Fiscal Policy in a Tractable Liquidity-Constrained Economy*

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Abstract

In this paper, we analyse the effects of transitory fiscal expansions when public debt is used as liquidity by the private sector. Aggregate shocks are introduced into a tractable flexible-price, incomplete-market economy where heterogenous, infinitely-lived agents face occasionally binding borrowing constraints and store wealth to smooth out idiosyncratic income fluctuations. Debt-financed increases in public spending facilitate self-insurance by bond holders and may crowd in private consumption. The implied higher stock of liquidity also loosens the borrowing constraints faced by firms, thereby raising labour demand and possibly the real wage. Whether private consumption and wages actually rise or fall ultimately depends on the relative strengths of the liquidity and wealth effects that arise following the shock. The expansionary effects of tax cuts are also discussed.

Keywords: Borrowing constraints; public debt; fiscal policy shocks.

JEL codes: E21; E62.

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Introduction

In this paper, we analyse the effects of transitory fiscal expansions when public debt is used as liquidity by the private sector. We conduct this analysis in an incomplete-market model where agents face uninsurable idiosyncratic income risk and have limited ability to borrow against future income (i.e., markets are 'liquidity-constrained' in the terminology of Kehoe and Levine, 2001, amongst others). Non-Ricardian models of this type have on occasion been used to analyse the aggregate and welfare effects of public debt in the steady state (see Woodford, 1990; Aiyagari and McGrattan, 1998).[1] To date, there have been surprisingly few attempts at clarifying how such economies respond to aggregate fiscal *shocks*. One important contribution is Heathcote (2005), who offers a quantitative assessment of the effect of tax cuts. In this paper, we attempt to characterise analytically and qualitatively the impact and dynamic effects of government spending shocks on macroeconomic aggregates.

The spending shocks of which we analyse the effects have one significant, and realistic, feature: they are at least partly financed by government bond issues in the short run, with public debt then gradually reverting to some long-run target value thanks to future tax increases.[2] Note that whether government spending is financed by taxes or debt does not matter in complete-markets, Ricardian economies with lump-sum taxation, because households' discounted disposable income flows are identical between alternative modes of government financing. Then, under reasonable assumptions about preferences and technology, the negative wealth effects associated with transitory spending shocks lead to falls in the demand for both private consumption and leisure, which in turn produces a drop in the real wage (e.g., Baxter and King, 1993).

The deficit financing of spending shocks can, however, have very different consequences

[1] Other important applications of the liquidity-constraint paradigm to macroeconomic issues include Bewley-type monetary models (e.g., Bewley, 1983; Scheinkman and Weiss, 1986), models of capital accumulation with precautionary savings (e.g., Aiyagari, 1994; Huggett, 1997), models of business cycles with heterogenous agents (e.g., Krusell and Smith, 1998) and asset-pricing models with borrowing constraints and short-sales constraints (e.g., Heaton and Lucas, 1996; Krusell and Smith, 1997).

[2] For example, Blanchard and Perotti (2002) document a limited impact response of taxes to spending shocks in the U.S., implying deficit financing in the short run. Bohn (1998) established that the U.S. debt-GDP ratio is mean-reverting due to the corrective action of the primary surplus.

when public debt is used as private liquidity, that is, as a store of value held by agents for precautionary, or 'self-insurance', purposes. Starting from a situation in which liquidity is scarce (in a sense that we specify below), such policies have the side effect of increasing the stock of assets available in the economy, thereby facilitating self-insurance by bond holders and effectively relaxing the borrowing constraints faced by households and firms. As we show, the liquidity effects associated with rising public debt tend to foster households' private consumption demand, along with the labour demand of borrowing-constrained firms. Whether and when such liquidity effects may offset wealth effects, and thus overturn the predictions of the complete-markets model regarding the effects of spending shocks on private consumption and wages, is the central theme of this paper.

It is perhaps surprising that the actual impact of our fiscal experiment is still subject to so much empirical controversy. In particular, the application of different identification strategies to U.S. data has either supported the Real Business Cycle prediction of a fall in private consumption and wages following an increase in public spending (Ramey and Shapiro, 1998; Ramey, 2009), or come to the opposite conclusion that both variables actually increase after the shock (e.g., Blanchard and Perotti, 2002; Perotti, 2007), which latter is consistent with the Old Keynesian model and with a version of the New Keynesian model endowed with a sufficient number of market imperfections (Gali *et al.*, 2007). Given this lack of consensus, our goal here is not to take any definitive position as to whether an adequate fiscal policy model should generate pro- or counter-cyclical responses of those variables to public spending shocks. Rather, we use our model to illustrate that both outcomes are theoretically possible (and not implausible quantitatively), depending on the relative strengths of the liquidity and wealth effects that arise following the shock. As we show, which effect actually dominates crucially depends on how quickly the fiscal rule followed by the government ensures the reversion of public debt towards its long-run target following the initial fiscal deficit. If taxes rise promptly after the increase in public spending, then public debt will not vary very much and liquidity effects will be weak; in this situation, wealth effects are likely to be dominant and private consumption and wages will fall. If, on the contrary, the slow reaction of taxes leads to a substantial growth of public debt in the short and the medium run, then liquidity effects may be strong enough to dominate wealth effects, causing private consumption and wages to rise. Overall, temporary increases in public spending are all the more effective at raising output when the simultaneous response of taxes is limited.[3]

The market incompleteness-cum-borrowing constraint assumption is the only departure from the frictionless neoclassical model considered here, the other aspects of our model remain fully standard in a stripped-down form. In contrast to several recent contributions on the effect of public spending shocks, we thus assume that the labour and goods markets are perfectly competitive, that both nominal prices and wages are fully flexible, that utility is separable over time as well as over consumption and leisure at any point in time, that all agents are utility-maximising, that there are no externalities associated with public spending, and that taxes are lump sum.[4] Our model thus provides an example of an economy wherein the pro-cyclical responses of private consumption and wages after a fiscal expansion arises from the non-Ricardian nature of the model *alone*.

Our model belongs to the growing literature on the consequences of market incompleteness and borrowing constraints for fiscal policy outcomes. Woodford (1990) derived the optimal level of steady-state public debt in a deterministic model in which borrowing-constrained agents hold government bonds for precautionary purposes. This work was subsequently extended by Aiyagari and McGrattan (1998) to incorporate idiosyncratic uncertainty, and then by Floden (2001) to take into account government transfers. Heathcote (2005) introduced aggregate uncertainty about taxes into this framework, while our paper focuses the effects of aggregate uncertainty about public spending (for the first time, as far as we are aware.) Methodologically, our paper is closest in spirit to Woodford's in that we derive a tractable equilibrium with limited agents' heterogeneity (despite the presence of uninsurable income shocks), which allows us to summarise the behaviour of the model by a small-dimensional

[3] This latter result is, of course, not inconsistent with the Old Keynesian view about the effectiveness of fiscal policy (e.g., the textbook 'Keynesian cross' model). It is, however, grounded on a very different set of assumptions here.

[4] Recent fiscal policy models include Ravn *et al.* (2006), who assume imperfect competition together with habit formation over individual varieties of the consumption good, Linnemann (2006), who assumes that consumption and leisure are nonseparable while consumption is an inferior good, Linnemann and Shabert (2003), who have imperfect competition and sticky nominal prices, and Gali *et al.* (2007), who combine ad hoc 'hand-to-mouth' households with imperfect competition and price rigidities in both goods and labour markets. Papers analysing the effects of distortionary taxation in the neoclassical growth model include Ludvigson (1996) and Burnside *et al.* (2004), while Baxter and King (1993) consider the effects of government spending shocks when the latter generate external productivity effects.

dynamic system. While this approach arguably limits the quantitative scope of the model, it has a number of advantages. One is that the wealth and liquidity effects triggered by fiscal shocks can be disentangled analytically. Another one is that the model can handle continuous variations of the fiscal policy variables, so that our theoretical impulse-response functions can be compared directly to their empirical counterparts, and notably to the wealth of evidence from recent VAR studies.[5] While our focus here is on the impact of fiscal policy shocks, the construction of a tractable general equilibrium model with heterogenous agents may be of interest in other contexts.

Finally, Angeletos and Panousi (2009) recently analysed the effect of changes in government spending in an incomplete-market economy with idiosyncratic production risk. There are at least three important differences between their work and ours. First, they study an economy in which Ricardian equivalence holds, and hence in which there is no liquidity role for government bonds. Second, they focus on permanent spending shocks (i.e., changes in the size of the government), whereas our analysis is chiefly motivated by the recent empirical puzzles pertaining to the effect of transitory fiscal shocks. Third, in their model the wealth effects associated with higher taxes lower firms' labour demand and lead, under standard preferences, to a fall in both wages and private consumption. While such supply-side effects may arguably be at work after a permanent increase in public spending, our purpose here is to understand when and why transitory spending shocks may generate *pro-cyclical* private consumption, labour demand and wages.

The rest of the paper is organised as follows. Section 1 presents our general framework with both liquidity-constrained workers (who face idiosyncratic unemployment risk) and entrepreneurs (who meet project opportunities randomly). It derives the optimal behaviour of all agents, describes the government budget constraint and policies, and spells out the market-clearing conditions in the general case. Section 2 builds on this framework to construct a tractable equilibrium with liquidity-constrained workers; it notably discusses the importance of wealth and liquidity effects in determining the response of aggregates to fiscal shocks, examines their dynamic impact via impulse-response analysis, and carries out a

[5] E.g., Blanchard and Perotti (2002), Burnside *et al.* (2004), Caldara and Kamps (2008), Fatas and Mihov (2001), Favero and Giavazzi (2007), Gali *et al.* (2007), Mountford and Uhlig (2008), Perotti (2007) and Ramey (2009).

number of sensitivity checks. Section 3 studies the impact of fiscal shocks with liquidity-constrained entrepreneurs, looking more specifically into how liquidity effects affect entrepreneurs' labour demand and hence the equilibrium real wage. While much of this Section abstracts from unemployment risk, it ends by constructing a tractable equilibrium in which both workers and entrepreneurs interact and jointly determine the economy-wide demand for liquidity. Section 4 concludes.

1 The Model

The present Section introduces our general set-up with liquidity constraints and incomplete markets. The specific classes of equilibria on which we shall focus –together with their associated transmission channels for fiscal shocks– are specified further in Sections 2 and 3.

1.1 Households

The economy is populated by a unit mass of infinitely-lived households as well as by a government, all interacting in perfectly competitive goods, labour and credit markets. The mass of households is divided into two subclasses, *workers* and *entrepreneurs* (think of the latter as holding entrepreneurial skills that the former do not). Entrepreneurs are in (exogenous) proportion $\mu \in [0, 1]$ in the population. Workers can be employed or unemployed, while entrepreneurs may run a project or not. More specifically, households are subject to idiosyncratic (i.e., uncorrelated) changes of status, which are modelled as follows.

Workers. Workers face unemployment risk: the status of workers in the labour market randomly switches between "employment", a time during which they freely choose their labour supply, and "unemployment", a status during which they are excluded from the labour market. Every employed worker has a constant probability $\pi^e \in [0, 1)$ of staying employed in the next period, and every unemployed worker stays unemployed in the next period with probability $\pi^u \in [0, 1)$. From their second period of continuous unemployment onwards, unemployed workers become home producers and get the (constant) income $\kappa > 0$.[6]

[6] It is analytically simpler, but by no means essential, to assume that home production income is available after a one-period lag.

Entrepreneurs. The source of idiosyncratic uncertainty faced by entrepreneurs is the random arrival of project opportunities that require funding. More specifically, entrepreneurs oscillate between two statuses: they may run a project or not. Entrepreneurs running a project at time t do not supply labour but have access to a constant-returns-to-scale production function $y_{t+1}^i = l_t^{f,i}$, where y_{t+1}^i is the number of goods produced by entrepreneur i at date $t+1$ resulting from having hired $l_t^{f,i}$ units of labour at date t. When they do not run a project, entrepreneurs rent out labour to the market and, as do workers, freely choose their labour supply. These project opportunities arrive randomly at the constant rate $1-\theta \in (0,1]$, and last for $\tau \geq 1$ periods.[7]

The individual labour-income fluctuations that result from these idiosyncratic status changes are assumed to be entirely uninsurable (i.e., agents cannot issue assets contingent on their future employment status, and there are no unemployment benefits). In addition, households face a debt limit that bounds their asset wealth below at all times. To allow for some, but limited, debt issuance by households, we follow the literature on limited commitment (e.g., Kiyotaki and Moore, 1997) and assume that only a quantity $\delta \geq 0$ of goods is pledgeable to outside lenders, with borrowers being able to perfectly commit to repay up to δ. Denoting by R_t the (riskless) interest rate between date t and date $t+1$, this implies that lenders will agree to lend a maximum amount of δ/R_t to any particular borrower at date t, and that the private bonds resulting from this operation will be perfectly safe –and hence perfect substitutes for government bonds. This debt limit hampers the ability of households to use private borrowing and lending to fully insulate individual consumption from idiosyncratic income fluctuations. However, privately-issued assets (i.e., 'inside liquidity') compete with government bonds (or 'outside liquidity') in households' portfolio, and both will facilitate the formation of buffer-stock saving by individual households in equilibrium.[8]

[7] Our environment generates tractable equilibria when exit from unemployment is stochastic but not when the length of entrepreneurs' projects is. This is because the optimal behaviour of entrepreneurs running a project involves a Euler equation with interior solution (see (6) below), and hence stochastic length would asymptotically generate infinitely many entrepreneur types. However, projects can in principle have any finite, deterministic length.

[8] Our definitions of 'inside' versus 'outside' liquidities follows Farhi and Tirole (2009), among others. See also Holmström and Tirole (1998) on these two forms of liquidity supply.

The generic budget and non-negativity constraints of a typical household i are given by:

$$c_t^i + a_t^i + \xi_t^i w_t l_t^{f,i} = a_{t-1}^i R_{t-1} + \left(1 - \xi_t^i - \zeta_t^i\right) w_t l_t^i + \xi_{t-1}^i y_t^i + \left(\zeta_t^i \times \zeta_{t-1}^i\right) \kappa - T_t, \tag{1}$$

$$c_t^i \geq 0,\ l_t^i \geq 0,\ l_t^{f,i} \geq 0,\ a_t^i \geq -\delta/R_t. \tag{2}$$

In equation (1)-(2), c_t^i, l_t^i and $l_t^{f,i}$are the consumption demand, labour supply and labour demand of household i at date t, a_t^i denotes the total quantity of bonds held by household i at the end of date t, T_t is a (possibly negative) lump-sum tax collected on all households at date t, and R_{t-1} is the (riskless) gross interest rate on bonds from date $t-1$ to date t, and w_t is the date-t real wage.

ξ_t^i and ζ_t^i are two indicator variables that summarise both the occupation and the status of household i. More specifically, $\xi_t^i = 1$ if the household is an entrepreneur currently running a project and equals zero otherwise, while $\zeta_t^i = 1$ if the household is an unemployed worker and is zero otherwise. Indeed, when $\xi_t^i = 1$ (and hence $\zeta_t^i = 0$ since the two occupations are mutually exclusive), the household demands labour (for a total wage bill $w_t l_t^{f,i}$) but enjoys no labour income (so that $(1 - \xi_t^i - \zeta_t^i)\, w_t l_t^i = 0$); note also that an entrepreneur who was running a project in the previous period (i.e., one for whom $\xi_{t-1}^i = 1$) currently enjoys the entrepreneurial income y_t^i. On the other hand, a worker for whom $\zeta_t^i = 1$ enjoys no labour income, while one for whom $\zeta_t^i = 0$ (so that $1 - \xi_t^i - \zeta_t^i = 1$) enjoys labour income $w_t l_t^i$ (as does an entrepreneur not running a project). Finally, the term $\left(\zeta_t^i \times \zeta_{t-1}^i\right)\kappa$ summarises the fact that the home production quantity κ is earned from the second period of continuous unemployment onwards. The inequalities in (2) reflect both the feasibility constraints (i.e., non-negative consumption levels and labour demands and supplies) and the borrowing limit faced by all households.

Households are assumed to maximise the following intertemporal utility:

$$E_t \sum_{j=0}^{\infty} \beta^j \left(u\left(c_{t+j}^i\right) - l_{t+j}^i\right), \tag{3}$$

where $\beta \in (0,1)$ is the subjective discount factor and $u(c)$ is a twice continuously differentiable utility function satisfying $u'(c) > 0$, $u'(0) = \infty$, $u''(c) < 0$. Note that linearity in the disutility of labour is key in the construction of our equilibrium with limited heterogeneity. As will become clear below, the implied high elasticity of labour supply means that workers who have just left unemployment and entrepreneurs whose project has just come to an end are willing to work as much as necessary to instantaneously replete their precautionary wealth. If this were not the case, the labour supply and asset holdings of these households would depend on their entire idiosyncratic history and the number of agent types (and associated Euler equations) would be very large. We analyse the robustness of our result with respect to this assumption in Section 2.4 below, where we develop a variant of the model with inelastic labour supply and partial risk-sharing.

We may now characterise any household i's optimal plans. Let us start with the intratemporal labour supply choice first. Neither entrepreneurs running a project nor unemployed workers derive income from supplying labour. Hence, any household for whom $\xi_t^i = 1$ or $\zeta_t^i = 1$ chooses $l_t^i = 0$. For those who sell some of their labour endowment to the market (i.e., employed workers or entrepreneurs not running a project), equations (1)–(3) imply that their optimal labour supply is l_t^i satisfying:

$$w_t u'\left(a_{t-1}^i R_{t-1} + w_t l_t^i + \xi_{t-1}^i y_t^i - T_t - a_t^i\right) = 1. \tag{4}$$

Turning to the intertemporal optimality condition, (1)–(3) imply that the Euler equation summarising household i's optimal asset holdings, a_t^i, is given by:

$$\begin{aligned} &u'\left(a_{t-1}^i R_{t-1} + \left(1 - \xi_t^i - \zeta_t^i\right) w_t l_t^i + \xi_{t-1}^i y_t^i - T_t - a_t^i - \xi_t^i w_t l_t^{f,i}\right) \geq \\ &\beta R_t E_t u'\left(a_t^i R_t + \left(1 - \xi_{t+1}^i - \zeta_{t+1}^i\right) w_{t+1} l_{t+1}^i + \xi_t^i y_{t+1}^i + \zeta_{t+1}^i \zeta_t^i \kappa - T_{t+1} - a_{t+1}^i - \xi_{t+1}^i w_{t+1} l_{t+1}^{f,i}\right), \end{aligned} \tag{5}$$

with (5) holding with strict inequality if the borrowing constraint is binding (so that the corner solution $a_t^i = -\delta/R_t$ prevails), and with equality otherwise (in which case $a_t^i > -\delta/R_t$ is an interior solution).

