \theta (pR_1^h - r) = -b^*(r, e), \\ V_r^{**}(e) &= pe - \theta p (pR_1^h - pr) = -pb^{**}(r, e). \end{aligned}$$

These expressions follow from the envelop theorem and have a straightforward interpretation. For the prudent intermediary, who never defaults and hence always repays  $r$  per unit of debt, the loss in value associated with a marginal rise in  $r$  is its total amount debt,  $b^*(r, e)$ . For the imprudent intermediary, who only repays in state 1, the loss in value is the relevant amount of debt,  $b^{**}(r, e)$ , *times* the probability that it will actually be repaid,  $p$ . For a rise in  $r$  to lower the threshold  $\tilde{e}$ , it must be the case that  $V^*(e)$  increases more than  $V^{**}(e)$  for the marginal intermediary, i.e., that for whom  $V^*(\tilde{e}) = V^{**}(\tilde{e})$  (i.e., that intermediary must turn prudent, rather than imprudent, following a rise in the interest rate). It

must be the case that  $V_r^*(\tilde{e}) > V_r^{**}(\tilde{e})$  or, equivalently by using the two expressions above,  $b^*(r, \tilde{e}) < pb^{**}(r, \tilde{e})$ : a switch by the marginal intermediary from the prudent to the imprudent investment strategy must involve a sufficiently large increase in leverage –as is always the case in our economy.

### 3.2 Aggregate demand for funds

Our key assumption here is that while the distribution of equity levels is perfectly known by outside lenders, financial opaqueness prevents lenders from observing the equity levels any particular intermediary. Hence, lenders cannot condition the loan rate on the specific equity level of an intermediary, so that a single borrowing rate  $r$  applies to the entire market.<sup>10</sup>

Then, we may define

$$g(r; \epsilon) \equiv \int_0^{\tilde{e}(r)} f(e; \epsilon) de = F(\tilde{e}(r); \epsilon) \quad (20)$$

as the proportion of imprudent intermediaries in the economy at a given interest rate  $r$ . Note that  $g_r(r; \epsilon) = -\theta(1+p)f(\tilde{e}(r); \epsilon)/2 < 0$ , that is, a lower face interest rate raises the proportion of imprudent intermediaries in the economy by increasing the threshold equity level  $\tilde{e}(r)$ . Moreover, we have  $g_\epsilon(r, \epsilon) = F_\epsilon(\tilde{e}; \epsilon) < 0$ , that is, an increase in  $\epsilon$  lowers the proportion of imprudent intermediary (for any given value of the cut-off  $\tilde{e}(r)$ ).

The total demand for funds aggregates the leverage choices of individual intermediaries, appropriately weighted by their shares in the economy. It is thus given by:

$$B^d(r; \epsilon) = \int_0^{\tilde{e}(r)} b^{**}(r, e) dF(e; \epsilon) + \int_{\tilde{e}(r)}^{\epsilon_{\max}} b^*(r, e) dF(e; \epsilon).$$

Then, substituting (13) and (17) into the latter expression, using (20) and rearranging, the total demand for funds is found to be:

$$B^d(r; \epsilon) = \theta [pR_1^h - r(1 - (1-p)g(r; \epsilon))] - E. \quad (21)$$

In the  $(B, r)$  plane, the  $B^d(r; \epsilon)$  curve lies to the left the  $\hat{B}^d(r; \epsilon)$  curve, its first-best counterpart. This is because, for any given value of  $r$ , the risk-shifting equilibrium includes

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<sup>10</sup>We assume for simplicity that intermediaries are completely identical from the point of view of the lender. Our result carry over in a set-up with partially segmented market involving different groups of intermediaries, with the members of each group facing the same interest rate. What matters for our results is the presence of an unobserved residual heterogeneity in intermediaries' equity stake.

a nonnegative fraction of imprudent intermediaries, whose demand for debt is larger than that of prudent intermediaries at any given interest rate  $r$  (see Figure 2(b)).

There are two properties of the aggregate demand for loanable funds that are worth discussing at this stage. First, it is continuous and decreasing in the borrowing rate, i.e.,

$$B_r^d(r; \epsilon) = -\theta(1 - (1 - p)g(r; \epsilon)) + \theta(1 - p)rg_r(r; \epsilon) < 0.$$

Two factors contribute to make the demand for funds a downward-sloping function of  $r$ . First, a lower interest rate raises the leverage of both prudent and imprudent intermediaries –see the optimal investment rules (12) and (16). Second, a lower interest rate induces “marginal” intermediaries (those which are close to the cut-off equity level  $\tilde{\epsilon}$  in (19)) to switch from prudent to imprudent behaviour, and those experience a discontinuous increase in their leverage –again, by (12) and (16). Hence, changes in the borrowing rate affect the “intensive” (i.e., conditional on not switching behaviour) and “extensive” (i.e., the number of intermediaries who switch behaviour) leverage margins in the same direction.

