Econ 509, *Introduction to Mathematical Economics I*
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Summer 2010

Exam 1

Please write in the space provided, and continue on the back of the page if needed, but mark "Continued on Back" clearly. All questions are of equal weight. You have 75 minutes.

Please print your name clearly in CAPITAL LETTERS: _____

Question 1
Verify both associative rules using Venn Diagrams:

$$\begin{aligned} \text{a} &: \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\ \text{b} &: \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \end{aligned}$$

Question 2
Write a definition of the linear independence of n vectors, $x_1, x_{2,}...x_n$.

Question 3
Find the derivative with respect to x of:

$$\begin{aligned} \text{a} &: \frac{f(x)\,x}{g(x)} \\ \text{b} &: \ln\left(\frac{f(x)}{x+1}\right) \\ \text{c} &: (1/x)^{f(x)} \end{aligned}$$

Question 4
Consider the transition matrix

$$P = \begin{bmatrix} p_{ee} & p_{eu} \\ p_{ue} & p_{uu} \end{bmatrix}$$

that pertains to the state vector $x_t' = \begin{bmatrix} e_t & u_t \end{bmatrix}$, where $e_t + u_t = 1$ and all elements are non negative. Find the steady state vector x.

Question 5
An elasticity of y with respect to x is defined as

$$\sigma_{y,x} = \frac{dy/y}{dx/x} \ . \tag{1}$$

A firm's demand function is: $Q = A - B \cdot P$, where $A > 0$ and $B > 0$. Use the definition of the elasticity (1) and the total differential approach to find the price elasticity of demand ($\sigma_{Q,P}$) and determine the price at which the elasticity of demand is exactly -1.

Question 6

Invert the following matrix using the formula $A^{-1} = \text{adj}A/\,|A|$ (show your work at each step). Then check your work by showing that the product of the inverse and the original matrix is the identity matrix.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Question 7
Define a base for a vector space. Does the matrix A from question 6 constitute a base for $\mathbb{R}^3$?

Question 8
Use Cramer's rule to solve the following equations for x, y and z as a function of the parameters, a, b and c:

$$\begin{aligned} -x + y + z &= a \\ x - y + z &= b \\ x + y - z &= c \end{aligned}$$

Question 9

A, B, and C are $n \times n$ matrices. Please simplify as much as possible the following expressions, showing intermediate steps.

$$\begin{aligned} \text{a} &: (A + C'B)' \\ \text{b} &: (C + BA)^{-1} \\ \text{c} &: (A'B'C')'(BA)^{-1} \end{aligned}$$

Question 10
Find the Jacobian matrix (not determinant) of

$$F(x,y,z) = \begin{bmatrix} 5x^{-4}y \\ e^{xz^3} \\ \ln(y+z) \end{bmatrix}.$$