The last relevant intertemporal choice in our model is that of entrepreneurs currently running a project (i.e., those for whom $\zeta_t^i = \zeta_{t+1}^i = 0$ and $\xi_t^i = 1$). Indeed, since their technology involves a production lag, these entrepreneurs trade off current consumption for current labour demand, which raises *future* production and consumption. Assuming that this choice is interior (which will always be the case under our assumed preferences and technology), we find that the optimal labour demand of entrepreneurs currently running a project, $l_t^{f,i}$, satisfies:

$$u'(a_{t-1}^i R_{t-1} + \xi_{t-1}^i y_t^i - T_t - a_t^i - w_t l_t^{f,i}) =$$
$$\beta w_t^{-1} E_t u'(a_t^i R_t + \left(1 - \xi_{t+1}^i\right) w_{t+1} l_{t+1}^i + l_t^{f,i} - T_{t+1} - a_{t+1}^i - \xi_{t+1}^i w_{t+1} l_{t+1}^{f,i}). \tag{6}$$

Comparing equations (5) and (6), we find that the borrowing constraint will be binding for these entrepreneurs if and only if:

$$1/w_t > R_t. \tag{7}$$

The interpretation of inequality (7) is straightforward. For entrepreneurs running a project, and given the production function $y_t^i = l_{t-1}^{f,i}$, any unit of funds used to raise labour inputs today will generate a payoff of $1/w_t$ in the next period. On the other hand, any unit of funds invested in bonds will yield R_t in the next period. Such entrepreneurs are borrowing-constrained if they never wish to hold assets, but instead would like to *borrow* as much as possible and to invest the borrowed funds in their own project (up to the point where the borrowing limit is reached). For this to be the case, the unit return on investing in the project must be higher than the unit borrowing cost, i.e., $1/w_t > R_t$.

1.2 Government

Let G_t and T_t denote government consumption and lump-sum taxes during period t, respectively, and B_t the stock of public debt at the end of period t. The government faces the budget constraint:

$$B_{t-1} R_{t-1} + G_t = B_t + T_t. \tag{8}$$

In equation (8), we think of transitory variations in G_t as being exogenously chosen by the government, of B_t as adjusting endogenously over time depending on the primary deficit and the equilibrium interest rate, and of T_t as obeying a fiscal rule with feedback from macroeconomic and/or fiscal variables. Following the observation by Bohn (1998) that the US debt-GDP ratio is stationary, we restrict our attention to rules ensuring that public debt reverts towards its (exogenous) long-run target B, at least asymptotically. Such rules, which exclude Ponzi schemes, are consistent with a wide variety of feedback mechanisms, including ones linking public debt to primary deficit as in Bohn (1998), output and debt to structural deficits (e.g., Gali and Perotti, 2003), as well as public debt and public spending to taxes (e.g., Gali *et al.*, 2007). Loosely speaking, stationarity requires that the tax feedback be sufficiently strong never to allow public debt to drift away from target forever.

Later on we shall illustrate the dynamics of the model in the context of a specific class of a fiscal rule and a shock process that satisfy this stationarity requirement. While our main focus is on the effects of government spending shocks, we will also study the impact of tax cuts, both for the sake of completeness and to compare the effectiveness of the two policies. The fiscal rule and shock processes that we consider are as follows:

$$T_t = T + \phi (B_t - B) - T_t^c, \tag{9}$$

$$G_t = \psi G_{t-1} + \epsilon_{1,t}, \; T_t^c = \chi T_{t-1}^c + \epsilon_{2,t}, \tag{10}$$

where T denotes steady-state taxes, B steady-state public debt (i.e., the long-run target), $\phi > 0$ and $(\psi, \chi) \in (0,1)^2$ constant parameters, T_t^c a transitory tax cut variable, and $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are innovations to public spending and tax cuts, respectively. Note that the qualitative properties of the model are robust to the inclusion of other feedbacks in (9) (e.g., from G_t to T_t), as well as to a lagged (rather than simultaneous) reaction of taxes to public debt. What matters for our results is the possibility that fiscal shocks may entail significant variations in the stock of public debt, at least in the short run.

Public debt will remain stationary as long as the policy parameter ϕ in (9) is sufficiently large.[9] Provided that this is the case, ϕ effectively indexes the way in which fiscal expansions are financed at various horizons. If ϕ is large, taxes rise quickly following a fiscal expansion, and public debt plays a relatively minor role in their short-run financing. Smaller values of ϕ, on the contrary, imply a muted short-run response of taxes and a more substantial role for public debt issuance in the short run; the ensuing rise in the stock of public debt then eventually triggers a rise in taxes in the medium run until the reversion of the public debt has been completed. Finally, the assumption that steady-state government consumption is zero in equation (10) is made for expositional clarity and entails no loss of generality; here it implies that in the steady state, tax revenues only just cover interest rate payments on public debt, i.e., $T = B(R-1)$.[10]

[9] For example, inequality (33) below ensures stationarity in the particular case where $\mu = \delta = 0$.

[10] The non-Ricardian nature of the model implies that $R-1$ may be negative if steady-state public debt,

1.3 Market clearing

There are two assets in the economy, public and private bonds and, as explained above, the two are perfect substitutes here. Then, denoting as $F_t(\tilde{a}, \zeta, \xi)$, with $\tilde{a} \in [-\delta/R_t, +\infty)$ and $(\zeta, \xi) \in \{0,1\} \times \{0,1\}$, the measure at date t of agents with beginning-of-period asset wealth $\tilde{a}$ and with current status summarised by (ζ, ξ), clearing of the bonds market requires that:[11]

$$\sum_{(\zeta,\xi)\in\{0,1\}^2} \int_{-\delta/R_t}^{+\infty} a_t dF_t(\tilde{a}, \zeta, \xi) = B_t. \tag{11}$$

This equality states that the sum of the bonds held by all agents at the end of date t adds up to the amount of public debt. Note that equation (11) reflects the fundamental difference between inside and outside liquidity from the point of view of the private sector. Namely, privately-issued assets enter individual wealth (i.e., a_t), but their quantity sums to zero since the private sector both issues and buys them. In contrast, government bonds are bought but not issued by the private sector. Thus, in the aggregate the private sector holds a net quantity of assets B_t.

Similarly, the labour market clears when total labour demand equals supply, i.e.,

$$\sum_{(\zeta,\xi)\in\{0,1\}^2} \int_{-\delta/R_t}^{+\infty} l_t dF_t(\tilde{a}, \zeta, \xi) = \sum_{(\zeta,\xi)\in\{0,1\}^2} \int_{-\delta/R_t}^{+\infty} l_t^f dF_t(\tilde{a}, \zeta, \xi) \equiv L_t. \tag{12}$$

Finally, denoting by Y_t aggregate output, the goods market clears if and only if:

$$\sum_{(\zeta,\xi)\in\{0,1\}^2} \int_{-\delta/R_t}^{+\infty} c_t dF_t(\tilde{a}, \zeta, \xi) + G_t = Y_t. \tag{13}$$

We may now define an *equilibrium* of our economy as a set of individual consumption levels, $\{c^i\}_{t=0}^{\infty}$, individual labour supplies and demands, $\{l_t^i, l_t^{f,i}\}_{t=0}^{\infty}$, individual bond holdings, $\{a_t^i\}_{t=0}^{\infty}$, and aggregate variables, $\{L_t, Y_t, B_t, R_t, w_t\}_{t=0}^{\infty}$ such that the optimality conditions (4)–(6) and the market-clearing conditions (11)–(13) hold for every agent and in every period, given the forcing sequence $\{G_t\}_{t=0}^{\infty}$ and a fiscal rule for T_t that ensures the stationarity of public debt.

B, is sufficiently low. In this case, the steady-state tax collection becomes a positive transfer of amount $-T$ (the bounds on R and the relation between R and B are detailed in Appendix A1).

[11] This formulation of the market-clearing conditions anticipates the recursive nature of our limited-heterogeneity equilibria, in which these conditions take very simple forms. See Heathcote (2005) for a general, non-recursive formulation.

1.4 Limited-heterogeneity equilibria

In general, uninsurable income uncertainty of the kind assumed here generates a very large number of household types, due to the dependence of current decisions on the household's entire history of individual shocks, and the distribution of types must be approximated numerically (e.g., Aiyagari, 1994; Heathcote, 2005). Here we focus on particular class of equilibria with a limited number of household types and a finite-state wealth distribution, allowing us to derive the model's dynamics in closed form. We construct these equilibria using a simple 'guess and verify' method based on two conjectures, and then derive sufficient conditions for both conjectures to hold in equilibrium once all their behavioural and market-clearing implications have been worked out. As stated in Propositions 1 and 2 below, the sufficient conditions for both conjectures to hold are that i) public debt trend-revert towards a sufficiently low long-run target, and ii) deviations of public debt from target be of limited magnitude.

The first conjecture (**C1**) is that the borrowing constraint is always binding for both unemployed workers and entrepreneurs who run a project. This is because the former expect to leave unemployment with positive probability in the next period, while the latter gather output from their current investment in the next period. Hence, both types face a rising income profile and, in the equilibria that we consider, exhaust the debt limit δ/R_t (i.e., they would like to *extend* borrowing beyond δ/R_t but are prevented from doing so). The second conjecture (**C2**) is that the borrowing constraint is never binding for labour-supplying households, which is to say, employed workers and entrepreneurs not currently running a project. This is because the former contemplate, and hence self-insure against, the possibility of falling into unemployment, while the latter hoard assets for future potential investment opportunities. In consequence, these households are willing to end the current period with non-negative asset wealth and hence to buy both government bonds and the assets issued by borrowing-constrained households. As we illustrate in the next sections, conjectures **C1**-**C2** together with the utility function (3) generically imply the existence of equilibria with finite-state, cross-sectional wealth distributions and hence with a finite number of agent types.

2 Fiscal policy shocks with liquidity-constrained workers

In the present section, we focus on the case where the only source of idiosyncratic uncertainty in the economy consists of unemployment risk. Consequently, we shut down the entrepreneurial sector (i.e., $\mu = 0$) and instead endow the economy with an external firm sector producing output with constant returns-to-scale technology $Y_t = L_t$ (so that $w_t = 1\ \forall t$). We first determine households' individual consumption, labour supply and asset holding rules under conjectures **C1-C2** (2.1). We then characterise the equilibrium that results from these rules and provide an existence proposition for our limited-heterogeneity equilibrium (2.2). Our next step is to derive the aggregate dynamics of the model under a number of specifications, highlighting in each case the central role of the dynamic liquidity effects triggered by fiscal shocks (2.3). Finally, we study a variant of the model which enables us to study how the elasticity of labour supply affects equilibrium outcomes (2.4).

2.1 Agent types

Consider first the consumption level of a worker who is unemployed both in the previous and the current period (and call this worker a 'uu worker'). Under conjecture **C1**, this worker left the previous period with asset wealth $-\delta/R_{t-1}$. At the end of the current period, this worker will have earned the home production income κ, repaid δ $(= (\delta/R_{t-1}) \times R_{t-1})$ to the lenders and, again by conjecture **C1,** renewed his debt up to the amount δ/R_t. We thus have:

$$uu : c_t^{uu} = \kappa - \delta + \delta/R_t - T_t. \tag{14}$$

Now consider the consumption level of a worker who is falling into unemployment in the current period. By definition this worker was employed, and thus unconstrained under **C2**, in the previous period, but is currently constrained under **C1**. Thus, for this worker equation (1) gives:

$$\left(\zeta_{t-1}^i, \zeta_t^i\right) = (0, 1) \Rightarrow c_t^i = a_{t-1}^i R_{t-1} - T_t + \delta/R_t, \tag{15}$$

where a_{t-1}^i is worker i's bond holdings inherited from the previous period and δ/R_t $(= -a_t^i)$ this worker's current debt. From (1)–(3), the intratemporal optimality condition for any employed household i imposes that the marginal rate of substitution between leisure and consumption be equal to the real wage, so that we obtain:

$$\zeta_t^i = 0 \Rightarrow c_t^i = c^e = u'^{-1}(1). \tag{16}$$

Any employed household stays employed in the next period with probability π^e and falls into unemployment with probability $1 - \pi^e$. Conjecture **C2** implies that employed households' consumption-savings plans are interior (i.e., $a_t^i > 0$ if $\xi_t^i = 1$) and, from (3), (15) and (16), that these plans obey the following Euler equation:

$$1 = \beta\pi^e R_t + \beta (1 - \pi^e) R_t E_t u'(a_t^i R_t - T_{t+1} + \delta/R_{t+1}). \tag{17}$$

The left-hand side of equation (17) is the current marginal utility of an employed household, $u'(c^e) = 1$. The first part of the right-hand side of (17) is the discounted utility of a marginal unit of savings if the household stays employed in the next period (in which case $u'(c_{t+1}^i) = u'(c^e) = 1$), while the second part is the marginal utility of the same unit when the household falls into unemployment in the next period (i.e., becomes unemployed, liquidates assets and, from equation (15), enjoys marginal utility $u'(c_{t+1}^i) = u'(a_t^i R_t - T_{t+1} + \delta/R_{t+1})$).

In equation (17), household i's current asset demand only depends on aggregate variables (R_t and T_t). The solution a_t^i to (17) is thus identical across employed households, and we can write:

$$\zeta_t^i = 0 \Rightarrow a_t^i = a_t^e \ (> 0) \ \forall i. \tag{18}$$

Equations (15) and (18) imply that workers currently falling into unemployment have identical asset holdings and consumption levels, so that we can write:

$$eu : c_t^{eu} = a_{t-1}^e R_{t-1} - T_t + \delta/R_t. \tag{19}$$

Employed workers can be of two different types, depending on whether or not they were employed in the previous period. Call the former 'ee workers' and the latter 'ue workers'. In the current period, ue workers consume c^e and save a_t^e. Moreover, since they were borrowing-constrained at date $t-1$ (by conjecture **C1**) and thus ended the previous period with debt δ/R_{t-1}, they must repay δ in the current period. Then, equations (1), (16) and (18) yield the labour supply of ue workers, l_t^{ue} (which is homogenous across such households) as the residual of the following equation:

$$ue : c^e + a_t^e = l_t^{ue} - T_t - \delta. \tag{20}$$

On the other hand, ee households consume c^e, save a^e_t, and enjoy the asset payoff $a^e_{t-1}R_{t-1}$. This also uniquely defines their labour supply, l^{ee}_t, through the equation:

$$ee : c^e + a^e_t = a^e_{t-1}R_{t-1} + l^{ee}_t - T_t. \tag{21}$$

To summarise, **C1** and **C2** imply that workers can be of four different types only (with budget constraints (14) and (19)–(21)), while the equilibrium wealth distribution is two-state (i.e., $a^i_t = a^e_t > 0$ or $-\delta/R_t \leq 0$). Note that it is almost sure, asymptotically, that any two randomly chosen workers have different individual income histories, due to the idiosyncratic nature of unemployment shocks. Nevertheless, under our conjectures workers' heterogeneity is limited by the fact that only last period's and current idiosyncratic shocks matter in determining workers' types. This is because, under **C1** and **C2,** i) workers falling into unemployment all liquidate their asset wealth and borrow δ/R_t, and ii) workers leaving unemployment adjust labour supply so as to reach their target level of precautionary wealth, a^e_t, instantaneously.[12] Given the assumed probabilities of changing employment status, the invariant proportions of each type of worker are:

$$\omega^{uu} = \frac{\pi^u(1-\pi^e)}{2-\pi^e-\pi^u},\ \omega^{eu} = \omega^{ue} = \frac{(1-\pi^e)(1-\pi^u)}{2-\pi^e-\pi^u} \text{ and } \omega^{uu} = \frac{\pi^u(1-\pi^e)}{2-\pi^e-\pi^u}, \tag{22}$$

and we denote the asymptotic unemployment rate by $\Omega = \omega^{uu}+\omega^{eu} = (1-\pi^e)/(2-\pi^e-\pi^u)$. For simplicity, we assume that the proportion of each type of worker is at the invariant distribution level from $t=0$ onwards (so that Ω is the unemployment rate at all dates).

2.2 Equilibrium

In our economy, only employed workers hold bonds, which are issued both by the government (to the amount B_t) and by the unemployed (for a total amount $\delta/R_t \times \Omega$). Given the distribution of workers types, the bond, labour and goods markets clearing conditions (11)–

[12] In reality individual asset depletion and repletion following changes in labour income are gradual rather than immediate. Our focus on a tractable analysis of aggregate fiscal shocks under incomplete markets and agents' heterogeneity requires that we abstract from this inertia in individual asset adjustments, except in Section 2.4 below where we analyse this issue explicitly. Of course, the individual wealth target itself, a^e_t, will vary over time following fiscal policy shocks.

(13) become:

$$(1-\Omega)\, a_t^e - \Omega\delta/R_t = B_t, \tag{23}$$

$$\omega^{ee} l_t^{ee} + \omega^{ue} l_t^{ue} = L_t, \tag{24}$$

$$(1-\Omega)\, c^e + \omega^{eu} c_t^{eu} + \omega^{uu} \left(c_t^{uu} - \kappa\right) + G_t = Y_t, \tag{25}$$

where in (25) $Y_t (= L_t)$ is production by the outside firm sector and $Y_t + \omega^{uu}\kappa$ is total output. Substituting (8), (16) and (23) into the Euler equation (17), we may write the relation between the interest rate and fiscal variables as follows:

$$1 = \beta R_t \left(\pi^e + (1-\pi^e)\, E_t u' \left(\frac{B_{t+1} - G_{t+1} + \Omega T_{t+1}}{1-\Omega} + \delta \left(\frac{\Omega}{1-\Omega} + \frac{1}{R_{t+1}} \right) \right) \right). \tag{26}$$

Note that when $\pi^e \to 1$ idiosyncratic uncertainty about labour income vanishes; the model then behaves in the same way as a (frictionless) Real Business Cycle model and $R_t \to 1/\beta$, the gross rate of time preference. We may now state the following existence proposition (the proof of which is found in Appendix A1).

Proposition 1. Assume that i) $\sigma(c) \equiv -cu''(c)/u'(c) \leq 1$, ii) π^u is small, iii) fluctuations of B_t around its steady-state value B are small, and iv) (B, δ) jointly satisfy:

$$0 < B + \beta\delta < \Sigma \equiv \frac{(1-\Omega)\, \beta u'^{-1}(1)}{(1-\Omega)\, \beta + \Omega}.$$

Then, the equilibrium with four worker types exists and has an interest rate R_t that is strictly lower than $1/\beta$ for all t.

In short, Proposition 1 indicates that our economy is liquidity-constrained if the stocks of public debt, as given by B in the steady state, and private debt, as indexed by δ, are both sufficiently low. In this case, the equilibrium interest rate is also low (relative to that prevailing in an unconstrained economy), due to the precautionary demand for bonds by high-income workers.[13] From here on, we shall proceed under the assumption that bonds are in limited supply at all dates, i.e., conditions iii) and iv) in Proposition 1 always hold, and we will make sure in our calibration exercises that π^u is sufficiently small for unemployed workers to be constrained –so that condition ii) also holds. Finally, condition i) is part of

[13] These properties essentially parallel those obtained by Woodford (1990) within a liquidity-constrained economy without inside liquidity and in which both aggregate and idiosyncratic uncertainties are shut down.

our set of sufficient conditions for the existence of a unique steady state, but it may be relaxed for particular ranges of parameters without compromising steady-state uniqueness. We illustrate this point below by performing sensitivity analysis with respect to $\sigma(c)$.

2.3 Liquidity versus wealth effects of fiscal expansions

In this section we begin by demonstrating how liquidity and wealth effects compete in determining the overall response of aggregate- and individual-level variables to fiscal shocks, and then illustrate the implied dynamic effects of these shocks under the fiscal rule (9).

Total consumption by employed households is $(1-\Omega)c^e$, while the total consumption of unemployed households is Ωc_t^u. Then, using (8), (19) and (23) and rearranging, total private consumption and total output may be written respectively as:

$$C_t = \Psi + (1-\pi^e)(B_t - G_t) + \Upsilon T_t + \Omega\delta R_t^{-1}, \tag{27}$$

$$Y_t = \Psi + (1-\pi^e)B_t + \pi^e G_t + \Upsilon T_t + \Omega\delta R_t^{-1}, \tag{28}$$

where $\Psi \equiv (1-\Omega)c^e + \Omega((1-\pi^e-\pi^u)\delta + \pi^u\kappa)$ and $\Upsilon \equiv \Omega(1-\pi^e-\pi^u)$ are constants.

