Econ 5090, *Introduction to Mathematical Economics I*
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**Exam 1**

Please write in the space provided, and continue on the back of the page if needed, but mark "Continued on Back" clearly. All questions are of equal weight. You have 90 minutes.

Please print your name clearly in CAPITAL LETTERS: ________________________________

**Question 1**

1. Write TWO definitions of the linear independence of $n$ vectors, $x_1, x_{2,}...x_n$.
2. The matrix $A_{n\times n}$ has an inverse. Explain why the columns of $A$ span $\mathbb{R}^n$.

**Question 2**

Show that the determinant of any lower triangular matrix is the product of diagonal elements (Hint: use the Laplace Expansion) (No need to prove it, but this also holds for any upper triangular or diagonal matrix).

**Question 3**

Consider the following market model

$$\begin{aligned} q^d &= d(\overset{-}{p}, \overset{+}{y}) \\ q^s &= s(\overset{+}{p}, \overset{+}{r}) \\ q^s &= q^d \ , \end{aligned}$$

where $y$ is consumer income, $r$ is rainfall and $-/+$ above an argument denotes the direction of the effect of an increase in that variable on the image.

Use a THREE function system of THREE endogenous variables $\left(q^d, q^s, p\right)$ and two exogenous variables $(y, r)$, $F\left(q^d, q^s, p; y, r\right) = 0$ and the implicit function theorem to determine the sign of $\partial q^d / \partial r$ and $\partial p / \partial r$. In doing so, state clearly the premises for the theorem and check them.

**Question 3 – continued**

**Question 4**

Consider the utility function $u(x, y)$ and the budget constraint $p_x x + p_y y = I$ (assume it is binding). What is the total effect of a small change in $x$ on utility?

**Question 5**

Find the stationary values (critical points) of the following functions, and determine whether they are maxima or minima:

1. $y = x \cdot \ln x$
2. $y = \frac{x}{e^x}$
3. $y = x^3 - 3x$

**Question 5 – continued**

**Question 6**

1. Suppose that the matrix product $AB$ is defined. We know that $(AB)' = B'A'$, if the product $B'A'$ exists. Under what conditions will the product $B'A'$ exist?

2. Let the *symmetric* matrix $A$ be

$$A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

   and let the matrix $B$ be

$$B = \begin{bmatrix} d & e \\ f & g \end{bmatrix}$$

   What must the matrix $B$ be in order to satisfy $AB = BA$? Can you generalize this result?

XXXX NEED TO MODIFY POINT 2 IN THIS QUESTION, IT IS NOT STRAIGHTFORWARD XXXX

**Question 6 – continued**