The second relevant property of the curve is that, holding  $r$  constant,  $B^d$  increases as the distribution of equity shifts leftwards. That is,

$$B_\epsilon^d(r; \epsilon) = \theta r(1 - p)g_\epsilon(r; \epsilon) < 0.$$

This is because, as the equity level of intermediaries decreases, some of them switch from prudent to imprudent behaviour. As imprudent intermediaries choose higher leverage than prudent ones, this composition effect translate into an upward shift in the aggregate demand for funds.

### 3.3 Aggregate supply of funds

The aggregate supply of funds depends on the expected return on loans,  $\rho$ , which under risk shifting not only depends on the face borrowing rate but also on both the share of imprudent intermediaries and the probability that they go bankrupt. In state 1, which occurs with probability  $p$ , all intermediaries repay the face interest rate  $r$  to the lenders: prudent intermediaries because they are always able to, imprudent ones because their risky bets turned out to be successful. In state 2, which occurs with complementary probability, only prudent intermediaries, which are in number  $1 - g(r; \epsilon)$ , are able to repay  $r$ . Imprudent intermediaries’ bets, on the contrary, turn out to be unsuccessful, leaving lenders with no

repayment at all. Summing up unit repayments across states and intermediaries types and rearranging, we find the ex ante gross return on loans to be:

$$\begin{aligned}\rho(r; \epsilon) &= p_1 r + (1 - p_1) (1 - g(r; \epsilon)) r \\ &= r (1 - (1 - p_1) g(r; \epsilon)),\end{aligned}\tag{22}$$

Note that this ex ante return is strictly increasing in the face interest rate, i.e.,

$$\rho_r(r; \epsilon) = 1 - (1 - p) g(r; \epsilon) - (1 - p) r g_r(r; \epsilon) > 0.\tag{23}$$

The increasingness of  $\rho(r; \epsilon)$  with respect to  $r$  occurs for two reasons. First, a higher face interest rate increases intermediaries' repayment if they do not default (the  $1 - (1 - p) g(r; \epsilon) > 0$  part of (23)). Second, a higher face interest rate favours prudent rather than risky behaviour by raising the threshold  $\tilde{e}$ , and hence by lowering the probability of default on a loan unit (the  $-(1 - p) g_r(r; \epsilon) > 0$  part). It follows that for  $(\epsilon, w)$  given the loan supply function is a nondecreasing, continuous function of  $r$ , which we may express as:

$$B^s(r; \epsilon, w) = w - u'^{-1}(r(1 - (1 - p) g(r; \epsilon))).\tag{24}$$

Let us briefly summarise the properties of the aggregate supply curve, before we analyse the equilibrium in the market for loanable funds. First,  $B^s(r; \epsilon, w)$  is strictly increasing in  $r$ , holding  $(\epsilon, w)$  constant; this follows from (5), the strict concavity of  $u(\cdot)$ , and the strict monotonicity of  $\rho$  w.r.t.  $r$  (see (23)). Second, from (5) it is strictly increasing in  $w$ , holding  $r$  and  $\epsilon$  constant. Third, it is increasing in  $\epsilon$ , holding  $r$  and  $w$  constant. The reason for this is that a higher overall level of equity in the economy raises the number of prudent intermediaries (i.e.,  $g_\epsilon(r, \epsilon) < 0$ ), and hence the expected return on loans (see (22)).

In the  $(B, r)$  plane, the  $B^s(r; \epsilon, w)$  curve lies to the left of its first-best analogue,  $\hat{B}^s(r; \epsilon, w)$ . This is because in the equilibrium with risk shifting lenders expect a nonnegative fraction of intermediaries to go bankrupt if state  $s_2$  occurs. Hence, any given value of the face interest rate  $r$  is associated with a lower expected return in the risk-shifting equilibrium than in the first-best –and hence with a lower supply of loanable funds (see Figure 2(b)).