These static, reduced-form equations provide a first insight into how liquidity effects alter the transmission of fiscal shock relative to that at work in the complete-markets model. To illustrate this point in the simplest possible manner, let us assume that $\delta = 0$ here and consider the following three prototypical fiscal experiments. Henceforth, we shall use hatted variables to denote level-deviations from the steady state (e.g., $\hat{B} = B_t - B$), and we will assume that all variables are at their steady-state values before the policy shock.

Fully debt-financed spending shock. Imagine first the effect of a purely transitory rise in public spending occurring at date t (of size $G_t > 0$) that is entirely financed by public debt (so that $\hat{T}_t = 0$ and $\hat{B}_t = G_t$), the implied increase in taxes necessary to satisfy the government's intertemporal budget constraint being left to some future periods. For concreteness, let us assume that this increase in taxes will take place only two periods after the policy change and will allow public debt to return to its steady state-level B at the end of date $t+2$ (i.e., $\hat{T}_{t+1} = 0$ and $\hat{T}_{t+2} > 0$ such that $\hat{B}_{t+2} = 0$). Equation (27) indicates that total private consumption does not change on impact (i.e., $\hat{C}_t = 0$), while by equation (28) $\hat{Y}_t = G_t$. Now looking one period ahead: by assumption, $\hat{G}_{t+1} = 0$ while $\hat{T}_{t+1} = 0$, which in turn implies that $\hat{C}_{t+1} = (1 - \pi^e)\,\hat{B}_{t+1}$. Then, using the government budget constraint (8) at dates t and $t+1$, we obtain:

$$\hat{C}_{t+1} = (1 - \pi^e)\,(B_t R_t - BR) = (1 - \pi^e)\,(B\hat{R}_t + R_t G_t).$$

Hence, unless the interest rate *falls* so much at the time of the policy impulse that the stock of public debt actually decreases, this policy generates a boom in private consumption one period after the shock. As we discuss later on, the 'crowding in' of private consumption by government spending occurs under much more realistic fiscal rules and policy changes. What is crucial here is the fact that public debt, which affects the stock of aggregate liquidity, is allowed to increase following the policy change; this increase raises the consumption level that agents hit by a bad idiosyncratic income shock can achieve, and hence raises aggregate consumption. The central role of public debt in this transmission channel is best understood when we look at the opposite situation of a full tax financing of the spending shock.

Fully tax-financed spending shock. Take exactly the same transitory increase in public spending, but assume instead that it is entirely financed by taxes (i.e., $\hat{T}_t = G_t$ and $\hat{B}_t = 0$), so that public debt never leaves its steady-state value. From equation (27), we have that

$$\hat{C}_t = (\Upsilon + \pi^e - 1)\,G_t\ (< 0),$$

with C_t returning to its steady-state value from date $t+1$ onwards. Hence, this policy triggers a drop in total private consumption. The cause of this decline is that such a policy does not change the aggregate amount of liquidity in the economy, and hence leaves workers' self-insurance possibilities unaffected. Consequently, the usual wealth effects dominate and lead to private consumption being crowded out by public spending –as in the baseline Real Business Cycle model.

Change in the timing of taxes. Finally, consider the textbook Ricardian experiment of a debt-financed cut in lump-sum taxes, financed by future tax increases, with the entire path of government consumption remaining at zero (so that $C_t = Y_t$ for all t). Again, for concreteness assume that this policy takes place at date t (i.e., $\hat{B}_t = -\hat{T}_t$) and that taxes will rise in the next period to ensure the reversion of public debt towards its steady state level (i.e., $\hat{T}_{t+1} > 0$ such that $\hat{B}_{t+1} = 0$). From equation (27)–(28), we have

$$\hat{C}_t = (1 - \pi^e - \Upsilon)\,\hat{B}_t\ (> 0),$$

so that the tax cut raises private consumption and output on impact. (Recall that this experiment would be neutral under Ricardian equivalence.) As we discuss further below, tax cuts leading to a persistently high stock of public debt also raise aggregate liquidity and workers self-insurance opportunities, which substantially *strengthens* the direct effect of the cut on the budget set of liquidity-constrained workers.

To obtain further insight into the underlying workings of these effects, we need to go beyond the reduced-form equations (27)–(28) and look at household-level variables, which describe how individual consumption (i.e. the private demand side of the model) and labour supply (the supply side) respond to fiscal shocks. The consumption of employed workers, c^e, is not affected by fiscal shocks. Now, substituting (23) into (19) we may write c_t^{eu} as follows:

$$c_t^{eu} = \underbrace{\frac{B_{t-1}R_{t-1}}{1-\Omega} + \frac{\Omega\delta}{1-\Omega}}_{\text{liquidated portfolio}} + \underbrace{\frac{\delta}{R_t}}_{\text{borrowing}} - \underbrace{T_t}_{\text{taxes}}. \tag{29}$$

The right-hand side of (29) is composed of four terms that all affect the consumption of eu workers. The sum of the first two terms is the total value of their liquidated portfolio in equilibrium, which depends on how much buffer-stock saving they were able to form in the previous period (as they were employed); this stock is affected by the quantity of outside liquidity in the economy (B_{t-1} in the $B_{t-1}R_{t-1}/(1-\Omega)$ term) as well as by that of inside liquidity, which depends on constrained workers' pledgeable income δ (the $\Omega\delta/(1-\Omega)$ term). Besides liquidating their asset portfolio, eu workers smooth consumption by borrowing (up to the debt limit δ/R_t). Finally, since these workers are borrowing-constrained, higher taxes reduce their attainable consumption level one for one (the $-T_t$ term). As we illustrate below, the interest rate responds positively to a rise in public debt. This in turn raises the liquidated value of workers' portfolio, but also generates some crowding out of private borrowing; hence, the economy's response to fiscal shocks will be smaller the higher is the share of private debt in the total stock of assets (that is, the higher is δ relative to B). The determinants of uu workers' consumption (see equation (14)) directly follows. Since they were constrained in the previous period (and hence liquidated their asset portfolio), they can only raise current consumption above home production by borrowing (up to δ/R_t). However, they must also repay δ to their creditors and pay the lump sum tax T_t.

Turning to the supply side of the model, we can substitute (16) and (23) into (20)–(21)

and write labour supply by employed households as follows:

$$l_t^{ee} = c^e + \frac{B_t - B_{t-1}R_{t-1}}{1-\Omega} + \frac{\Omega\delta}{1-\Omega}\left(\frac{1}{R_t} - 1\right) + T_t, \tag{30}$$

$$l_t^{ue} = c^e + \frac{B_t}{1-\Omega} + \frac{\Omega\delta}{(1-\Omega)}\frac{1}{R_t} + T_t. \tag{31}$$

Equations (30)–(31) show that labour supply responds not only to taxes, as is predicted by the standard complete-markets model, but also to the stock of liquidity that households acquire as self-insurance against unemployment risk. *ue* workers, who have just moved out of unemployment and have zero beginning-of-period wealth, will seize any extra opportunity to save by raising labour supply; *ee* workers, who are partly self-insured when they enter the current period, adjust their labour supply depending on the new stock of government and private bonds available for purchase relative to the current value of their previously-accumulated portfolio. In both cases, the growth of public debt that may result from higher public spending generates liquidity effects that *strengthen* the wealth effects on labour supply.

As is shown in Appendix A2, under (9)–(10), the behaviour of the model with liquidity-constrained workers can be approximated by a two-dimensional dynamic system with endogenous state vector $[R_t, B_t]$. To gain further insight into these dynamics, and notably about the role of ϕ in determining the stability of the system, it may be useful to look further into our baseline scenario, in which $\delta = 0$ (so that the only source of liquidity in the economy is from government bonds). When this is the case, the dynamics of the model become univariate and are summarised by the following linearised debt process (see Appendix A2 for details):[14]

$$B_t = (1-\gamma)B + \gamma B_{t-1} + \mu G_t + \nu G_{t-1} + \mu T_t^c + \upsilon T_{t-1}^c, \tag{32}$$

where G_t and T_t^c are given by (10), $\gamma > 0$, $\mu, \upsilon > 0$, $\nu < 0$ are constants that depend on the deep parameters of the model and the target debt level B, and where $\partial\gamma/\partial\phi < 0$ (i.e., a stronger tax reaction speeds up the reversion of public debt towards target). Finally, equations (8) and (27)–(28) give the values of R_t, C_t and Y_t as functions of B_t and G_t.

Since $\gamma > 0$, stationarity of public debt requires that $\gamma < 1$. As is shown in Appendix A2, this condition is equivalent to:

[14] The univariate debt dynamics (32) is obtained by combining the (backward-looking) government budget constraint (8) and the (forward-looking) Euler equation (26). Both are nonlinear and need to be linearised to be merged into (32).

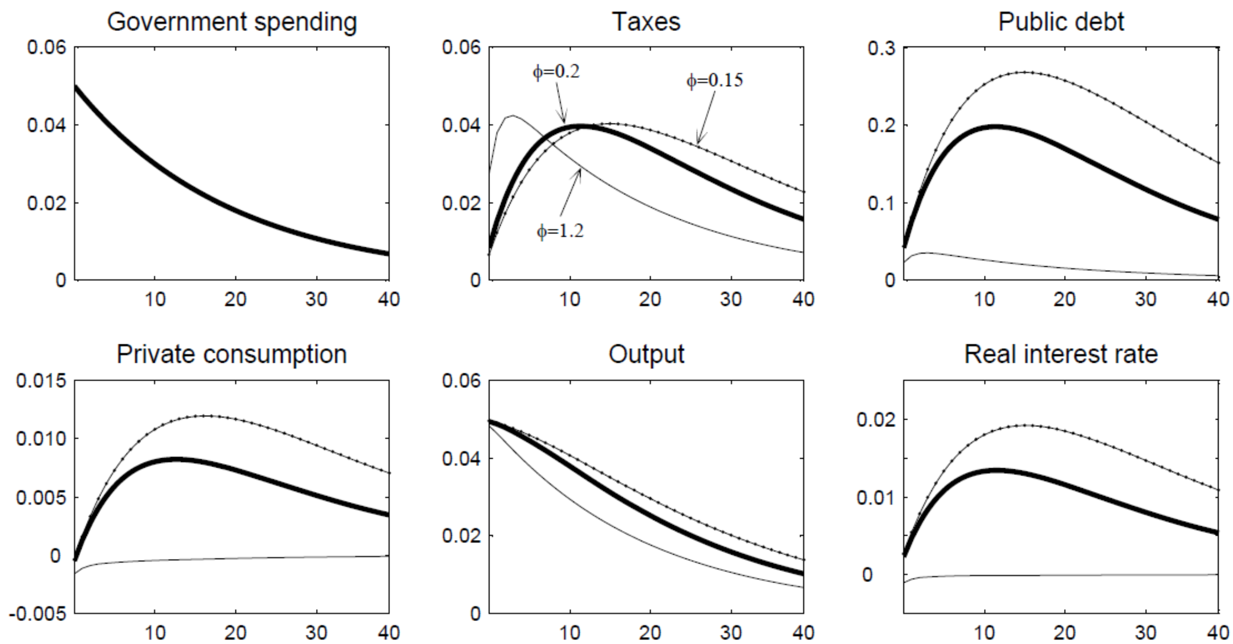
$$\phi > \phi_{\min} \equiv \frac{R-1+\rho}{1-\rho\Omega}, \text{ with } \rho \equiv \frac{(1-\pi^e\beta R)\,\sigma\,(c^{eu})\,R}{1-\Omega+\Omega R} > 0, \tag{33}$$

and where $1-\rho\Omega > 0$ and $R > 0$ is uniquely defined by the target debt level B.

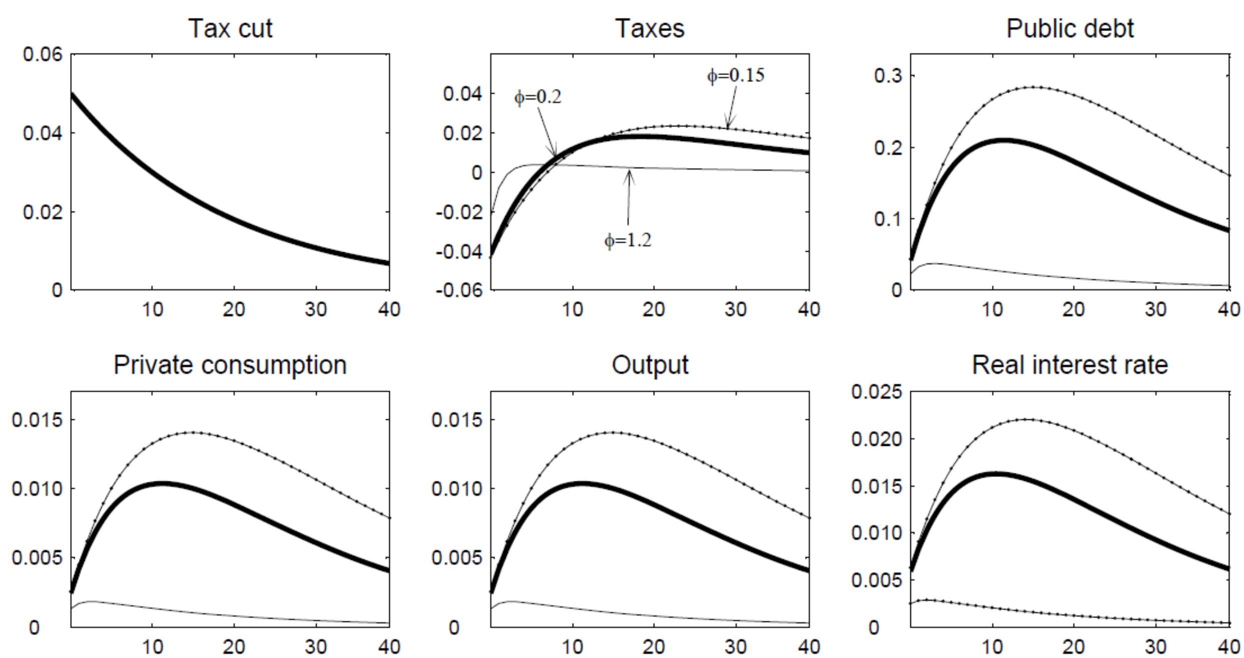
To illustrate the dynamic impact of liquidity and wealth effects in our economy, we draw impulse-response functions for all relevant variables using equation (32) together with (8), (9)-(10) and (27)–(28). Our benchmark (quarterly) parameters are $\beta = 0.98$, $\pi^e = 0.95$, $\pi^u = 0.20$ (this generates an unemployment rate of $\Omega \simeq 5.88\%$), $\delta = 0$, $\kappa = 0.6$, $\psi = \chi = 0.95$, the (unique) value of B such that $R = 1.01$, and $u(c) = \ln c$.

Figure 1 displays the responses of our variables under study to government spending and tax cut shocks. Time-series evidence on the dynamic behaviour of public debt reports a very slow reversion of the debt-GDP ratio towards its long-run mean (e.g., Bohn, 1998; Gali *et al.*, 2007). We take $\phi = 0.2$ as our benchmark for the responsiveness of taxes, which produces such a slow reversion, and we also study the cases in which $\phi = 0.15$ and $\phi = 1.2$. Unsurprisingly, liquidity effects are stronger when $\phi = 0.15$, and hence so are the responses of the aggregates. While setting $\phi = 1.2$ is clearly unrealistic, it is useful as a counterfactual experiment since, as argued above, a quick tax reaction and a small increase in public debt takes our economy's response to the shocks close to that which would be implied by a baseline RBC model.

Let us take government spending shocks first. The case in which $\phi = 0.2$ illustrates a situation where liquidity effects dominate wealth effects on total private consumption, except at the very moment of the shock, due to the substantial increase in public debt and the implied improvement in households' self-insurance opportunities. (Note that private consumption tracks public debt, and is thus far more persistent than the shock itself.) As a result, the output effect of a spending shock is large, in the sense that the spending multiplier is greater than one almost all along the adjustment path. In contrast, wealth effects dominate when $\phi = 1.2$, due to the limited increase in public debt and the rapid reaction of taxes, resulting in a negative response of private consumption all along the transition path; in consequence, the government-spending multiplier is always smaller than one in this case. Holding other parameters constant, values of ϕ between 0.2 and 1.2 (not represented here) cause private consumption to start falling below its steady-state level for several periods (during which public debt and implied liquidity effects are still limited), and then rise above



A. Government spending shock



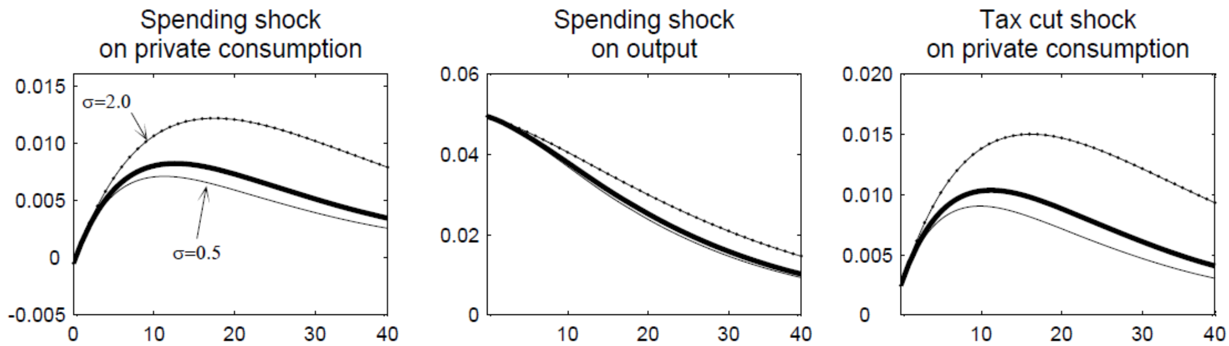
B. Tax cut shock

FIGURE 1. LIQUIDITY-CONSTRAINED HOUSEHOLDS: BASELINE CASE. The figure displays the level deviation from steady state of taxes (T_t), public debt (B_t), private consumption (C_t), output (Y_t) and the real interest rate (R_t), following a public spending shock (Panel A) or a tax cut shock (Panel B) of 5% of steady-state output. Three values of the policy responsiveness parameter are considered (the benchmark is $\phi = 0.2$).

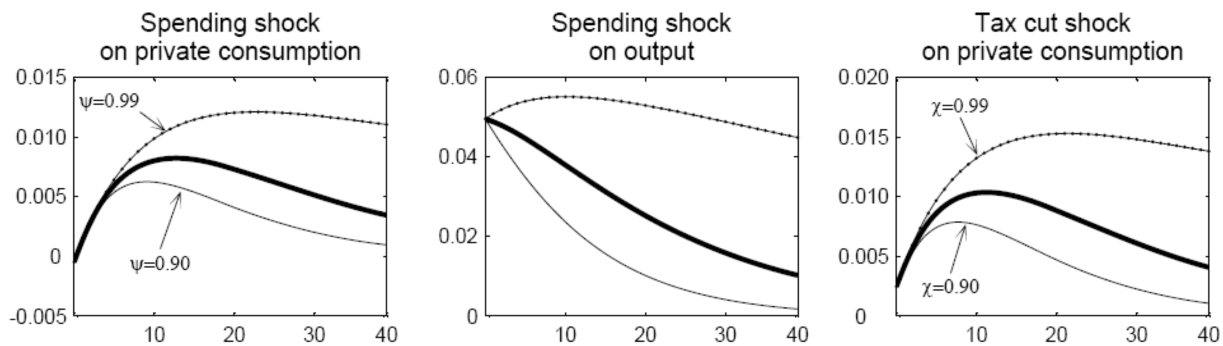
its steady-state level for the rest of the adjustment period (after public debt has risen enough to make the liquidity effects prevalent).

