Fritz John Theorem For One Equality Constraint

Let f, $g \in C^1$ be functions of $x \in \mathcal{R}^n$. Suppose that x^* is a maximizer of f on the constrained set defined by $g(x) = c$.

Form the Lagrangian

$$\mathcal{L}(x, \lambda) = \lambda_0 f(x) + \lambda_1 [c - g(x)] \ .$$

There exists λ_0^* and λ_1^* such that

1. $\frac{\partial \mathcal{L}(x^*, \lambda^*)}{\partial x_i} = 0$, $i = 1, 2, \ldots n$
2. $\frac{\partial \mathcal{L}(x^*, \lambda^*)}{\partial \lambda_1} = 0 \ \Leftrightarrow \ g(x^*) = c$
3. $\lambda_0^* = 0$ or 1
4. $(\lambda_0^*, \lambda_1^*) \neq (0, 0)$.

This theorem generalizes to many equality constraints.