### 3.4 Market clearing

In equilibrium, the total demand for funds by the intermediary sector must equal the total supply of funds provided by outside lenders. In other words, the face interest rate that clears

the market for loanable funds must satisfy

$$B^s(r; \epsilon, w) = B^d(r; \epsilon) \quad (25)$$

Since  $B^d(r; \epsilon)$  is continuously decreasing in  $r$  while  $B^s(r; \epsilon, w)$  is continuously increasing in  $r$ , the equilibrium is unique provided that it exists. Again, we are focusing on risk-shifting equilibria in which all intermediaries are leveraged, the conditions under which this is the case being summarised in the following lemma.

**Lemma 2.** Assume that i.  $\theta pR_1^h > e_{\max}$  and ii.

$$w > \max \left[ u'^{-1} (pR_1^h - E/\theta), \theta \bar{r} (1 - p) g(\bar{r}; \epsilon) - E + u'^{-1} (\bar{r} (1 - (1 - p) g(\bar{r}; \epsilon))) \right],$$

where  $\bar{r} = pR_1^h - e_{\max}/\theta$ . Then, the equilibrium with risk shifting is unique and such that  $b^*(e)$  and  $b^{**}(e)$  are positive fall all  $e \in [0, e_{\max}]$ .

To summarise, the equilibrium is well behaved provided that lenders' income,  $w$ , is sufficiently large. The existence conditions stated in Lemma 2 are slightly more stringent than those stated in Lemma 1, so the former also ensure the existence of the first-best outcome characterised in Section 2.3.

The equilibrium in the market for loanable funds is depicted in Figure 2(b). The intersection of the two curves gives the equilibrium contracted loan rate  $r$ , given the exogenous parameter set  $(\epsilon, w)$ . The loan rate in turn determines the equilibrium share of imprudent intermediaries  $g(r; \epsilon)$  (by equation (20)), as well as the equilibrium expected return on loans to intermediaries,  $\rho(r; \epsilon)$  (by (22)). Note that despite differences in the implied equilibrium interest rate in the two economies, the equilibrium amount of aggregate lending is the same. Indeed, the interest rate in the first-best equilibrium satisfies  $\theta (pR_1^h - \hat{r}) - E = w - u'^{-1}(\hat{r})$ , while the expected rate of return in the risk-shifting equilibrium satisfies  $\theta [pR_1^h - \rho] - E = w - u'^{-1}(\rho)$ . This implies that  $\rho = \hat{r}$  (i.e., lenders' expected compensation for their loans in the same across the two equilibria), so that  $B^s(\rho; w) = \hat{B}^s(\hat{r}; w)$  (i.e., they lend the same amount). From (22) and the fact that  $\rho = \hat{r}$ , we find the interest rate premium generated by the presence of imprudent intermediaries to be:

$$\frac{r}{\hat{r}} = \frac{g(r; \epsilon)}{(1 - p)^{-1} - g(r; \epsilon)},$$

which is positive and increasing in the both the number of such intermediaries,  $g(r; \epsilon)$ , and the probability that they go bust,  $1 - p$ .