Tax cuts also have strong expansionary effects, whether $\phi = 0.15$ or 0.2, for two reasons. First, liquidity-constrained workers consume the tax rebate one for one (see the $-T_t$ part in (29)). Second, the cut raises public debt and hence aggregate liquidity (the $(2 - \pi^e) B_{t-1} R_{t-1}$ part in (29)). Both channels are much weaker (but still active) when $\phi = 1.2$.

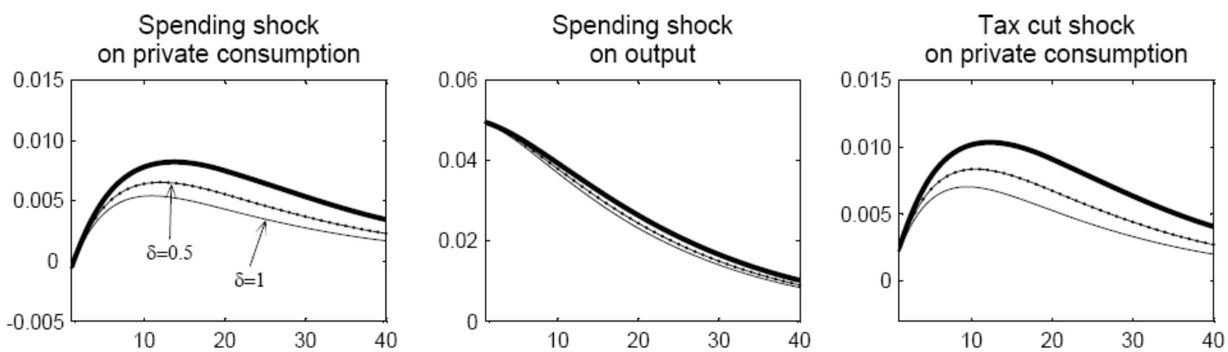
Sensitivity. Figure 2 shows how changes in some key parameters of the model alter the dynamic responses of private consumption and output to fiscal shocks. Panel A considers different degrees of risk aversion, with $u(c) = c^{1-\sigma}/(1-\sigma)$, $\sigma > 0$. As discussed above, the requirement that $\sigma \leq 1$ is not necessary for our equilibrium to remain well behaved; we verified numerically that it is so when $\sigma = 2$ and the other parameters are at their baseline values. Interestingly, the responses of consumption and output are larger when risk aversion rises (or, equivalently, when the intertemporal elasticity of substitution –IES– falls). The reason for this is that high risk aversion/low IES make agents less willing to substitute current consumption for future consumption following the shock, leading bonds to command a higher return in equilibrium. For a given value of the tax rule parameter, this stronger reaction of the interest rate induces a larger response of public debt and hence stronger liquidity effects. Panel B studies the impact of alternative persistence parameters. Note that in the case of a spending shock, higher persistence leads to both greater wealth effects (since the present value of total taxes is higher) and greater liquidity effects (since, for a given tax rule, higher public spending leads to a stronger debt response to the shock); the first two graphs indicate that liquidity effects are more affected than wealth effects by an increase in the persistence parameter. Panel C relaxes the assumption that the unemployed have no borrowing capacity. There is now a whole range of pairs (B, δ) consistent with both our conjectured equilibrium and the requirement that $R = 1.01$, of which $\delta = 0$ (i.e., our baseline economy without inside liquidity, the bold line) is one particular instance. Intuitively, by setting the same value of R for the three specifications we force them to share similar levels of steady-state aggregate (i.e., inside plus outside) liquidity, but allow the *composition* of aggregate liquidity to vary across specifications. Panel C shows that raising the share of private debt in total liquidity weakens the responses of all variables. The reason for this is



A. Alternative degrees of risk aversion



B. Alternative shock persistence



C. Alternative liquidity composition

Figure 2. Liquidity-constrained households: sensitivity. The figure displays the responses of private consumption (C_t) and output (Y_t) to government spending and tax cut shocks (note that output equals consumption in the case of a tax cut). The baseline calibration (bold curves) is $\sigma = 1$, $\psi = \chi = 0.95$, $\delta = 0$ and B such that $R = 1.01$. Each panel show how changes in each parameter alter impulse responses, holding other parameters at their baseline values.

the crowding out of private debt by public debt that takes place after either type of fiscal expansion. Recall that, in the economy without inside liquidity (Figure 1), these shocks raise the real interest rate. With inside liquidity, this higher rate reduces unemployed workers' ability to borrow (since δ/R_t is smaller) and hence their consumption demand moves less than in the baseline economy. Moreover, fewer assets are issued by the private sector, which reduces households' ability to self-insure when employed.

It may be useful at this stage to compare our results with those in Gali *et al.* (2007), who show that a variant of the dynamic New Keynesian model can produce a positive consumption response to spending shocks. While both models put the emphasis on liquidity-constrained households and deficit financing, the channels underlying the procyclicality of consumption differ substantially between the two models. In Gali *et al.*, both aggregate output and employment are demand-determined, due to sticky prices and real wage rigidities, and a share of the population is made of 'hand-to-mouth' workers who consume all of their extra disposable income. Since government spending raises total demand and output, it raises the wage bill and hence the consumption of these workers one for one; then, private consumption rises if these workers are in sufficiently large number. By contrast, in the model described above all prices are fully flexible, so that such (Keynesian) aggregate demand effects are inoperative; moreover, liquidity-constrained agents are unemployed, which makes their labour income unresponsive to fiscal shocks. What ultimately matters here for the procyclical response of private consumption is the ability of these agents to have built up their precautionary wealth *when they were employed*, which is in turn determined by the stock of public debt.

2.4 Imperfectly-elastic labour supply and gradual asset accumulation: an economy with partial risk sharing

As discussed above, our assumption of linear labour disutility is crucial in generating an equilibrium with a finite-state, cross-sectional distribution of wealth, for it implies that, at the individual level, workers leaving unemployment are willing to work as much as necessary to reach their target level of precautionary wealth instantaneously. However, this functional form also tends to magnify the *aggregate* response to fiscal shocks, relative to an economy with lower labour-supply elasticity. Consider, for example, the extreme situation in which labour supply would be completely inelastic, so that output would be entirely unresponsive to fiscal shocks (since labour is the only variable input here). In this situation, tax cuts would not affect total private consumption or output (although they could have significant cross-sectional effects). Since spending shocks would not affect output either, private consumption would necessarily be *crowed out*, rather than crowded in, by public spending. In short, the responses of consumption, output and other aggregates to fiscal shocks depend crucially on both the size of liquidity effects and the willingness of private agents to alter their labour supply after the policy impulse.

To assess the robustness of our results with respect to the elasticity of labour supply, whilst maintaining both tractability and continuity with our previous analysis, we construct an economy with partial risk sharing that has the property of nesting our baseline model with liquidity-constrained workers (and no risk sharing at all) as a special case. For the sake of conciseness, we discuss the implications of this partial risk sharing arrangement mostly informally here and leave much of the corresponding algebra in Appendix A3.

We assume that full risk sharing can take place between employed workers, but only from the second period of continuous employment onwards. This risk-sharing arrangement is akin to the 'family' interpretation of the representative agent model when the underlying agents (i.e., the family 'members') are heterogenous (e.g., Lucas, 1990, Andolfato, 1996); the difference is that we restrict family membership to a subclass of workers, depending on their labour market history: they leave the family when they fall into employment, taking their fair share of the family's assets with them, and re-enter the family when they have been employed for two consecutive periods. All resources (that is, asset and labour income net of taxes) are pooled within the family.

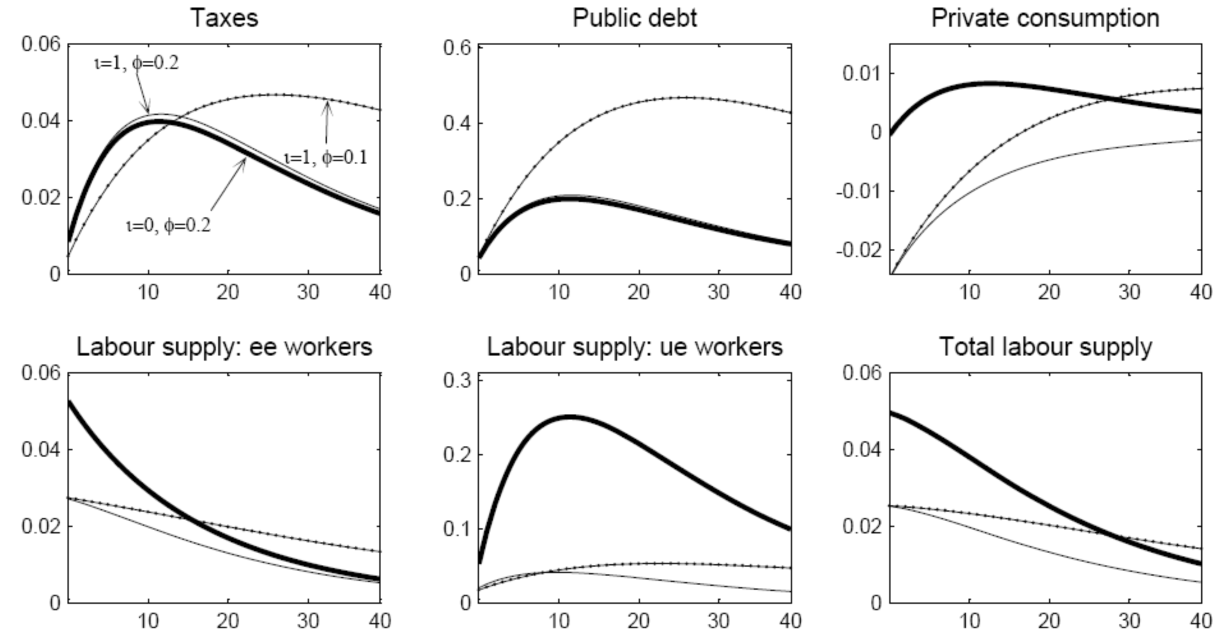
This simple risk-sharing structure has, under conjectures **C1**-**C2**, the following properties. First, the precautionary saving motive is maintained by the threat of family exclusion. Second, tractability is maintained (despite the imperfect elasticity of labour supply) since, even though workers gradually accumulate assets (i.e., those just leaving unemployment hold less asset than after two employment periods), all family members have the same consumption and saving rules. Third, the economy becomes exactly identical to one without risk sharing when labour supply is perfectly elastic. Indeed, when such is the case, workers leaving unemployment work as much as necessary to acquire the same asset wealth as that of family members; this in turn implies that risk sharing within the family becomes redundant (see Appendix A3 for details). As a consequence, we can study how changes in the elasticity of labour supply alter the effectiveness of fiscal policy by continuity with our baseline economy.

We assume the following parametric form for the instant utility function here:

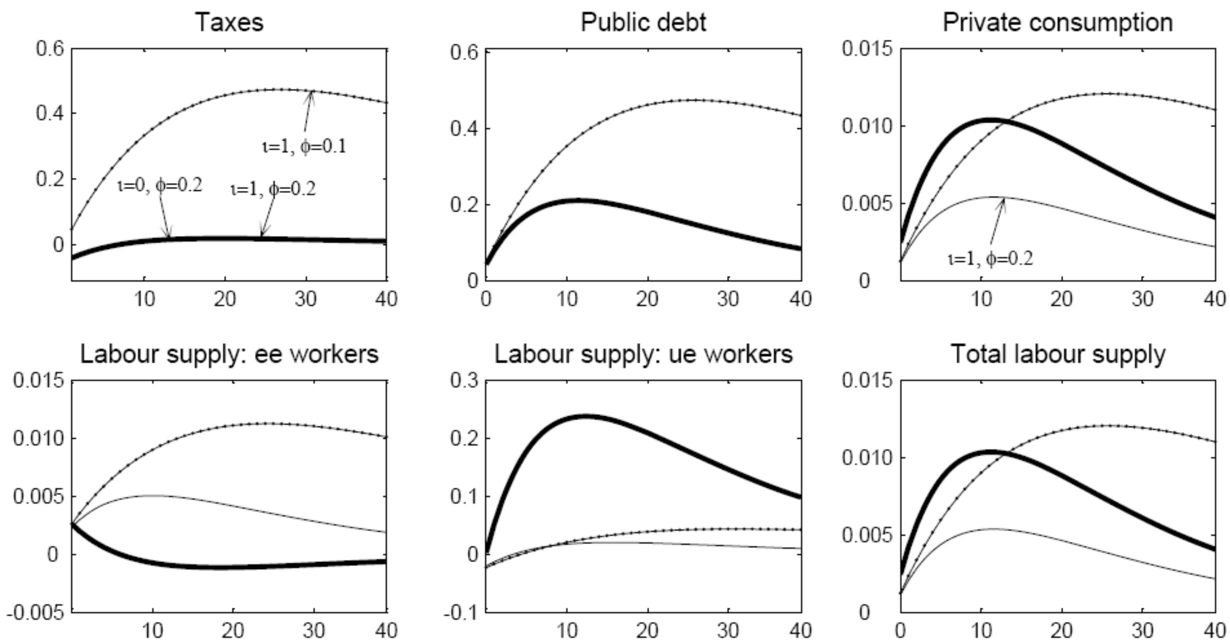
$$u(c) - v(l) = \ln c - \frac{l^{1+\iota}}{1+\iota}, \ \iota \geq 0,$$

and we compare the behaviour of our baseline specification (i.e., $\iota = 0$) to one in which the labour elasticity parameter ι takes the higher value of 1 (as in, for example, Christiano *et al.*, 2005).

Figure 3 shows the paths of taxes, debt, private consumption, as well as the components of labour supply, under the same paths for public spending and tax cuts as in Figure 1. Unsurprisingly, a higher value of ι, holding the policy rule parameter ϕ constant, is associated with a smaller response of aggregates to both public spending and tax cut shocks (again, recall that under fully inelastic labour supply those shocks would not affect output at all). The reason for this is that a value of ι higher than zero makes agents less willing to supply labour to purchase the available stock of liquidity. Consequently, asset accumulation is gradual (see equations (A11)–(A12) in Appendix A3), and output is less responsive to the shocks. Since the path of government spending is exogenous, a muted output response implies that a spending shock is more likely to lead to a crowding out of private consumption (this is notably the case when we set $\iota = 1$ and $\phi = 0.2$). Let us note, however, that in this case a strengthening of liquidity effects may restore crowding in, though later in time (e.g., when $\iota = 1$ and $\phi = 0.1$). To summarise, while output always rises after a spending shock provided that $\iota < \infty$, the overall effect of the shock on consumption depends on both the willingness of workers to supply labour (as indexed ι) and on the intensity of liquidity effects (as determined by the policy rule parameter ϕ). In contrast, tax cut shocks always have expansionary effects on both output and private consumption (again, as long as $\iota < \infty$).



A. Government spending shock



B. Tax cut shock

Figure 3. Liquidity-constrained households: imperfectly elastic labour supply. The figure displays the responses of taxes (T_t), public debt (B_t), private consumption (C_t) as well as aggregate (L_t) and disaggregated (l_t^{ee}, l_t^{ee}) labour supplies for different values of the labour elasticity (ι) and the policy rule (ϕ) parameters. In all calibrations we set $\sigma = 1$, $\psi = \chi = 0.95$, and choose the appropriate value of B such that $R = 1.01$.

3 Fiscal policy shocks with liquidity-constrained entrepreneurs

Our analysis has thus far focused on the way in which liquidity effects may affect the labour supply and consumption demand of private agents. We now wish to study how the quantity of aggregate liquidity may affect labour demand and the equilibrium real wage, in addition to determining individual consumption levels. So that the channels we emphasise will remain transparent, we proceed in three steps. We first derive in Section 3.1 the properties and the conditions for existence of the simplest model of entrepreneurial liquidity demand; this derivation is done by abstracting from unemployment risk and private debt issuance, and by considering one-period projects (that is, we set $\mu = \tau = 1$ and $\delta = 0$). We then characterise in Section 3.2 the dynamics of fiscal shocks in this economy and carry out a number of sensitivity checks, notably with respect to the fiscal policy rule and shocks, as well as the length of project and the severity of the borrowing constraint. Finally, we study in Section 3.3 the case in which both liquidity-constrained workers and entrepreneurs interact (i.e., $\mu \in (0,1)$), so that the two sources of idiosyncratic risk (unemployment risk and random project opportunities) determine the economy-wide demand for liquidity.

3.1 Agents' behaviour and equilibrium

The optimality conditions for entrepreneurs are given by equations (4)–(6), with $\zeta_t^i = 0\ \forall t$. For entrepreneurs currently running a project (i.e., those for whom $\xi^i = 1$), equation (4) is inoperative (since they do not supply labour), the optimal labour *demand* (6) applies, and the optimality condition (5) holds with strict inequality (by conjecture **C1**). For those who do not run projects (that is, for whom $\xi^i = 0$), equation (6) is inoperative (since they do not demand labour), but equations (4)–(5) both hold with equality (by conjecture **C2**). As in Section 2, an equilibrium with a limited number of household type/asset states results from conjectures **C1**-**C2** and the assumed utility function (3). For the sake of conciseness, we simply describe the properties of this equilibrium here and then establish the sufficient conditions for its existence in Proposition 2 below (see also Appendix B1 for details).

With one period-lived projects (i.e., $\tau = 1$), the model generates the following three types of entrepreneurs: i) 'f entrepreneurs' who currently run a project but were supplying labour in the previous period; ii) 'ee entrepreneurs', who do not currently run a project and did not in the previous period either (i.e., they have been supplying labour in both periods); and iii) 'fe entrepreneurs', who are currently employed after having run a project in the previous period. By conjecture **C2**, entrepreneurs who do not run a project are not borrowing-constrained, which under the utility function (3) implies that they all choose the same consumption and asset holding levels, denoted by $\tilde{c}_t^e$ and $\tilde{a}_t^e$ (note that $\tilde{c}_t^e$ will be time-varying, due to changes in the real wage). By conjecture **C1**, entrepreneurs who do run a project are borrowing-constrained, and we denote by c_t^f and l_t^f their consumption and labour demands. The budget constraints of each type of entrepreneur are:

$$ee : \tilde{c}_t^e + \tilde{a}_t^e = \tilde{a}_{t-1}^e R_{t-1} + w_t \tilde{l}_t^{ee} - T_t, \tag{34}$$

$$fe : \tilde{c}_t^e + \tilde{a}_t^e = w_t l_t^{fe} + l_{t-1}^f - T_t - \delta, \tag{35}$$

$$f : c_t^f + w_t l_t^f = \tilde{a}_{t-1}^e R_{t-1} - T_t + \delta / R_t. \tag{36}$$

Equation (34) is the same as (21), except for the fact that the consumption of entrepreneurs who do not run a project, $\tilde{c}_t^e$, is now time-varying (due to time-variations in the equilibrium wage). In equation (35), fe entrepreneurs earn the labour income $w_t l_t^{fe}$ plus production output $y_t = l_{t-1}^f$, and this total income is used to pay for consumption, $\tilde{c}_t^e$, asset accumulation, $\tilde{a}_t^e$, taxes , T_t, and the repayment of their debt obligations, δ. Equation (36), the budget constraint of entrepreneurs running a project, states that they entirely liquidate their (beginning-of-period) assets, $\tilde{a}_{t-1}^e R_{t-1}$, and borrow up to the borrowing limit δ / R_t, in order to finance current consumption, c_t^f, taxes, T_t, and the wage bill $w_t l_t^f$.

Finally, we denote by $\tilde{\omega}^{ee}$, Γ and $\tilde{\omega}^{fe}$ the asymptotic shares of entrepreneurs of type ee, f, and fe, respectively, which are assumed to prevail from date 0 onwards (note that by construction Γ is also the number of projects being run in the economy). Given the transitions of entrepreneurs among individual states, these shares are given by:

$$\tilde{\omega}^{ee} = \theta / (2 - \theta), \ \Gamma = \tilde{\omega}^{fe} = (1 - \theta) / (2 - \theta). \tag{37}$$

From (4)–(5) and (34)–(36), the intratemporal and intertemporal optimality conditions for entrepreneurs not currently running a project but supplying labour to the market are:

$$w_t u'(\tilde{c}_t^e) = 1, \tag{38}$$

$$u'(\tilde{c}_t^e) = \beta R_t E_t(\theta u'\left(\tilde{c}_{t+1}^e\right) + (1 - \theta) u'(c_{t+1}^f)). \tag{39}$$

From (36), entrepreneurs who run a project allocate their after-tax resources, $\tilde{a}^e_{t-1}R_{t-1} - T_t + \delta/R_t$, to current consumption, c^f_t, and the wage bill, $w_t l^f_t$, taking the real wage as given. From (6) and (36), together with the fact that these entrepreneurs exit active entrepreneurship after one period, the solution to their optimal labour demand, l^f_t, satisfies:

$$w_t u'(c^f_t) = \beta E_t u'\left(\tilde{c}^e_t\right). \tag{40}$$

The optimality condition (40) simply sets equal the utility fall implied by a decrease in current consumption necessary to hire an extra unit of labour to the utility gain that is expected from increasing current labour input (and thus future production) by that unit.

Given that entrepreneurs running a project are in proportion Γ, clearing of the bond, labour and goods markets now requires:

$$(1-\Gamma)\,\tilde{a}^e_t - \Gamma\delta/R_t = B_t, \tag{41}$$

$$(1-2\Gamma)\,\tilde{l}^{ee}_t + \Gamma l^{fe}_t = \Gamma l^f_t, \tag{42}$$

$$(1-\Gamma)\,\tilde{c}^e_t + \Gamma c^f_t + G_t = \Gamma y_t. \tag{43}$$

Equation (41) is similar to (23). Equation (42) is like (24), except for the fact that total labour demand, $L_t = \Gamma l^f_t$, now emanates from the entrepreneurial sector. In equation (43), y_t is output per entrepreneur and thus $Y_t = \Gamma y_t$ is total output. Finally, the government's behaviour is described by the budget constraint (8), together with our fiscal rule and shock processes (9)–(10), where again ϕ must be large enough for public debt to be stationary. Proposition 2, whose proof is found in Appendix B1, parallels Proposition 1 by stating the conditions under which our limited-heterogeneity entrepreneurial equilibrium exists.

Proposition 2. Assume that i) $\sigma(c) \leq 1$, ii) fluctuations of B_t around B are small, and iii) (B,δ) jointly satisfy:

$$0 < B + \beta\delta < \tilde{\Sigma} \equiv \left(\frac{\beta^2}{\beta+1-\theta} + \frac{\beta}{1-\theta}\right) u'^{-1}\left(\beta^{-1}\right).$$

Then, the equilibrium with three types of entrepreneurs exists and has an interest rate R_t strictly lower than $1/\beta$ for all t.

Just as in the case of liquidity-constrained workers, the existence of a limited-heterogeneity equilibrium with liquidity-constrained entrepreneurs relies on steady-state public debt being sufficiently low. Importantly, here again the requirement that $\sigma(c) \leq 1$ is meant to guarantee steady-state uniqueness for all possible parameter configurations, but one can easily construct economies where $\sigma > 1$ and verify numerically that uniqueness still prevails.

Entrepreneurs who encounter a project opportunity play a central role in our analysis, so it may be instructive to decompose their budget set in equilibrium as we did earlier for workers falling into unemployment (see (29)). Substituting (41) into (36), we obtain:

$$\underbrace{c_t^f}_{\text{consumption}} + \underbrace{w_t l_t^f}_{\text{wage bill}} = \underbrace{\frac{B_{t-1}R_{t-1}}{1-\Gamma} + \frac{\Gamma\delta}{1-\Gamma}}_{\text{liquidated portfolio}} + \underbrace{\frac{\delta}{R_t}}_{\text{borrowing}} - \underbrace{T_t}_{\text{taxes}}.$$

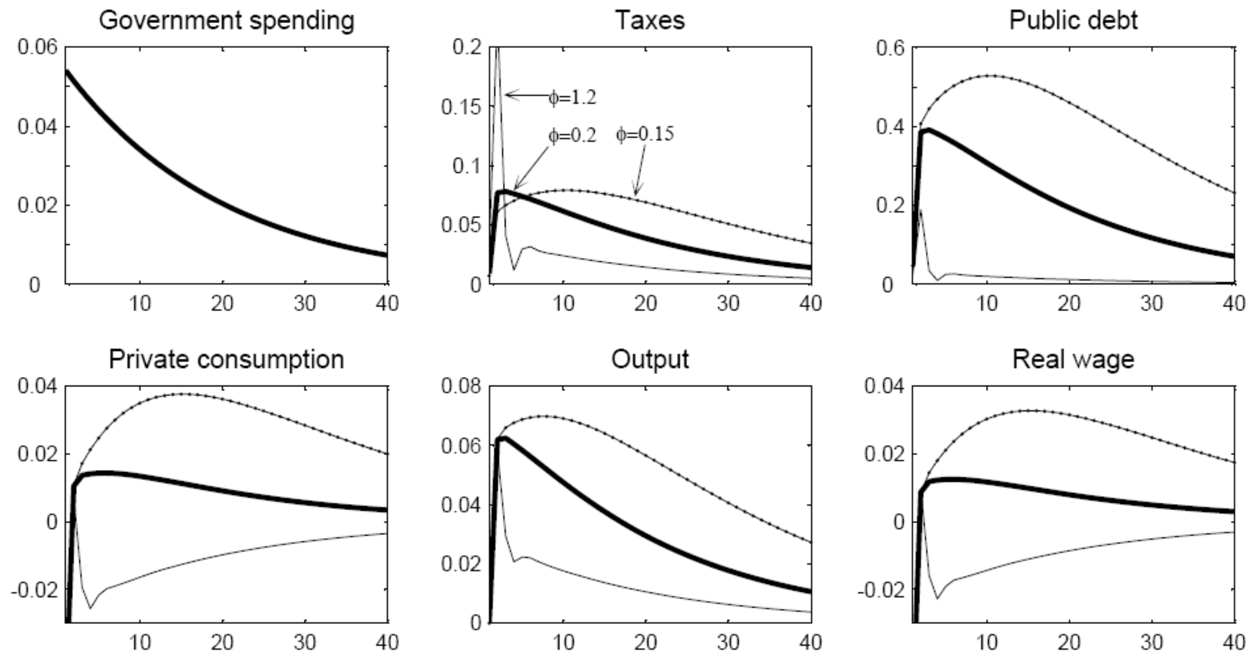
In short, these entrepreneurs allocate their after-tax resources between current consumption and the wage bill, with the optimal trade-off between the two characterised by equation (40). These resources consist of their liquidated asset portfolio, whose value depends on the stocks of outside and inside liquidity available in the economy (and hence on B_{t-1} and δ), as well as the corporate debt they are able to issue (up to δ/R_t), minus taxes.

3.2 The dynamic effects of fiscal shocks

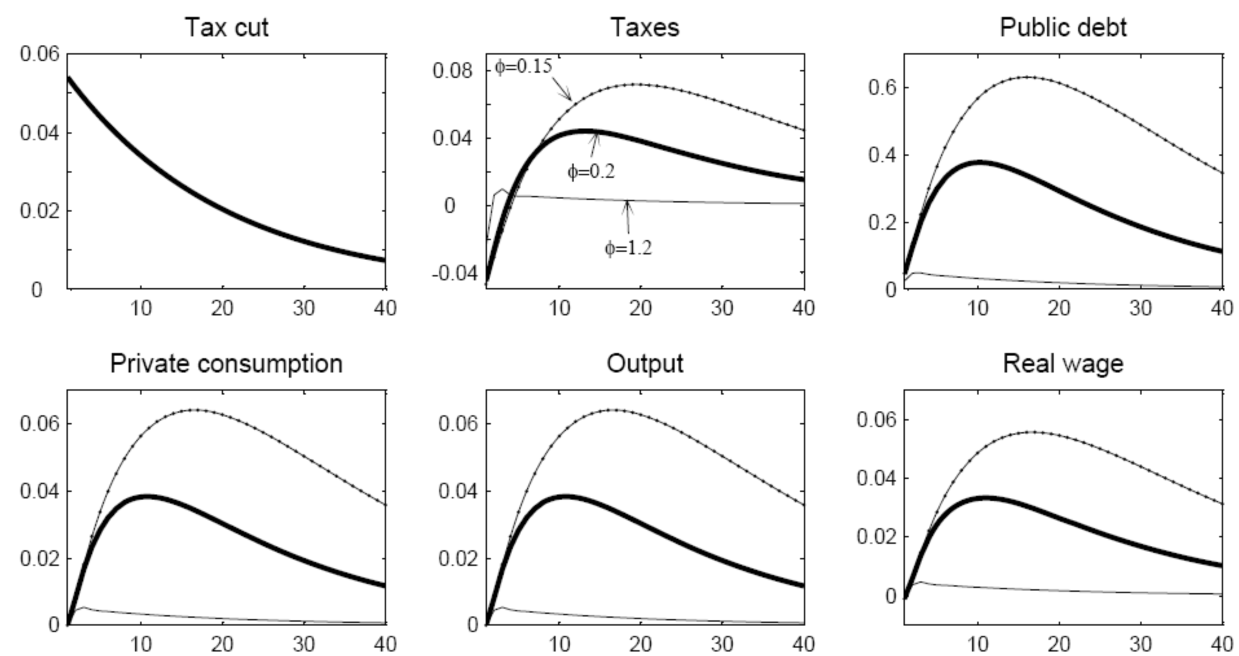
The dynamic system characterising the entrepreneurial model involves more lags and more interactions between variables than the basic model (the equations forming this dynamic system are described in Appendix B1). For the sake of comparability, we run policy experiments with exactly the same parameter values as in the previous section, except for θ, which is now set to 0.80 (implying a share of entrepreneurs of $\Gamma \simeq 16.67\%$).[15] As is summarised in Appendix B2, the dynamics of the entrepreneurial model yield an expectational dynamic system that can be solved numerically for the vector of relevant variables and for the stationarity condition.

Figure 4 displays the responses of fiscal and aggregate variables to either type of fiscal shock generated by our baseline entrepreneurial model. (Note that $\tilde{c}_t^e$ and l_t^f, although not represented, are tracked by w_t and Y_{t+1}, respectively). Let us start with government spending shocks again. Since liquidity effects on labour demand take one period to be operative (as

[15] This value is roughly equal to the number of U.S. firms, from The Census Bureau's 2002 Survey of Business Owners (23 million firms) divided by total employment by the end of the same year from the BLS Current Population Survey (136.5 million people).



A. Government spending shock



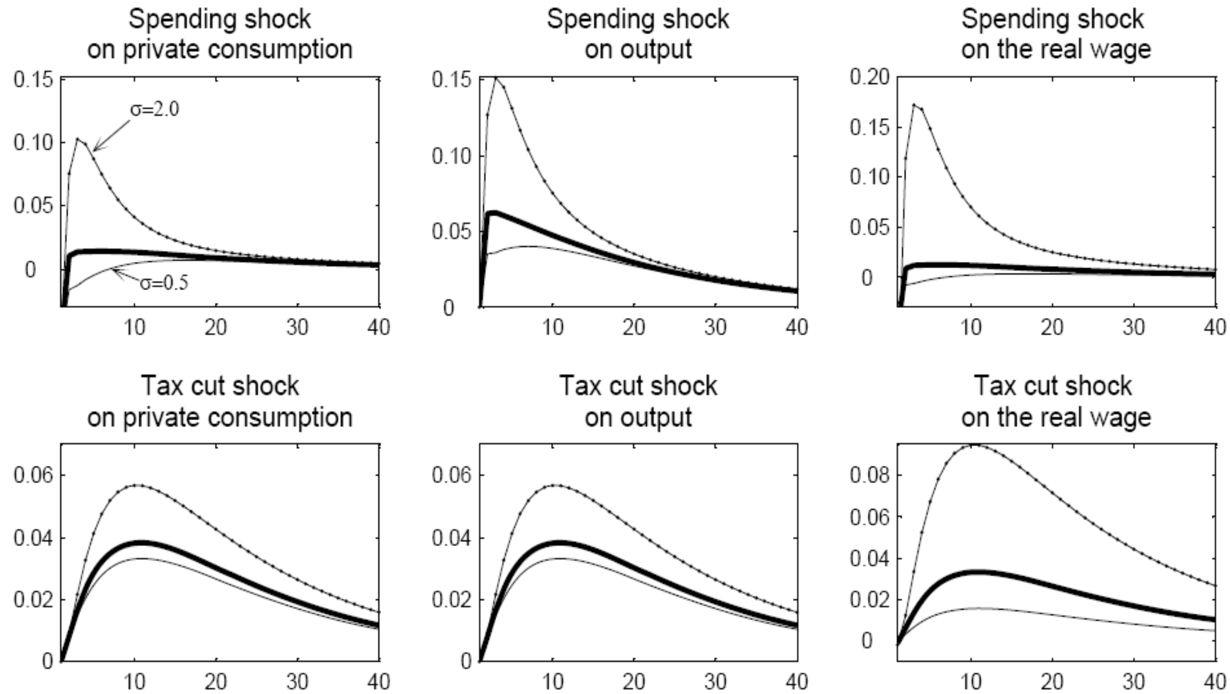
B. Tax cut shock

FIGURE 4. LIQUIDITY-CONSTRAINED ENTREPRENEURS: BASELINE CASE. The panels display the linear deviations from the steady state of taxes (T_t), public debt (B_t), private consumption (C_t), output (Y_t) and the real wage (w_t) following a government spending shock (Panel A) or a tax cut shock (Panel B) of 5% of steady-state output. Three values of the policy responsiveness parameter are considered (the benchmark is $\phi = 0.2$).

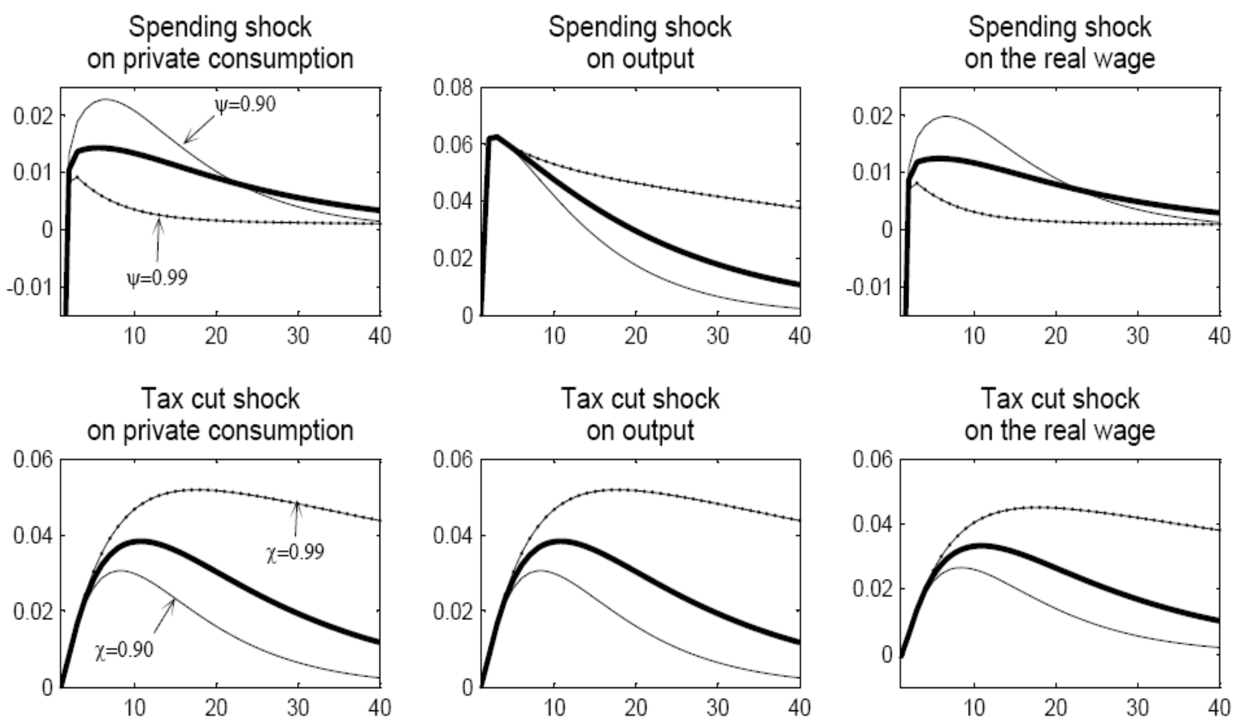
some employed households having increased their savings turn into entrepreneurs), wealth effects on labour supply dominate on impact for all values of ϕ. The ensuing increase in labour supply leads to a sharp fall in the real wage and the consumption of employed households, causing total private consumption to fall. However, when $\phi = 0.2$ liquidity effects on labour demand become dominant (in the sense of leading to higher-than-steady-state wages) for the entire adjustment path starting from one period after the shock, leading to a persistent boom in private consumption. While these positive wage and private consumption responses are magnified when $\phi = 0.15$, they are inverted when $\phi = 1.2$. In this latter case, the strong reaction of taxes and limited growth of public both act to weaken the liquidity effects on labour demand whilst strengthening wealth effects on labour supply. This leads to a limited increase in labour demand relative to the contemporaneous increase in labour supply, and thus to a fall in the real wage and a crowding-out of private consumption by public spending. Here again, values of ϕ between 0.2 and 1.2 (not represented here) generate a more mixed picture with dominance of either effect at different points on the transition path.

The response of private consumption to tax cut shocks looks qualitatively similar to that generated by the model with liquidity-constrained households, but labour market adjustments play a central role here. More specifically, a tax cut loosens the borrowing constraint of entrepreneurs (equation (36)) both directly through its effect on T_t and indirectly through it effect on $a_{t-1}R_{t-1}$. This in turn contributes to raise both entrepreneurs' consumption, c_t^f, and their labour demand, l_t^f. This higher labour demand then raises the equilibrium real wage and hence the consumption of employed households, $\tilde{c}_t^e$ (see (38)). Unsurprisingly, these effects are larger the smaller is the policy responsiveness parameter.