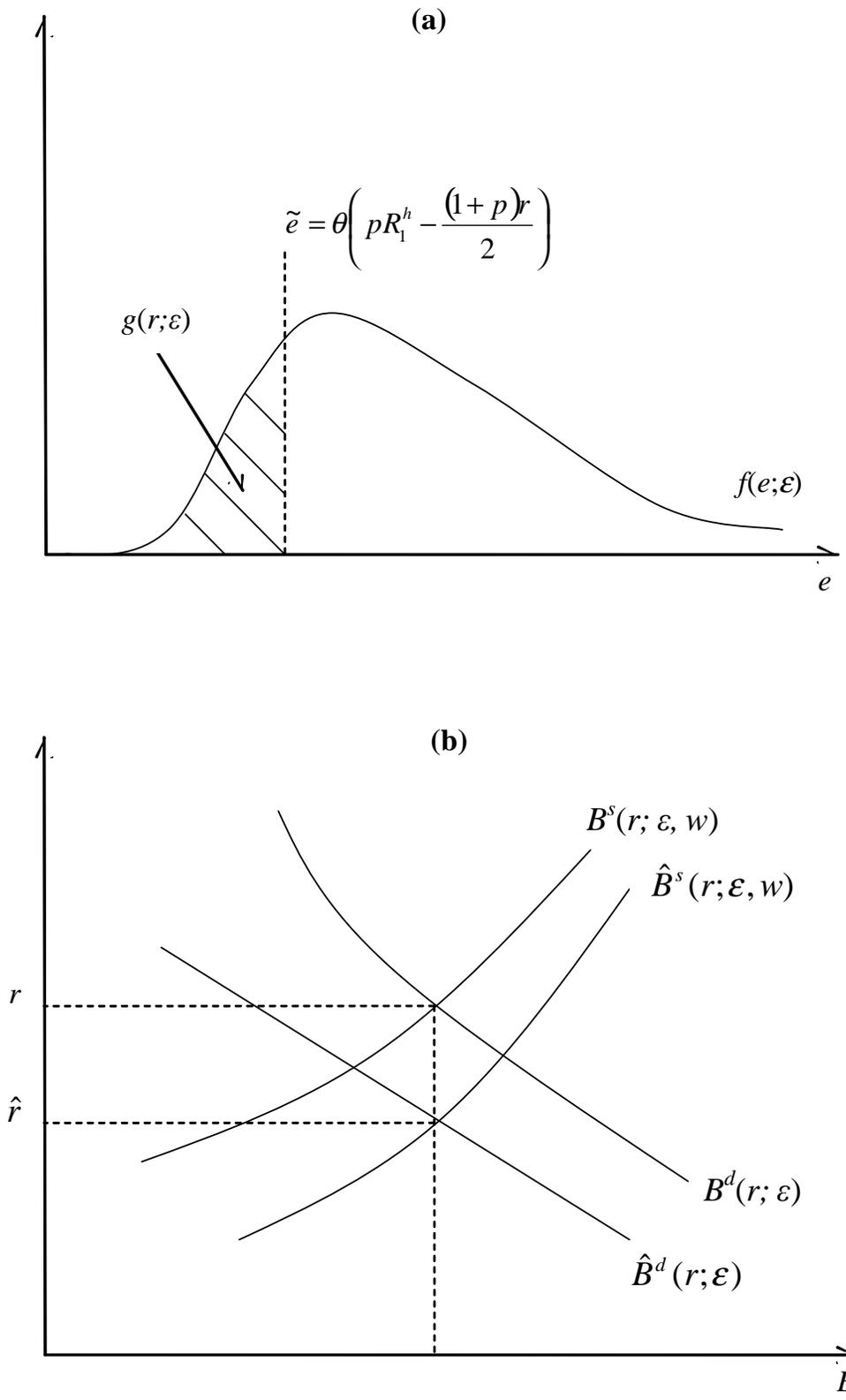


Figure 2. Share of imprudent intermediaries (a) and loanable funds equilibrium (b).

## 4 Impact of aggregate shocks

We may now state the main predictions of the model about how shifts in the underlying fundamentals (the supply of funds and the distribution of intermediaries' capital) affect the three key equilibrium variables,  $r$ ,  $\rho(r; \epsilon)$  and  $g(r; \epsilon)$ . These predictions are summarised in the following propositions.

**Proposition 1 (Lending boom).** An exogenous increase in the supply of funds (i.e.,  $dw > 0$ ) i) lowers the equilibrium contracted rate,  $r$ , ii) lowers the expected return on loans,  $\rho(r; \epsilon)$ , and iii) raises the share of imprudent intermediaries in the economy,  $g(r, \epsilon)$ .

Proposition 1 essentially states that easier financing conditions for intermediaries tend to fuel systemic risk by inducing an increasing number of intermediaries to take larger and riskier bets; conversely, tighter credit raises the interest rate and discipline banks' risk-taking behaviour. The effect of the boom shift in the funds supply curve is depicted in Figure 3(a). More specifically, the boom is associated with a rightward shift in the  $B^s$  locus, whose direct effect is to lower the equilibrium contracted loan rate. Holding  $\epsilon$  constant, the new value of  $r$  is associated with a lower value of the equity cutoff  $\tilde{e}(r)$  in (19), so that an increasing number of intermediaries turn from prudent to imprudent –i.e.,  $g(r, \epsilon)$  rises. Both the lower value of  $r$  and the higher value of  $g(r, \epsilon)$  contribute to lower the expected return on loans,  $\rho(r; \epsilon)$ .