Sensitivity. Figure 5 shows how modifying either risk aversion or shock persistence alters impulse-response functions. As in the model with liquidity-constrained consumers, a higher degree of risk aversion tends to magnify liquidity effects (see Panel A), since it triggers a larger reaction of the equilibrium real interest rate and thus of the stock of public debt. Note, however, that with $\sigma = 0.5$ and after a spending shock liquidity effects on labour demand are so weak that they are dominated by wealth effects on labour supply over much of the adjustment path; consequently, both the real wage and private consumption lie below their steady-state value most of the time (first row of Panel A). The conflict between wealth and liquidity effects is even more apparent when one looks at the effect of changing the persis-



A. Alternative degrees of risk aversion



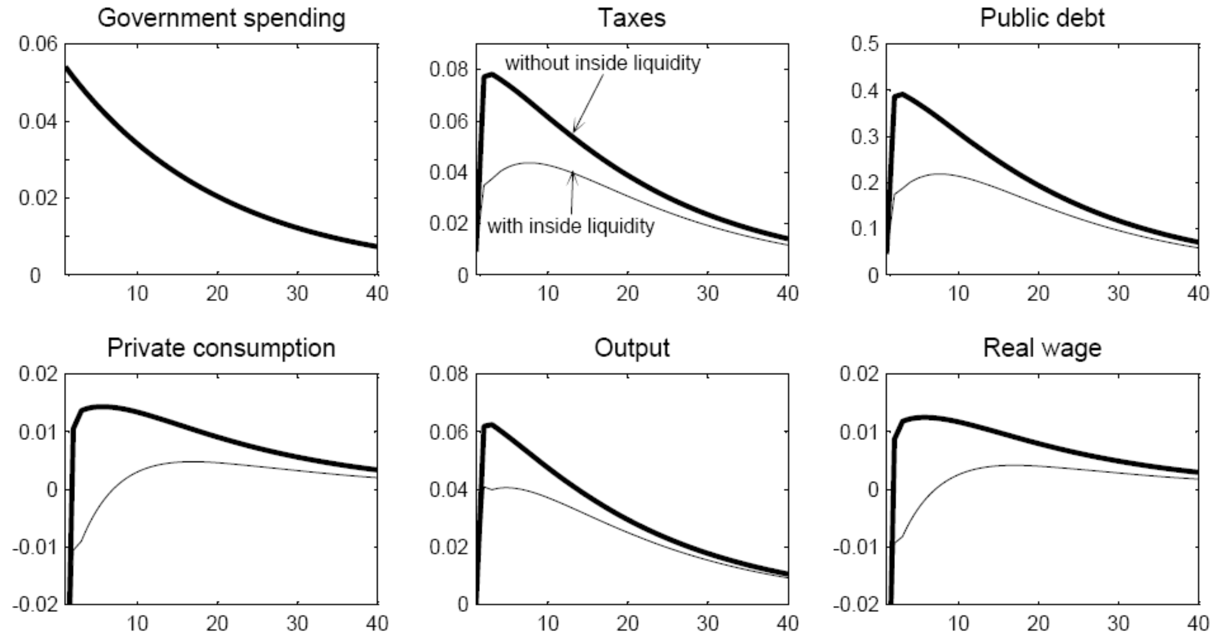
B. Alternative shock persistence

Figure 5. Liquidity-constrained entrepreneurs: sensitivity. The figure displays the responses of private consumption (C_t), output (Y_t) and the real wage (w_t) to spending and tax cut shocks. The baseline calibration (bold curves) is $\sigma = 1$ and $\psi = \chi = 0.95$.

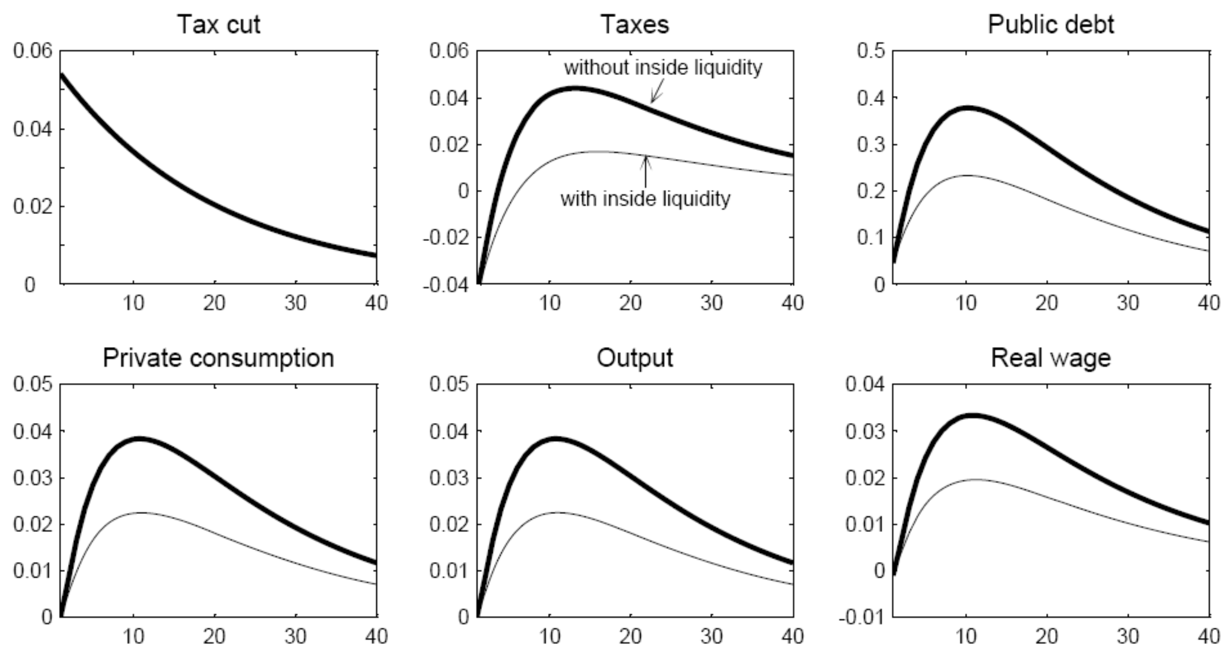
tence of spending shocks (first row of Panel B). On the one hand, such shocks imply that taxes are higher than their steady-state value for a sustained amount of time, which leads to a prolonged increase in labour supply. On the other hand, these shocks raise public debt and hence the labour demand of entrepreneurs. Since raising the persistence of the shock strengthens both effects, the way it will affect the equilibrium real wage and private consumption at different point of the adjustment path is a priori ambiguous. In the case of tax cut shocks (second row of Panel B), lower taxes and higher liquidity both contribute to generate a short-run boom in the real wage and private consumption.

Figure 6 compares our baseline entrepreneurial model (in which $\delta = 0$) with one with both inside and outside liquidity (i.e., $\delta > 0$), and impose for the latter the unique value of B that generates a B/Y ratio of 8/3 (since R, and thus Y, are interpreted as quarterly values, the corresponding yearly debt-output ratio would be 2/3); given our requirement that the steady-state gross interest rate be 1.01, this uniquely pins down δ. Again, a higher share of private debt turns out to weaken the responses of all variables, due to the interest rate increase that follows fiscal shocks. More specifically, in the economy with inside liquidity, this higher rate reduces entrepreneurs' ability to borrow (since δ/R_t is smaller), and hence their consumption and labour demand move less than in the baseline economy; this in turn translates into a smaller reaction of the real wage and thus a muted increase in entrepreneurs' consumption. The impact of this crowding out on total output naturally follows. Of course, we should expect a similar crowding out of private demand to take place if we were to introduce other assets into the economy, such as claims on the capital stock. In the latter case, the higher interest rate induced by fiscal expansions would deter investment demand and thus tune down the economy's reaction to fiscal shocks (see Aiyagari and McGrattan, 1998, for an analysis of this crowding out in the steady state). Here again, which channel is likely to dominate ultimately depends on the relative strengths of the crowding-in and crowding-out effects on private demand.

In Figure 7, we consider the case in which entrepreneurial projects last for more than one period (see Appendix B3 for details). To understand how this modifies the responses to fiscal shocks, take the simple example where $\tau = 2$. Entrepreneurs running a project in the current period now include two types of entrepreneurs in equal numbers: those who start a project in the current period and those whose project started in the previous period and are



A. Government spending shock



B. Tax cut shock

Figure 6. Liquidity-constrained entrepreneurs: with and without inside liquidity. The panels are as in Figure 4, with the impulse-response functions now showing the responses of the variables in the economy with and without inside liquidity.

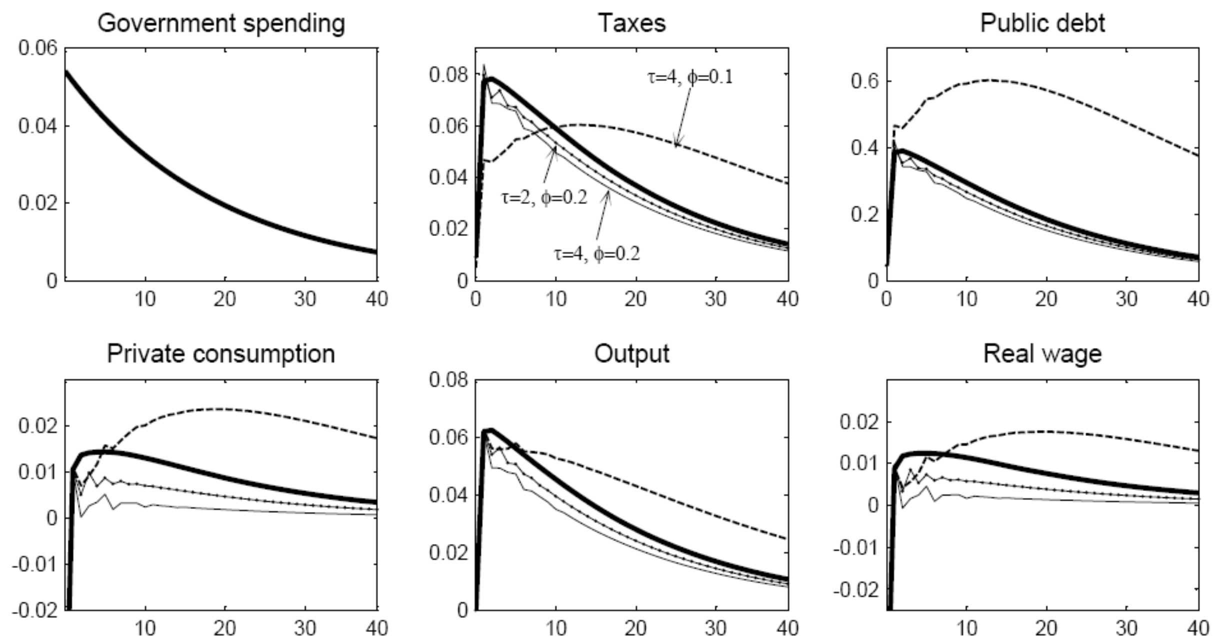
are still ongoing. Importantly, both are borrowing-constrained provided that $w_t R_t < 1$ (see our discussion of condition (7) above). Extending the project length has two conflicting effects here. One the one hand, debt-financed fiscal shocks increase the stock of liquidity in the economy. This increased liquidity relaxes the borrowing constraint of entrepreneurs who encounter a project opportunity and boosts their labour demand. Hence, they will produce more output in the next period, which will again enable them to raise their labour demand in the next period too and to produce more output two periods ahead. Thus, long-lived projects generate *intertemporal spillovers* of fiscal-policy shocks. On the other hand, if we maintain, as we do, the share of active entrepreneurs at the same value as in the baseline model (about 17%), the probability of meeting an investment opportunities, $1 - \theta$, must be smaller (0.1 instead of 0.2); this tends to reduce the immediate impact of liquidity shocks. As shown in Figure 7 this latter effect slightly dominates the former under our parameterisation. Note, however, that this dominance is small, in the sense that the overall impact of fiscal shocks is primarily dominated by the fiscal rule rather than by the length of projects (at least for the lengths that we are considering). For example, if we set $\varphi = 0.1$ –a value that still generates a plausible debt response to the shocks–, then liquidity effects remain largely dominant even with $\tau = 4$. We may thus conclude that our basic qualitative results about the expansionary effects of fiscal shocks are robust to the inclusion of long projects.

3.3 The economy with constrained workers and entrepreneurs

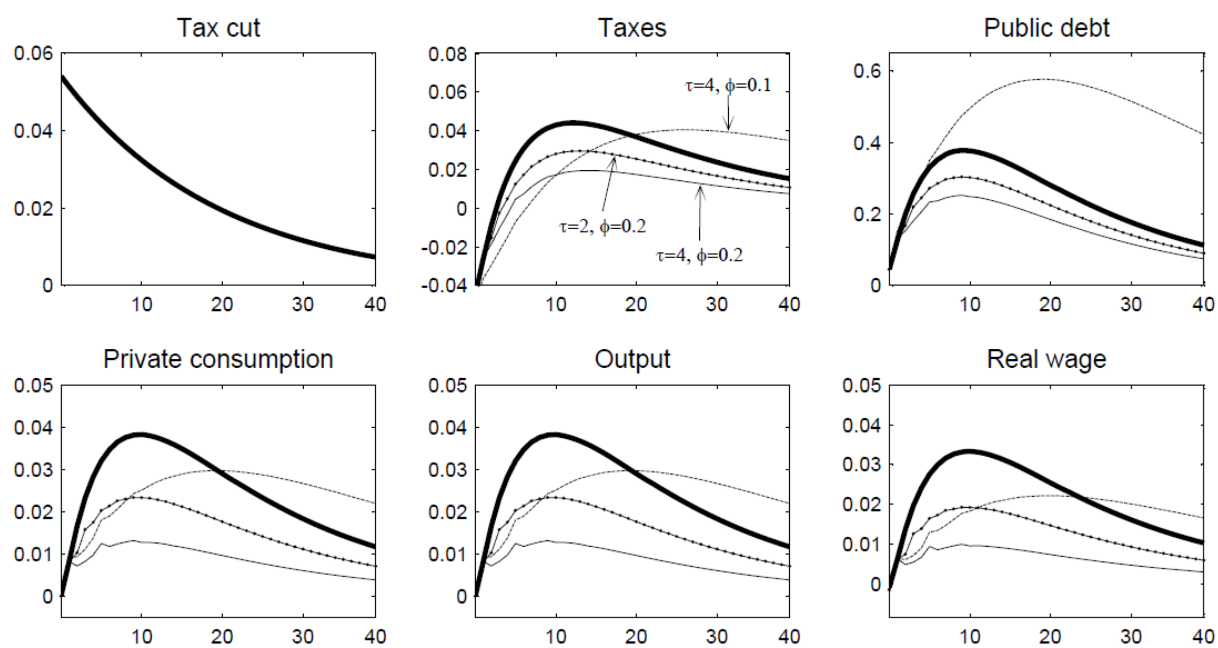
Having disentangled how the liquidity effects induced by rising public debt affect liquidity-constrained workers (who self-insure against unemployment risk) and liquidity-constrained entrepreneurs (who hoard wealth to finance stochastic project opportunities), it is now straightforward to consider the more general and realistic case in which households of both occupations interact, i.e., $\mu \in (0, 1)$.

The budget constraints and optimality conditions for entrepreneurs are given by (34)–(40), as before. Regarding workers, their budget constraints are still given by equations (14) and (19)–(21), but their optimality conditions must be modified slightly to account for the fact that they face a potentially time-varying wage payment in the labour market.

The first thing to note is that the optimal consumption level of employed workers is no longer given by equation (16), but by (38) instead, and is thus equal to that of entrepreneurs



A. Government spending shock



B. Tax cut shock

FIGURE 7. LIQUIDITY-CONSTRAINED ENTREPRENEURS: IMPACT OF LONG PROJECTS. The panels are as in Figure 4, with the impulse-response functions now showing the variables' responses for different values of the project length (τ) and the fiscal policy responsiveness parameter (ϕ).

not currently running an investment project. This in turn implies that their optimal asset demand is now given by:

$$u'\left(\tilde{c}_t^e\right) = \beta \pi^e R_t E_t u'\left(\tilde{c}_t^e\right) + \beta\left(1-\pi^e\right) R_t E_t u'(a_t^e R_t - T_{t+1}). \tag{44}$$

The market-clearing conditions must be modified to account for the interactions of all types of agents in the economy. For example, equilibrium in the bonds market now requires:

$$(1-\mu)\left(\omega^{ee}+\omega^{ue}\right) a_t^e + \mu\left(\tilde{\omega}^{ee}+\tilde{\omega}^{fe}\right)\tilde{a}_t^e = B_t, \tag{45}$$

where the ωs are those in (22) and (37) above.

The first part in the right-hand side of (45) is the total asset demand emanating from employed workers, which is in turn given by their total mass in the population $(1-\mu)\left(\omega^{ee}+\omega^{ue}\right)$ *times* their (common) individual asset demand a_t^e, with the latter given by (44). The second part of the equation is the total liquidity demand by entrepreneurs contemplating the possibility of having an investment opportunity in the next period; those are in numbers $\mu\left(\tilde{\omega}^{ee}+\tilde{\omega}^{ue}\right)$ in the population, while each of them hoards a quantity $\tilde{a}_t^e$ of liquidity (with $\tilde{a}_t^e$ being determined by (39)). With $\delta = 0$, the right-hand side of the inequality, B_t, is the aggregate supply of liquidity in the economy (as before, setting $\delta > 0$ would lower the impact of fiscal shocks but would not alter our results qualitatively).

Similarly, the equilibrium condition in the market for goods is now given by:

$$\begin{aligned}\left[(1-\mu)\left(\omega^{ee}+\omega^{ue}\right)+\mu\left(\tilde{\omega}^{ee}+\tilde{\omega}^{fe}\right)\right]\tilde{c}_t^e \\ +(1-\mu)\,\omega^{ue}c_t^{eu}+(1-\mu)\,\omega^{uu}c_t^{uu}+\mu\Gamma c_t^f+G_t=\mu\Gamma l_{t-1}^f.\end{aligned} \tag{46}$$

Take the right-hand side of (46), and recall that $\tilde{c}_t^e$ is now the consumption level of both employed workers and entrepreneurs waiting for a project opportunity; again, the former and the latter are in numbers $(1-\mu)\left(\omega^{ee}+\omega^{ue}\right)$ and $\mu\left(\tilde{\omega}^{ee}+\tilde{\omega}^{fe}\right)$ in the population, respectively. The other relevant consumption levels (i.e., c_t^{eu}, c_t^{uu} and c_t^f) are similarly weighted by their respective population shares. As in our baseline economy, total output is produced by those who encountered a project opportunity in the last period (because of the production lag); those demanded a quantity of labour l_{t-1}^f, and are now in number $\mu\Gamma$ in the population. Finally, equilibrium in the labour market requires:

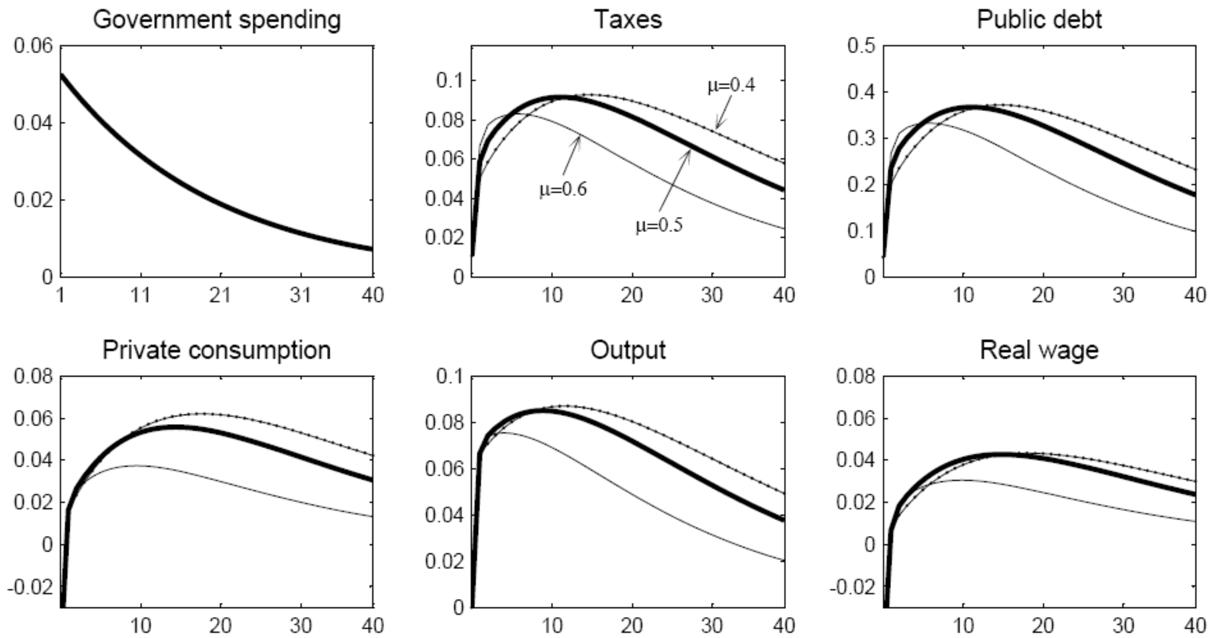
$$(1-\mu)\left(\omega^{ee}l_t^{ee}+\omega^{ue}l_t^{ue}\right)+\mu(\tilde{\omega}^{ee}\tilde{l}_t^{ee}+\tilde{\omega}^{fe}l_t^{fe})=\mu\Gamma l_t^f. \tag{47}$$

Figure 8 illustrates the dynamic effects of fiscal shocks for several values of the share of entrepreneurs in the economy. Our calibration strategy here is as follows: given the coexistence of workers and entrepreneurs, we adjust the probabilities of changing status, $1-\pi^e$ and $1-\theta$, in such a way that the unemployment and active entrepreneurship rates in the population, here $(1-\mu)\,\Omega$ and $\mu\Gamma$, take the same values as in the basic scenarios of Sections 2 and 3, namely 5.88% and 16.67% (the transition probability π^u is left at 0.20, but adjusting it within realistic bounds jointly with π^e only changes the IRFs marginally). Then, the requirements that $\pi^e, \theta > 0$ impose bounds on μ, given our chosen values of $(1-\mu)\,\Omega$ and $\mu\Gamma$; here it implies that we must have $0.33 < \mu < 0.89$.

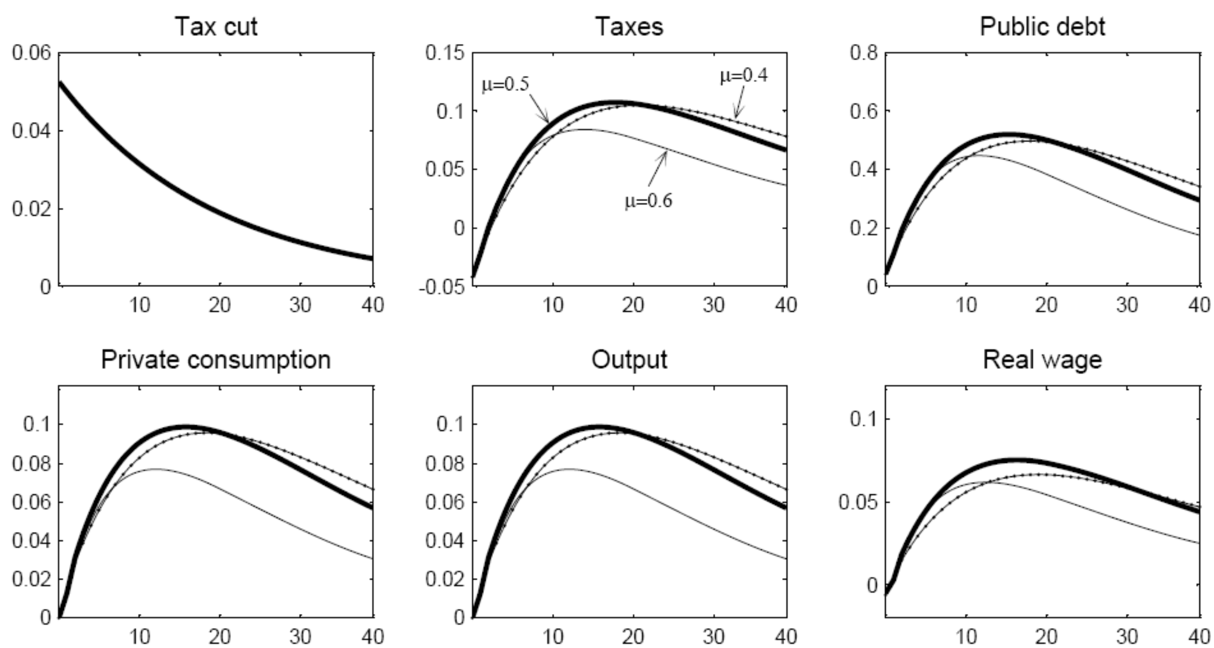
The fact that we cannot continuously move from $\mu = 1$, in which case the value of π^e is irrelevant, to a value of μ consistent with $\pi^e > 0$ implies that the dynamic effects of fiscal shocks evolve substantially across the two specifications. In particular, the economy with both workers and entrepreneurs displays a stronger and more persistent reaction of public debt to both kinds of fiscal policy shocks, and hence stronger liquidity effects on all aggregates (this arises because the interest rate interest rate response to the shock is substantially stronger when $0.33 < \mu < 0.89$ than when $\mu = 1$, and hence by equation (8) public debt rises more and more persistently in the former case than in the latter. In this context, values of ϕ below $\phi = 0.25$ generate either a highly (and unrealistically) persistent public debt response to the shocks, or even lack of stationarity; we thus impose $\phi = 0.25$ here. Aside from this required fiscal-rule adjustment, the economy with both types of households inherits the salient qualitative features of the two basic specifications (i.e., $\mu = 0$ and $\mu = 1$). In particular, private consumption and the real wage start falling below steady state before rising above it in a hump-shaped manner after a spending shock (this is because wealth effects are set in motion before liquidity effects here), while tax cuts are expansionary all along the transition path.

4 Concluding remarks

We have presented in this paper the predictions of a tractable liquidity-constrained economy regarding the effects of debt-financed fiscal expansions, with particular attention being paid to the effects of spending shocks on private consumption and the real wage. Our main goal



A. Government spending shock



B. Tax cut shock

FIGURE 8. THE LIQUIDITY-CONSTRAINED ECONOMY WITH WORKERS AND ENTREPRENEURS. The panels are as in Figure 4, with $\phi = 0.25$ and for different values of the proportion of entrepreneurs in the economy (μ).

has been to illustrate that the liquidity effects induced by temporary changes in the stock of public debt can drastically alter the predictions of the baseline complete-markets model, in which changes in public spending affect aggregates only through intertemporal wealth effects. To summarise, our main results are as follows:

First, debt-financed increases in public spending generate potentially powerful liquidity effects when agents face uninsurable idiosyncratic uncertainty. This effect occurs because aggregate liquidity facilitates self-insurance by households facing unemployment risk, while at the same time helping potential entrepreneurs to hoard asset wealth for future investment needs. As a result, such policies may have strong expansionary effects on private consumption, labour demand, and the equilibrium real wage.

Second, both spending shocks and tax cuts have stronger effects on macroeconomic aggregates when the response of taxes necessary to ensure the solvency of the government is delayed. This property arises because the extent of liquidity effects is indexed by the aggregate supply of assets in the economy. The latter is directly affected by the dynamics of public debt, and hence by the tax rule adopted by the government.

Third, fiscal expansions are more effective the tighter the borrowing constraints faced by private agents. Tight borrowing constraints make agents highly dependent on government-issued assets to self-insure against idiosyncratic income risk. In contrast, looser constraints reduce their dependence, while at the same time crowding out private asset issuances when borrowing limits are interest-rate dependent.

Our model relied on two assumptions that we are planning to dispose of in future research. First, we abstracted from capital accumulation. As discussed above, when public debt and capital are substitutes as outside liquidity instruments then debt-financed fiscal shocks are bound to crowd out private investment demand and thus to reduce the impact of fiscal shocks; further investigation is thus needed to assess the extent of this crowding out and how it would affect the effectiveness of fiscal policy. Second, we have assumed throughout that there was no distortionary cost associated with high levels of government debt, so that raising liquidity supply could only be beneficial in the liquidity-constrained equilibrium; again, incorporating such distortions is likely to qualify (but also enrich) our results on the dynamic effects of fiscal shocks. More generally, the relative tractability of our model may make it useful for understanding more complicated fiscal-policy issues such as the international transmission of fiscal shocks. In particular, it has been argued that scarce world liquidity and heterogenous financial development are crucial in determining the direction and size of international capital flows (e.g., Caballero *et al.*, 2008). Inasmuch as domestic public debt provides liquidity to foreign savers, our framework may offer new insights into how fiscal shocks are transmitted across financially integrated economies.

Appendix A. Liquidity-constrained workers

A1. Proof of Proposition 1

If fluctuations around the steady state are sufficiently small, then **C1** and **C2** hold in every period provided that they hold in the steady state. The condition $B+\beta\delta>0$ implies that either $B>0$ or $\delta>0$ or both, and hence that $a^e>0$ from equation (23); thus, conjecture **C2** holds provided that $B+\beta\delta>0$. What is left to establish is that conjecture **C1** holds in the steady state provided that conditions i-iv in the proposition are satisfied. We proceed in two steps. First we show that $\beta R<1$ if and only if $B+\beta\delta<\Sigma$ (step 1). Second, we show that **C1** holds whenever $\beta R<1$ and conditions i-iii are satisfied (step 2).

Step 1. To prove that $R\beta<1$ if and only if $B<\Sigma-\beta\delta$, we show that $B(R)$ is a continuous, strictly increasing function of R over the appropriate interval and that $B(1/\beta)=\Sigma-\beta\delta$ (so that $B(R)<\Sigma-\beta\delta\Leftrightarrow R<1/\beta$). First, let us rewrite the steady-state counterpart of (17) as follows:

$$u'\left(c^{eu}\right)=\left((\beta R)^{-1}-\pi^e\right)/\left(1-\pi^e\right), \tag{A1}$$

By assumption $G=0$, implying that $T=B(R-1)$. Thus, after some manipulations the steady-state counterpart of (26) can be written as:

$$B=\tilde{B}\left(R\right)-\delta/R\equiv B\left(R\right), \tag{A2}$$

where

$$\mathring{B}\left(R\right)\equiv\left(\frac{1-\Omega}{1+\Omega\left(R-1\right)}\right)u'^{-1}\left(\frac{(\beta R)^{-1}-\pi^e}{1-\pi^e}\right).$$

The term $-\delta/R$ in (A2) is strictly increasing in R when $\delta>0$. Moreover, we have:

$$\mathring{B}'\left(R\right)=\frac{-\left(1-\Omega\right)\Omega}{\left[1+\Omega\left(R-1\right)\right]^2}u'^{-1}\left(\frac{(\beta R)^{-1}-\pi^e}{1-\pi^e}\right)+\left(\frac{1-\Omega}{1+\Omega\left(R-1\right)}\right)\frac{\partial}{\partial R}u'^{-1}\left(\frac{(\beta R)^{-1}-\pi^e}{1-\pi^e}\right). \tag{A3}$$

Equation (A1) implies that $u'(c^{eu}(R)) = ((\beta R)^{-1} - \pi^e)/(1-\pi^e)$, so the $\partial u'^{-1}(.)/\partial R$ term above is:

$$\frac{\partial}{\partial R} u'^{-1}\left(\frac{(\beta R)^{-1} - \pi^e}{1-\pi^e}\right) = \frac{1}{u''(c^{eu})} \times \frac{\partial u'(c^{eu})}{\partial R} = \frac{1}{u''(c^{eu})} \times \frac{-1}{(1-\pi^e)\beta R^2}.$$

After rearranging, this allows us to rewrite (A3) as follows:

$$\begin{aligned}
\mathring{B}'(R) &= \frac{-(1-\Omega)\Omega c^{eu}}{[1+\Omega(R-1)]^2} + \left(\frac{1-\Omega}{1+\Omega(R-1)}\right) \times \frac{-R^{-2}}{(1-\pi^e)\beta} \times \frac{1}{u''(c^{eu})} \\
&= \frac{(1-\Omega)\Omega u'(c^{eu})}{[1+\Omega(R-1)]^2 u''(c^u)} \left(-\frac{c^{eu}u''(c^{eu})}{u'(c^{eu})} - \frac{(1+\Omega(R-1))}{\Omega R^2(1-\pi^e)\beta u'(c^{eu})}\right) \\
&= \frac{(1-\Omega)\Omega u'(c^{eu})}{[1+\Omega(R-1)]^2 u''(c^{eu})} \left(\sigma(c^{eu}) - \frac{1+\Omega(R-1)}{(1-\pi^e\beta R)\Omega R}\right).
\end{aligned}$$

The term inside the pair of large brackets must be negative for $\mathring{B}'(R)$ to be positive. Since $\sigma(c) \leq 1$ by assumption, a sufficient condition for this is $(1+\Omega(R-1))/(1-\pi^e\beta R)\Omega R > 1$, which is always true. Thus, $\mathring{B}(R)$ is continuous and strictly increasing in over $(0, 1/\beta\pi^e)$ and, from the definition of $\mathring{B}(R)$, we have the boundaries:

$$\lim_{R\to 0} \mathring{B}(R) = u'^{-1}(\infty)\ (=0), \quad \lim_{R\to 1/\beta\pi^e} \mathring{B}(R) = \frac{\beta\pi^e(1-\Omega)u'^{-1}(0)}{1+\Omega(1-\beta\pi^e)}\ (\leq \infty).$$

This in turn implies that $B(R)$ in (A2) is continuous and strictly increasing in R.

When $\delta = 0$, we have that $B(R) = \mathring{B}(R)$, and the inequality $B(R) < \Sigma$ is recovered by evaluating $\mathring{B}(R)$ at $R = 1/\beta$. When $\delta > 0$, the maximum possible value of R is still $1/\beta\pi^e$, with $\lim_{R\to 1/\beta\pi^e} B(R) = \lim_{R\to 1/\beta\pi^e} \mathring{B}(R)$. The lowest possible value of R, denoted $R_{\min}$, corresponds to the point at which $B(R_{\min}) = 0$. Hence, from (A2), $R_{\min}$ is the (unique) solution to $\mathring{B}(R) = \delta/R$, and by construction we have that $\lim_{R\to R_{\min}} B(R) = 0$. Again, the equivalence between $\beta R < 1$ and $B + \beta\delta < \Sigma$ follows from the increasingness of the $B(R)$ function and its evaluation at $R = 1/\beta$. (Note also that $R_{\min} < 1/\beta$ since $\mathring{B}(1/\beta) > \delta\beta$ under condition iv in the proposition).

Step 2. We must now show that $\beta R < 1$ is a sufficient condition for conjecture **C1** to hold when conditions i-iii in the proposition also hold. For **C1** to hold, both eu and uu workers must be borrowing-constrained in the steady state, so that we must have:

$$u'(c^{eu}) > \beta R(1-\pi^u)u'(c^e) + \beta R\pi^u u'(c^{uu}), \tag{A4}$$

$$u'(c^{uu}) > \beta R(1-\pi^u)u'(c^e) + \beta R\pi^u u'(c^{uu}), \tag{A5}$$

with $u'(c^e) = 1$ (see equation (16)). The right-hand side of (A4) and (A5) are the expected marginal utility of future consumption for an unemployed worker, which in our conjectured equilibrium is the same whether the worker is *eu* or *uu*. Hence, there are two cases to consider. If $c^{uu} \geq c^{eu}$, then $u'(c^{uu}) \leq u'(c^{eu})$ and (A5) is a sufficient for (A4)–(A5) to hold; on the contrary, if $c^{eu} > c^{uu}$, then $u'(c^{eu}) < u'(c^{eu})$ and (A4) is a sufficient condition for (A4)–(A5) to hold. *Case 1.* Assume that $c^{uu} \geq c^{eu}$, so that (A5) is the relevant sufficient condition. The inequality holds for $\pi^u \to 1$ whenever $\beta R < 1$. Then, since $u'(c^{uu}) > u'(c^e)$ (because $c^{uu} < c^e$, otherwise the employed would be constrained), it follows that the inequality holds for all $\pi^u \in [0,1)$. *Case 2.* Assume that $c^{eu} > c^{uu}$, so that (A4) is the relevant sufficient condition. Using (A1), we may rewrite (A4) as follows:

$$\frac{1 - \beta\pi^e R - (1-\pi^e)\beta^2 R^2}{(1-\pi^e)\beta^2 R^2} > \pi^u \left(u'(c^{uu}) - u'(c^e)\right).$$

The fact that $\beta R < 1$ ensures that the left hand side of this inequality is positive, while $c^{uu} < c^e$ implies the right hand side also is. Thus, the inequality holds provided that π^u is sufficiently small (condition ii in Proposition 1).

Maximum public debt-output ratio. Note that the condition according to which $B < \Sigma - \delta\beta$ can be expressed equivalently as a condition on the maximum level of the steady-state public debt-output ratio, B/Y, consistent with the liquidity-constrained equilibrium. Using (28), (22), the fact that $T = B(R-1)$ and the definitions of Ψ and Υ in Section 2.3, we find that the output-public debt ratio can be expressed as follows:

$$\frac{Y}{B} = \Omega + \frac{1}{B}\left(\Psi + \frac{\Omega\delta}{R}\right) + \Upsilon R.$$

Recall that R increases with B, so the second term in the right-hand side of the latter equation falls with B while the third term rises with B. However, as π^u becomes small (condition ii) in the proposition), ΥR becomes small relative to $(\Psi + \Omega\delta/R)B^{-1}$, thus the second term determines how B affects Y/B. This implies that the public debt-output ratio B/Y rises with B, so that we can find the maximum value of B/Y consistent with our limited-heterogeneity equilibrium by evaluating B/Y at $R = 1/\beta$. After some rearrangements, we obtain:

$$\left(\frac{B}{Y}\right)_{R=1/\beta} = \left(\frac{\Omega\Sigma + \Psi}{\Sigma - \delta\beta} + \frac{\Upsilon}{\beta}\right)^{-1}.$$

A2. Dynamics and stability

We use hatted variables to denote level-deviations from steady state (i.e., $\hat{X}_t = X_t - X$). First, substitute (9)–(10) into the linearised versions of (8) and (26) to obtain:

$$\hat{B}_t = \left(\frac{B}{1+\phi}\right)\hat{R}_{t-1} + \left(\frac{R}{1+\phi}\right)\hat{B}_{t-1} + \left(\frac{1}{1+\phi}\right)G_t + \left(\frac{1}{1+\phi}\right)T_t^c, \tag{A6}$$

$$\hat{R}_t = \frac{-R^2\beta\left(1-\pi^e\right)u''\left(c^{eu}\right)}{1-\Omega}\left[\left(1+\Omega\phi\right)E_t(\hat{B}_{t+1}) - \psi G_t - \Omega\chi T_t^c\right] - \left(1-\pi^e\right)\beta u''\left(c^{eu}\right)\delta E_t(\hat{R}_{t+1}), \tag{A7}$$

where, from (A1)–(A2),

$$c^{eu} = u'^{-1}\left(\frac{\left(\beta R\right)^{-1} - \pi^e}{1-\pi^e}\right) = \frac{\left(B+\delta/R\right)\left(1+\Omega\left(R-1\right)\right)}{1-\Omega}.$$

Equations (A6)–(A7) define a two-dimensional backward/forward dynamic system, with sequences of unknowns $\{\hat{B}_t\}_{t=0}^{\infty}$ and $\{\hat{R}_t\}_{t=0}^{\infty}$ and forcing sequences $\{G_t\}_{t=0}^{\infty}$ and $\{T_t^c\}_{t=0}^{\infty}$. The solution to this system takes the form of a VAR whose coefficients can be recovered by the method of undetermined coefficients. More specifically, let $X_t \equiv [\ \hat{B}_t \quad \hat{R}_t\ ]'$ and $Z_t \equiv [\ G_t \quad T_t^c\ ]'$. Leading (A6) one period, taking expectations and substituting (10) and (A7) into the resulting equation, we can express the dynamics of the model in matrix form as $X_t = ME_t\left(X_{t+1}\right) + NZ_t$, where M and N are conformable matrices whose coefficients are functions of the deep parameters of the model. There are two cases to consider, depending on whether $\delta = 0$ or $\delta > 0$.