While our analysis remains formal, several interpretations may be given to the shift in credit supply leading to easier financing conditions. According to Bernanke (2005), for example, a supply-driven shift in funding occurred in the first half of the last decade due to recycled balance-of-payment surpluses from China and oil-exporting countries; in this interpretation, systemic risk in the U.S. was closely related to the “global imbalances” problem, which was itself rooted in the willingness of surplus countries to hoard wealth in the form of U.S. assets. Another view has it that exceptionally loose monetary policy leading to exceedingly low real interest rates in the wake of the 2001 recession in the U.S. would have given rise to a “risk-taking” channel of monetary policy, thereby fostering widespread systemic risk in the U.S. financial sector (see Taylor, 2009, Adrian and Shin, 2010, as well as Altunbas et al. (2010) for a survey and some evidence).<sup>11</sup> Be it the consequence of either or both, the

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<sup>11</sup>As argued by Obstfeld and Rogoff (2009), these two views are likely more complementary than substitutes.

model unambiguously predicts that falling interest rates raise risk-taking by an increasing number of banks and hence the economywide level of risk. Moreover, the model predicts that this increase in aggregate risk is rooted in changes in the portfolio choices of less capitalised intermediaries –i.e., those to the left of, but close to, the equity cutoff  $\tilde{e}(r)$ . This channel is consistent with the findings of Jiminez et al. (2010), who study the risk-taking behaviour of a panel of Spanish banks and find that falling short-term rates increase risk-taking low-capital banks (rather than the “average” bank.)

**Proposition 2 (Equity squeeze).** A downward shift in the distribution of equity (i.e.,  $d\epsilon < 0$ ) raises the equilibrium interest rate,  $r$ . If the elasticity of the credit supply with respect to  $\rho$  is sufficiently high, then it also raises the share of imprudent intermediaries,  $g(r, \epsilon)$ .

Proposition 2 reflects the three effects at work following a downward shifts in the distribution of equity. First, for a given value of the cut-off  $\tilde{e}$ , the shift directly increases the number of imprudent banks in the economy by lowering the stake of “marginal” intermediaries (i.e., those who are initially to the right of, but close to,  $\tilde{e}$ ); those intermediaries then discontinuously raise their leverage while engaging in imprudent behaviour (see (12) and (16)), thereby raising the demand for funds. Second, to the extent that this shift lowers the overall equity base of the intermediary sector,  $E$ , all intermediaries, which have a target portfolio size, seek to offset the loss in internal funding by external debt, again raising the economywide demand for funding. Both of these effects shift the  $B^d$ -curve rightwards and exert an upward pressure on the equilibrium borrowing rate,  $r$ . Third, this increase in the borrowing rate has a disciplining effect on the intermediary sector by shifting the cut-off equity level  $\tilde{e}$  leftwards. Hence, while the effect of the equity squeeze on the borrowing rate is not ambiguous, that on the share of imprudent intermediaries is. However, if the supply of funds is sufficiently elastic, the adjustment of the borrowing rate after the shock and its disciplining effect will be limited, causing  $g(r; \epsilon)$  to rise.

This situation is depicted in Figure 3(b). The initial distributional shift causes the  $g(r, \epsilon)$  curve to shift leftwards. The direct impact of higher risk (holding  $r$  fixed) is to lower the expected return on loans,  $\rho(r, \epsilon)$ , which in the  $(B, r)$  plane manifests itself as an exogenous reduction in lending (i.e., an inwards shift of the  $B^s$  curve). Finally, the increase in the demand for funding causes the  $B^d$ -curve to shift rightwards. If the supply of funds

is sufficiently elastic (that is, the slope of the  $B^s$  curve is sufficiently low), then the overall effect of the three shifts is to raise the equilibrium value of  $g(r; \epsilon)$

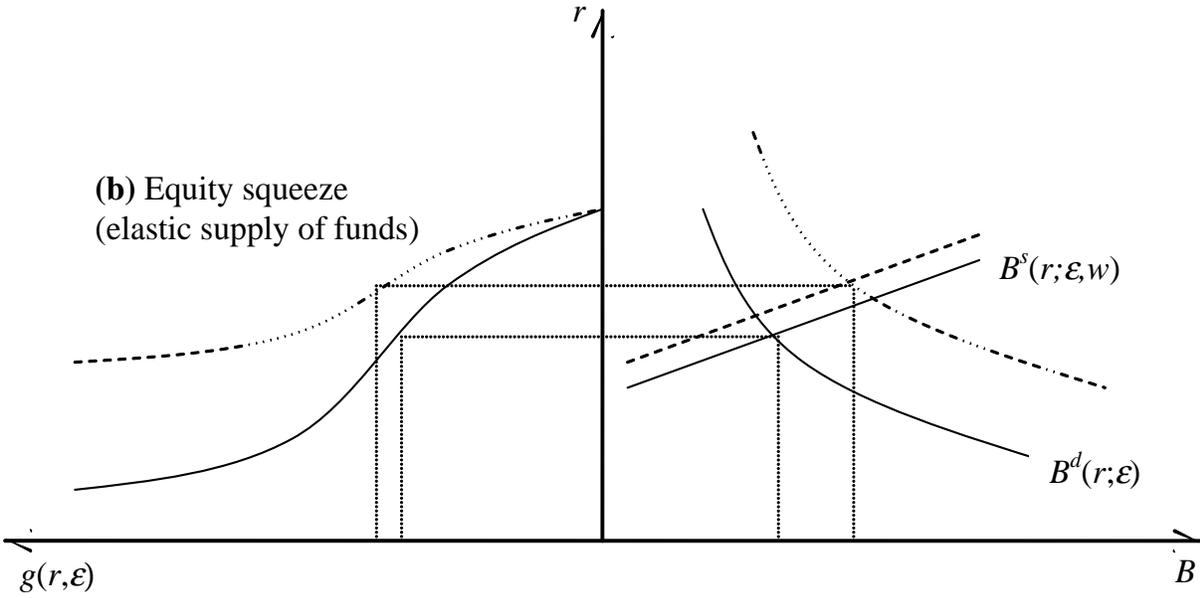
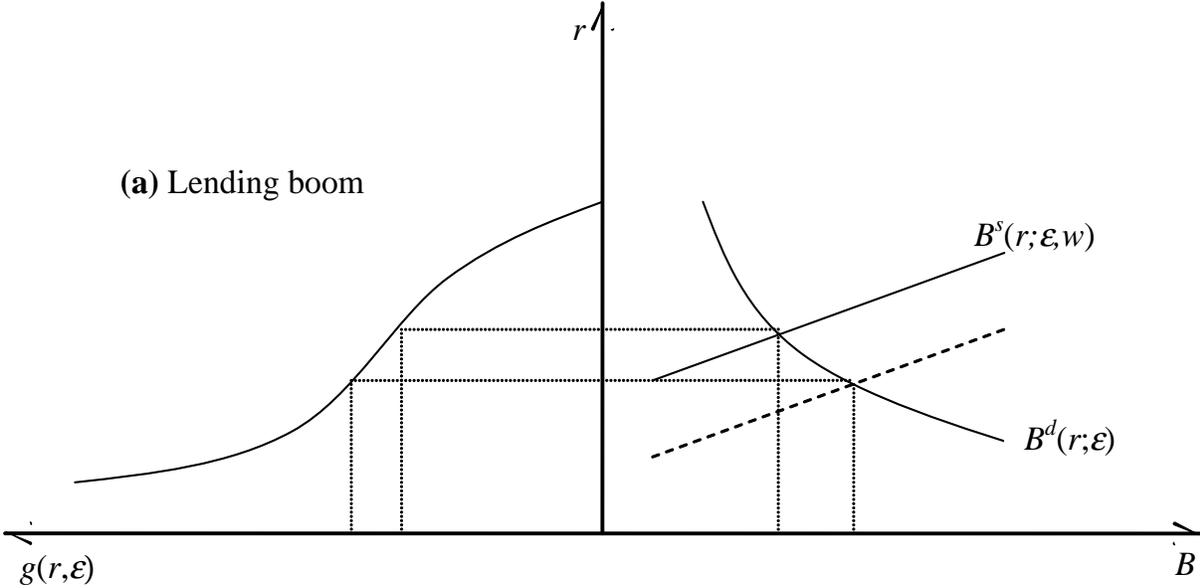


Figure 3. Impact of a lending boom (a) and a equity squeeze (b) on the interest rate,  $r$ , and the share of imprudent intermediaries,  $g(r; \epsilon)$ .

## 5 Concluding remarks

In this paper, we have analysed the portfolio and leverage choices of limited-liability intermediaries and their implications for the level of aggregate risk and the way it responds to changes in economic conditions. The novelty of our framework relative to earlier analysis of intermediaries' risk-shifting behaviour is twofold. First, we emphasise the disciplining role of shareholders' inside equity stake and the heterogeneities that it implies for their equilibrium balance sheets –both on the asset and liability sides. Second, and relatedly, we explicitly model changes in economywide risk-shifting along the “extensive margin” –i.e., due to changes in the number of intermediaries endogenously choosing to expose themselves to the risk of default–, in addition to the usual intensive margin –i.e., related to changes in their individual balance-sheet choices.