Case 1. When $\delta = 0$, M is singular, implying that the solution dynamics of the model is univariate. To see why this is the case, let us rewrite (A7) as follows, making use of (A1)–(A2) and the definition of ρ in (33):

$$\left(\rho/B\right)E_t\left(\left(1+\Omega\phi\right)\hat{B}_{t+1} - \psi G_t - \Omega\chi T_t^c\right) = \hat{R}_t. \tag{A8}$$

Leading (A3) one period and taking expectations, solving (A5) for $E_t(\hat{B}_{t+1})$, and then equating the two expressions, we obtain:

$$R\hat{B}_t + \psi\left(1 - \frac{\left(1+\phi\right)}{1+\Omega\phi}\right)G_t + \chi\left(1 - \frac{\left(1+\phi\right)\Omega}{\left(1+\Omega\phi\right)}\right)T_t^c = B\left(\frac{\left(1+\phi\right)}{\rho\left(1+\Omega\phi\right)} - 1\right)\hat{R}_t.$$

Now, lagging the latter equation one period, solving it for $\hat{R}_{t-1}$and substituting the resulting expression into (A3), one finds equation (32) with coefficients:

$$\gamma = \frac{R}{1+\phi-\rho\left(1+\phi\Omega\right)},\ \mu = \frac{1}{1+\phi},\ \nu = -\frac{\gamma\rho\psi\phi\left(1-\Omega\right)}{R\left(1+\phi\right)},\ \upsilon = \frac{\gamma\rho\chi\left(1-\Omega\right)}{R\left(1+\phi\right)},$$

where R is uniquely defined by B (see Appendix A1).

The sign of γ is related to the stationarity requirement that $|\gamma| < 1$. In the case where $\gamma > 0$, then a necessary and sufficient condition for stationarity is (33) in the body of the paper, given that $1 - \rho\Omega > 0$. Does a stationary path for B_t exist consistent with the case where $\gamma < 0$? If $\gamma < 0$, the necessary and sufficient condition for stationarity becomes $\phi < (-1 - R + \rho) / (1 - \Omega\rho)$, but the right-hand side of this inequality is negative. Since this is inconsistent with $\phi > 0$, it must be the case that (33) holds, which in turn implies that $\gamma > 0$. By implication $\nu < 0$, and obviously $\mu > 0$ and $\upsilon > 0$ since $\phi > 0$. Finally, with $\gamma > 0$ and $1 - \rho\Omega > 0$ we have $\partial\gamma/\partial\phi < 0$.

Case 2. When $\delta > 0$, M is invertible and the solution dynamics are bivariate. First, rewrite the forward-looking dynamics $X_t = ME_t(X_{t+1}) + NZ_t$ as follows:

$$E_t(X_{t+1}) = M^{-1}X_t - M^{-1}NZ_t. \tag{A9}$$

We know from the literature on expectational linear systems (e.g., Uhlig, 2001) that the solution to (A9) has the following VAR representation:

$$X_t = \tilde{M}X_{t-1} + \tilde{N}Z_t, \tag{A10}$$

where $\tilde{M}$ and $\tilde{N}$ are matrices to be determined. Leading (A10) one period, taking expectations and using (A9) enables us to fully identify $\tilde{M}$ and $\tilde{N}$. We may then verify numerically that for the parameter configurations considered when running impulse-response functions ϕ is sufficiently large for $\{X_t\}_{t=0}^{\infty}$ to remain stationary.

A3. Imperfectly elastic labour supply

For simplicity we assume here that $\delta = 0$, but nothing peculiar hinges on this assumption. With imperfectly inelastic labour supply, asset accumulation is gradual, and not all working agents end the period with the same asset wealth. We denote by a_t^{ue} and a_t^{ee} the end-of-period asset wealth of ue and ee workers, respectively, and by c_t^{ue} and that c_t^{ee} the corresponding individual consumption levels. By assumption, all ee workers pool their asset wealth at the beginning of the period, so any ee worker turns out starting the period with individual asset $\left(\omega^{ee}a_{t-1}^{ee} + \omega^{ue}a_{t-1}^{ue}\right) / (\omega^{ee} + \omega^{ue})$ –there are $\pi^e\omega^{ee}$ and $\pi^e\omega^{ue}$ workers entering date t with wealth a_{t-1}^{ee} and a_{t-1}^{ue}, respectively, and the total number of ee workers is $\omega^{ee} = \pi^e(\omega^{ee} + \omega^{ue})$.

Since $\omega^{ee}/\left(\omega^{ee}+\omega^{ue}\right)=\pi^{e}$ and $\omega^{ue}/\left(\omega^{ee}+\omega^{ue}\right)=1-\pi^{e}$, we may rewrite the budget constraint of an ee worker as follows:

$$ee: c_t^{ee}+a_t^{ee}=\left(\pi^{e} a_{t-1}^{ee}+\left(1-\pi^{e}\right) a_{t-1}^{ue}\right) R_{t-1}+l_t^{ee}-T_t. \tag{A11}$$

Since eu workers hold end-of-period wealth level a_t^{ue}, their budget constraint is:

$$ue: c_t^{ue}+a_t^{ue}=l_t^{ue}-T_t. \tag{A12}$$

Note that workers who fall into unemployment at date t can now be of two different types, depending on their asset holdings at the end of date $t-1$ with (i.e., a_{t-1}^{ee} or a_{t-1}^{ue}), which in turn depends on their labour statuses at dates $t-1$ and $t-2$. These two types (that is, 'eeu workers' and 'ueu workers') have the following budget constraints:

$$eeu: c_t^{eeu}=a_{t-1}^{ee} R_{t-1}-T_t, \;\; ueu: c_t^{ueu}=a_{t-1}^{ue} R_{t-1}-T_t.$$

Finally, the budget constraint of uu workers is unchanged (i.e., $uu: c_t^{uu}=\kappa-T_t$).

The optimal asset demand and labour supply decisions of employed workers are as follows: ee workers, who end the current period with wealth a_t^{ee}, remain employed with π^{e}, in which case they remain of the ee type, or fall into unemployment, in which case they become of the eeu type. Similarly, ue workers stay employed and become ee or fall into unemployment and become ueu. Hence the optimal asset demand and labour supply decisions of ee and ue workers must satisfy:

$$\begin{aligned}
ee&: u'\left(c_t^{ee}\right)=\beta \pi^{e} R_t u'\left(c_{t+1}^{ee}\right)+\beta\left(1-\pi^{e}\right) R_t E_t u'(c_{t+1}^{eeu}), \;\; u'\left(c_t^{ee}\right)=v'\left(l_t^{ee}\right), \\
ue&: u'\left(c_t^{ue}\right)=\beta \pi^{e} R_t u'\left(c_{t+1}^{ee}\right)+\beta\left(1-\pi^{e}\right) R_t E_t u'(c_{t+1}^{ueu}), \;\; u'\left(c_t^{ue}\right)=v'\left(l_t^{ue}\right).
\end{aligned}$$

Finally, the bond market clearing condition must be modified to account for the fact that $a_t^{ee} \neq a_t^{ue}$ whenever $v'(l) \neq 1$. It is now given by $\omega^{ee} a_t^{ee}+\omega^{ue} a_t^{ue}=B_t$.

Note that when labour supply becomes perfectly elastic (i.e., $v'(l)=1\ \forall l$, as in our baseline utility function), the intratemporal optimality conditions for ee and ue workers give $u'\left(c_t^{ee}\right)=u'\left(c_t^{ue}\right)=1$, so that $c_t^{ee}=c_t^{ue}=u'^{-1}(1)=c^{e}$. Then, their intertemporal optimality conditions, combined with the budget constraints of eeu and ueu workers, give

$$1=\beta \pi R_t+\beta(1-\pi) R_t E_t u'(a_t^{se} R_t-T_{t+1}), \; s=u, e,$$

so that $a_t^{ee}=a_t^{ue}=a_t^{e}$ and $c_t^{eeu}=c_t^{ueu}=c_t^{eu}$. Hence the economy with partial risk sharing nests our baseline model as a special case.

Appendix B. Liquidity-constrained entrepreneurs

B1. Proof of Proposition 2

We must first derive the dynamic system characterising the entrepreneurial equilibrium under the joint conjecture that entrepreneurs are always borrowing-constrained while employed households never are, and then derive from the steady-state relations the range of debt levels compatible with this joint conjecture. Equations (38) and (40) give:

$$\tilde{c}_t^e = u'^{-1}\left(w_t^{-1}\right), \; c_t^f = u'^{-1}\left(\beta w_t^{-1} E_t\left(w_{t+1}^{-1}\right)\right) \tag{B1}$$

Substituting (B1) into (43), the goods-market equilibrium can be written as:

$$u'^{-1}\left(w_t^{-1}\right) + (1-\theta)\, u'^{-1}\left(\beta w_t^{-1} E_t\left(w_{t+1}^{-1}\right)\right) + (2-\theta)\, G_t = (1-\theta)\, l_{t-1}^f \tag{B2}$$

Substituting (8), (41) and (B1) into the budget constraint of f-households, (36), gives:

$$u'^{-1}\left(\beta w_t^{-1} E_t\left(w_{t+1}^{-1}\right)\right) + w_t l_t^f = (2-\theta)\left(B_t - G_t + \Gamma T_t\right) + \frac{\delta\left(1 + (1-\theta) R_t\right)}{R_t}. \tag{B3}$$

Finally, substituting (B1) into (39), the Euler equations for employed households is:

$$w_t^{-1} = \beta R_t\left(\theta E_t\left(w_{t+1}^{-1}\right) + (1-\theta)\, E_t\left(\beta w_{t+1}^{-1} E_{t+1}\left(w_{t+2}^{-1}\right)\right)\right) \tag{B4}$$

Since shocks are small by assumption, the dynamic system just derived is an equilibrium if, in the steady state, i) all employed households hold positive assets at the end of the current period (which, from (23), is ensured by $B > 0$), and ii) entrepreneurs are always borrowing-constrained, i.e., $u'(c^f) > \beta R u'\left(\tilde{c}^e\right)$. From (B1), this latter condition is equivalent to $wR < 1$. Now, the steady state counterpart of (B4) gives:

$$w = \beta^2 (1-\theta)\, R / \left(1 - \beta\theta R\right), \tag{B5}$$

so that $\partial w / \partial R > 0$. Substituting (B5) into the inequality $wR < 1$, we find that entrepreneurs are borrowing-constrained if and only if $R < 1/\beta$.

We may now compute B^{**}, the unique upper debt level ensuring that $R \in (0, 1/\beta)$ whenever $B \in (0, B^{**})$. First, use the facts that $G = 0$ and $T = B\,(R-1)$ to write the steady-state counterparts of (B2) and (B3) as follows:

$$wl^f = \frac{wu'^{-1}\left(w_t^{-1}\right)}{1-\theta} + wu'^{-1}\left(\beta w^{-2}\right),$$

$$wl^f = \left(B + \frac{\delta}{R}\right)\left(1 + (1-\theta) R\right) - u'^{-1}\left(\beta w^{-2}\right).$$

Equating the two, using (B5) and rearranging, we can write steady-state public debt as:

$$\tilde{B}(R)=\left(\frac{R}{1/R+1-\theta}\right)\left(\frac{\beta^{2}(1-\theta)}{1-\beta\theta R}\right)^{2}\left(\frac{u'^{-1}(w^{-1})}{(1-\theta)w}+\frac{u'^{-1}(\beta w^{-2})}{w}+\frac{u'^{-1}(\beta w^{-2})}{w^{2}}\right)-\frac{\delta}{R}, \tag{B6}$$

where w is itself a function of R (see (B5)). The term $-\delta/R$ in (B6) is continuously increasing in R over $(0,\infty)$. The terms inside the first two pairs of large brackets in (B6), as well as w in (B5), are all continuously increasing in R over $[0,1/\theta\beta)$. Hence, if the term inside the third pair of large brackets is non-decreasing in w, then $\tilde{B}(R)$ will be continuous and increasing in R over $(0,1/\theta\beta)$; we now show that this is the case provided that $\sigma(c)\leq 1$ for all c. By making use of (B1), we can compute the following derivatives:

$$\frac{\partial}{\partial w}\left(\frac{u'^{-1}(w^{-1})}{w}\right)=\frac{\tilde{c}^{e}}{w^{2}}\left(\frac{1}{\sigma(\tilde{c}^{e})}-1\right),\quad \frac{\partial}{\partial w}\left(\frac{u'^{-1}(\beta w^{-2})}{w}\right)=\frac{c^{f}}{w^{2}}\left(\frac{2}{\sigma(c^{f})}-1\right),$$
$$\text{and}\quad \frac{\partial}{\partial w}\left(\frac{u'^{-1}(\beta w^{-2})}{w^{2}}\right)=\frac{2c^{f}}{w^{3}}\left(\frac{1}{\sigma(c^{f})}-1\right).$$

All three are non negative if $\sigma(c)\leq 1$, implying that $\tilde{B}(R)$ is continuously increasing over $(0,1/\theta\beta)$. With $\delta\geq 0$, the lower bound for R consistent with $\tilde{B}>0$ is $\tilde{R}_{\min}$ that solves $\tilde{B}(R)=0$, and by construction we have that $\lim_{R\to\tilde{R}_{\min}}\tilde{B}(R)=0$ and $\tilde{R}_{\min}<1/\beta$. Moreover, equation (B6) implies that $\lim_{R\to 1/\theta\beta}\tilde{B}(R)=\infty$. Thus, we may compute the joint condition on (B,δ) stated in Proposition 2 by evaluating $\tilde{B}(R)$ at $R=1/\beta$.

As in the model with liquidity-constrained households, one may also compute the equivalent maximum steady-state public debt-output ratio consistent with the bindingness of the borrowing constraint by evaluating steady-state output Y at $R=1/\beta$. At $R=1/\beta$, we have $w=\beta$ (see (B5)) and hence $\tilde{c}^{e}=c^{f}=u'^{-1}(\beta^{-1})$ (see (B1)). In this situation, the market clearing condition (43) gives $Y=u'^{-1}(\beta^{-1})$ and an upper value for the ratio of:

$$\left(\frac{B}{Y}\right)_{R=1/\beta}=\frac{\beta^{2}}{\beta+1-\theta}+\frac{\beta}{1-\theta}.$$

B2. Dynamics and stability

The dynamic system characterising the behaviour of the entrepreneurial model is derived as follows. First, substitute the linear counterparts of (9) and (B1) and into the linearised versions of (8) and (B2)–(B4). The latter equations then form a four-dimensional expectational dynamic system with forcing terms G_t and T_t^c and vector of unknowns:

$$X_t=\left[\begin{array}{cccc} B_t & l_t^f & R_t & w_t \end{array}\right]'$$

This system can be solved numerically for its auto-regressive representation using standard methods once values have been assigned to all deep parameters of the model and to the target debt level B. (Here again the latter is chosen so as to generates a steady state value of R of 1.01, but equation (B6) rather than (A2), is used.) Finally, total private consumption is $C_t = (1 - \Gamma)\, \tilde{c}_t^e + \Gamma c_t^f$, with $\tilde{c}_t^e$ and c_t^f given by (B1), and aggregate output is $Y_t = \Gamma l_{t-1}^f$. For our baseline parameters, the stationarity requirement is $\phi > \phi_{\min} \simeq 0.134$

B3. Long projects

Let us here assume that $\delta = 0$ and that all parameters apart from the length of projects are at their baseline value. For any value of τ, there are ee entrepreneurs and fe entrepreneurs in the economy, with budget constraints (34)–(35) and optimal labour supply and asset demand choices characterised by (38)–(39). However, values of τ higher than one imply that there are several types of entrepreneurs running a project –as many as the number of periods that a project lasts. For the sake of conciseness we only show how to construct the equilibrium when $\tau = 2$ here, but the approach can be applied straightforwardly to higher values of τ.

With two-period projects there are two types of active entrepreneurs, those who are currently starting a project and those who did so in the previous period. Let us call the former 'f entrepreneurs' (by analogy with the $\tau = 1$ case) and the latter 'ff entrepreneurs'. Under **C1**-**C2**, their budget constraints are as follows:

$$f : c_t^{ef} + w_t l_t^f = \tilde{a}_{t-1}^e R_{t-1} - T_t, \quad ff : c^{ff} + w_t l_t^{ff} = l_{t-1}^f - T_t.$$

In short, these two equations indicate that entrepreneurs who meet a project opportunity entirely liquidate their asset wealth to invest in it, and will be using the implied output to re-invest in the project (and consume) in the next period; we may then check numerically that when B is sufficiently small, then $w_t R_t < 1$, so that these entrepreneurs would like to borrow rather hold positive assets at the end of every period (that is, the economy is liquidity-constrained). There are now two optimal labour demand conditions (rather than the unique condition (40) in the one-stage case), depending on the stage of the project:

$$w_t u'(c_t^f) = \beta E_t u'(c_{t+1}^{ff}), \;\; w_t u'(c_t^{ff}) = \beta E_t u'\left(\tilde{c}_{t+1}^e\right).$$

Finally, given that entrepreneurs not running a project will meet a project opportunity with probability θ in the next period and, in this case, run their project for exactly two periods, the (asymptotic) shares of each type of entrepreneur in the economy are:

$$\tilde{\omega}^{ff} = \tilde{\omega}^{f} = \tilde{\omega}^{fe} = (1-\theta)/(3-2\theta), \quad \tilde{\omega}^{ee} = \theta/(3-2\theta),$$

while the total number of entrepreneurs running a project is $\tilde{\omega}^{ef} + \tilde{\omega}^{ff} = 2(1-\theta)/(3-2\theta)$ here. Total output at date t results from the labour inputs of the two types of active entrepreneurs in the previous period, $\tilde{\omega}^{f} l_{t-1}^{f} + \tilde{\omega}^{ff} l_{t-1}^{ff}$. Hence, the model is closed once the following market-clearing conditions are imposed in bonds and goods markets:

$$\left(1-\tilde{\omega}^{f}-\tilde{\omega}^{ff}\right) a_t = B_t,$$
$$\left(1-\tilde{\omega}^{f}-\tilde{\omega}^{ff}\right) \tilde{c}_t^{e} + \tilde{\omega}^{f} c_t^{f} + \tilde{\omega}^{ff} c_t^{ff} + G_t = \tilde{\omega}^{f} l_{t-1}^{f} + \tilde{\omega}^{ff} l_{t-1}^{ff}.$$

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