A important property of the model's equilibrium is that it jointly determines the (common) borrowing rate faced by intermediaries and the level of aggregate risk in the economy, due to the endogenous sorting of intermediaries along the equity dimension. Unsurprisingly, intermediaries with low shareholders' stake are more likely to behave imprudently than those with high inside equity stake. More interestingly, the sorting of intermediaries is itself affected by the interest rate, with falling interest associated with a rising number of imprudent intermediaries and aggregate risk. For this reason, exogenous factors that affect the market for loanable funds (e.g., international capital flows) have a direct impact on the level of risk generated by the financial sector. Similarly, exogenous changes in the distribution of intermediaries' capital affect the equilibrium interest rate, aggregate risk, and the return that ultimate lenders can expect from entrusting the financial sector with their funds.

While we have focused on two specific financial fragility channels (the risk-taking channel of low interest rates and the gambling-for-resurrection channel of falling equity), our model could be elaborated further to analyse the impact on intermediaries risk taking of other changes in macroeconomic conditions. For example, it is frequently argued that booms are times of low risk aversion, thereby affecting investors' portfolio choices (e.g., Bernanke and Kuttner, 2005; Campbell and Cochrane, 1999). Analysing the impact of changes in risk aversion for intermediaries' risk taking would require departing from the risk neutral assumption, with nontrivial implications for both intermediaries' choices and the implied aggregate welfare.

## Appendix

**Proof of Lemma 1.** From (??), the best capitalised intermediary (i.e., that for whom  $e = e_{\max}$ ) has positive leverage if and only if the equilibrium interest rate is not too low, i.e.,  $r < \bar{r} = pR_1^h - e_{\max}/\theta$ ; were inequality i. not to hold, then no positive equilibrium interest rate would satisfy this property. Moreover,  $\hat{B}^s(r; w) > 0$  provided that  $r > \underline{r} = u'(w)$ . Hence, the relevant domain of  $r$  is  $(\underline{r}, \bar{r})$ , and we need  $\underline{r} < \bar{r}$ , that is,  $u'(w) < pR_1^h - e_{\max}/\theta$ . For the two curves to cross, we need  $\hat{B}^d(\underline{r}; w) = \theta(pR_1^h - u'(w)) - E > \hat{B}^s(\underline{r}; w) = 0$  and  $\hat{B}^d(\bar{r}; w) = e_{\max} - E < \hat{B}^s(\bar{r}; w) = w - u'^{-1}(pR_1^h - e_{\max}/\theta)$ . Assumption ii. ensures that both are satisfied (and hence that  $\underline{r} < \bar{r}$ ).

**Proof of Lemma 2.** As before we need  $r < \bar{r} = pR_1^h - e_{\max}/\theta$  for all intermediaries (including the best capitalised amongst the prudent ones) to be leveraged, so inequality i. is required for this to be possible at positive interest rate levels. Lending is positive whenever  $B^s(r; \epsilon, w) = w - u'^{-1}(\rho) > 0$ , that is, using (22), whenever

$$\rho = r(1 - (1 - p)g(r; \epsilon)) > \underline{\rho} = u'(w).$$

Letting  $\underline{r}'$  solve  $\underline{r}'(1 - (1 - p)g(\underline{r}'; \epsilon)) = u'(w)$ , the relevant domain of  $r$  is now  $(\underline{r}', \bar{r})$ , where  $\underline{r}' > \underline{r}$ . For the two curves to cross, the following conditions must be satisfied:

$$\begin{aligned} B^d(\underline{r}'; \epsilon) &= \theta(pR^h - \underline{\rho}) - E > B^s(\underline{r}'; \epsilon, w) = w - u'^{-1}(\underline{\rho}) = 0, \\ B^d(\bar{r}; \epsilon) &= \theta\bar{r}(1 - p)g(\bar{r}; \epsilon) - E < B^s(\bar{r}; \epsilon, w) = w - u'^{-1}(\bar{r}(1 - (1 - p)g(\bar{r}; \epsilon))). \end{aligned}$$

Solving both conditions for  $w$ , we obtain:

$$\begin{aligned} w &> u'^{-1}(pR^h - E/\theta), \\ w &> \theta\bar{r}(1 - p)g(\bar{r}; \epsilon) - E + u'^{-1}(\bar{r}(1 - (1 - p)g(\bar{r}; \epsilon))), \end{aligned}$$

which is the second condition of Lemma 2.

**Proof of Proposition 1.** First, substitute (22) into (21) so as to write (25) as follows:

$$B^s(\rho(r; \epsilon); w) + \theta\rho(r; \epsilon) = p\theta R_1^h - E. \quad (26)$$

Total differentiating (26) whilst holding  $\epsilon$  constant, we find:

$$\frac{dr}{dw} = -\frac{B_w^s(\rho(r; \epsilon), w)}{[\theta + B_\rho^s(\rho(r; \epsilon), w)]\rho_r(r; \epsilon)} < 0, \quad (27)$$

which in turn implies, by the properties of  $\rho(r, \epsilon)$  and  $g(r, \epsilon)$ :

$$\begin{aligned}\frac{d\rho(r, \epsilon)}{dw} &= \rho_r(r; \epsilon) \frac{dr}{dw} < 0, \\ \frac{dg(r, \epsilon)}{dw} &= g_r(r, \epsilon) \frac{dr}{dw} > 0.\end{aligned}$$

**Proof of Proposition 2.** Total differentiating (26) again but now holding  $w$  rather than  $\epsilon$  constant, we find

$$\frac{dr}{d\epsilon} = - \frac{[\theta + B_\rho^s(\rho(r; \epsilon); w)] \rho_\epsilon(r; \epsilon) + E_\epsilon}{[\theta + B_\rho^s(\rho(r; \epsilon); w)] \rho_r(r; \epsilon)}.$$

From (22), we have  $\rho_\epsilon(r; \epsilon) = -r(1-p)g_\epsilon(r; \epsilon) > 0$  (i.e., an increase in  $\epsilon$  raises the return on loans by inducing more intermediaries to adopt a prudent behaviour), while  $E_\epsilon = \frac{\partial}{\partial \epsilon} \int_0^{\epsilon_{\max}} ef(e; \epsilon) di \geq 0$ . Hence, the numerator in the ratio is positive. Since the denominator is also positive, we have  $dr/d\epsilon < 0$ , i.e., a downward shift in the economywide equity base raises the equilibrium interest rate (as intermediaries seek to substitute equity for debt).

The equilibrium share of imprudent intermediaries is  $g(r; \epsilon)$ , and from above we have

$$\frac{dg(r; \epsilon)}{d\epsilon} = g_r(r; \epsilon) \frac{dr}{d\epsilon} + g_\epsilon(r; \epsilon). \quad (28)$$

The direct effect of  $\epsilon$  on  $g(r; \epsilon)$ , as measured by  $g_\epsilon(r; \epsilon)$  and holding  $r$  constant, is negative –i.e., a leftward shift in the distribution of equity raises the number of imprudent banks. The ambiguity in the overall response of  $g(r; \epsilon)$  comes from the reaction of the equilibrium interest rate. We have just seen that  $dr/d\epsilon < 0$ , and we know from (20) that  $g_r(r; \epsilon)$ , hence  $g_r(r; \epsilon) dr/d\epsilon > 0$  –i.e., a leftward shift in the distribution of equity raises the aggregate demand for funds, which raises the interest rate and ultimately disciplines bank behaviour.

The second part of the proposition can be established by continuity with the limiting case, where  $B^s(\rho(r; \epsilon); w)$  is infinitely elastic to changes in  $\rho(r; \epsilon)$ . In this case,  $\rho(r; \epsilon)$  is constant, so that total differencing it and using (22), we obtain:

$$\begin{aligned}\rho_r(r; \epsilon) dr + \rho_\epsilon(r; \epsilon) d\epsilon = \\ [1 - (1-p)g(r; \epsilon) - (1-p)rg_r(r; \epsilon)] dr - r(1-p)g_\epsilon(r; \epsilon) d\epsilon = 0,\end{aligned}$$

or

$$\frac{dr}{d\epsilon} = \frac{rg_\epsilon(r; \epsilon)}{(1-p)^{-1} - g(r; \epsilon) - rg_r(r; \epsilon)}.$$

Finally, substituting this latter expression into (28) gives:

$$\frac{dg(r; \epsilon)}{d\epsilon} = \frac{g_\epsilon(r; \epsilon) [(1-p)^{-1} - g(r; \epsilon)]}{(1-p)^{-1} - g(r; \epsilon) - rg_r(r; \epsilon)}.$$

Since  $g_\epsilon(r; \epsilon) < 0$  while  $(1-p)^{-1} > 1 \geq g(r; \epsilon)$ , the numerator is necessarily negative. By (23), the denominator is positive, so  $dg(r; \epsilon)/d\epsilon < 0$ . By continuity, if the supply of funds is sufficiently elastic then a leftward shift in the distribution of equity raises  $g(r; \epsilon)$ .

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