Production Function Estimation with Multi-Destination Firms

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Abstract

We develop a production function estimator for the case when firms endogenously select into multiple destination markets where they compete imperfectly, and when output is denominated only in value. The estimator exploits a novel source of variation to identify demand curvature: firm-level export shares. We demonstrate that ignoring the multi-destination dimension (i.e., exporting) yields biased and inconsistent inference. We estimate in French Manufacturing data increasing total returns to scale, decreasing returns to flexible inputs, and demand elasticities between -19.42 and -3.56. These estimates imply 5 to 8 times larger effects of a U.S. tariff increase compared to estimates from alternative estimators.

Keywords: production function, productivity, returns to scale, non-segmented markets, trade, learning by exporting

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1 Introduction

Production function estimation is a central component of many economic analyses.[1] While early work relied on restrictive assumptions with respect to the evolution of unobserved shocks to supply, remarkable progress has been made in the last 30 years towards specifying flexible conditions under which structural elements of supply can be identified from firm or plant-level data (Olley & Pakes, 1996; Levinsohn & Petrin, 2003; Wooldridge, 2009; Ackerberg et al., 2015; Gandhi et al., 2020). Nevertheless, significant gaps remain between the conditions assumed by the literature and the real-world datasets confronted by practitioners (De Loecker & Goldberg, 2014). This paper's objective is to close part of this gap.

A fundamental problem that is emphasized in the literature is that since outputs are usually observed in monetary terms—i.e., sales, not physical quantities—unobserved pricing decisions of firms directly influence the outcome variable, sales. In such cases, unobserved shocks to demand can generate bias in the estimation of output elasticities, even if unobserved shocks to supply are adequately controlled for (Klette & Griliches, 1996; Foster et al., 2008; De Loecker & Goldberg, 2014; De Loecker & Syverson, 2021).

When estimating production functions with outputs denominated in value, researchers usually follow one of three approaches: (1) deflate revenues with an industry-wide price deflator and treat the resulting series as if they were quantities, (2) control for an industry-wide aggregate demand shock, or (3) impose constant returns to variable inputs. It is well known that the first approach fails to neutralize omitted variable bias if firms charge different prices (De Loecker & Goldberg, 2014). The second approach yields consistent estimates under parametric restrictions on the inverse demand function, and if theory-consistent price and quantity indices are observed for *all* markets served by firms. Lacking this information, researchers typically include aggregate controls for the domestic market only (Klette & Griliches, 1996; Melitz & Levinsohn, 2006; De Loecker,

[1] Researchers estimate production functions for a wide array of purposes. The structural coefficients can be used to compute returns to scale (Caballero & Lyons, 1992) or markups (Hall, 1986). Alternatively, the estimated residuals can be used as controls to address omitted variable bias (Almunia et al., 2021), or as an dependent variable (Harrigan et al., 2026). For a recent review of the literature, see De Loecker & Syverson (2021).

2011), which is insufficient to address the bias if firms serve multiple markets. Under constant returns to variable inputs, the elasticity of demand can be identified from the ratio of variable input costs to revenues, and the elasticity of output to quasi-fixed factors can be estimated via standard GMM techniques (Aw et al., 2011; Almunia et al., 2021); but the assumption of constant returns to variable inputs is hard to defend *a priori*; moreover, constant returns to variable inputs rules out many mechanisms of interest, such as cross-market interactions.[2]

In this paper, we develop a procedure for estimating structural elements of supply and demand from firm-level data when outputs are only observed in monetary terms, firms serve potentially multiple markets, and returns to variable inputs are potentially non-constant. As in previous work, we substitute for missing price data with assumptions on demand and market structure. But instead of identifying the demand curvature parameter exclusively from time-series variation in aggregate quantity indices, we exploit a novel time- and cross-sectional source of variation: the *export share of sales* at the firm level. We show that this readily-observable metric encodes valuable information about the demand conditions faced by firms on all destination markets, given standard assumptions on the profit maximizing behavior of firms. We combine this metric with information derived from the first order necessary conditions of the firm's optimization problem to build model-consistent moment conditions, and thus estimate the curvature of demand, factor output elasticities, and the elasticity of productivity to observed determinants, such as contact with foreign markets.

We start by specifying a data generating process in which heterogeneous firms serve potentially multiple destination markets. Within each destination market, firms face an isoelastic demand curve, with a price elasticity of demand that may vary by destination.[3] Firms face on each market two types of idiosyncratic demand shocks. First, there are idiosyncratic demand shocks that are revealed to the firm prior to making production plans—which we call "*ex ante*" shocks. Second,

[2]For example, Almunia et al. (2021) propose a model in which decreasing returns to variable inputs leads firms to substitute sales away from particular markets when those markets experience downturns. Constant returns to variable inputs are inconsistent with such a mechanism. See also Arkolakis et al. (2023) for cross-market complementarities in multinational location choices.

[3]This is a more general framework than is usually employed in trade models. Researchers often allow that the elasticity of substitution varies by product, but rarely by destination within product (e.g., Broda & Weinstein 2006; Caliendo & Parro 2015; Shapiro 2016; Fontagné et al. 2022). A notable exception is Imbs & Méjean (2017), who identify destination-specific elasticities of substitution from trade flows and price data.

there are idiosyncratic demand shocks that are realized at the point of sale, and hence are unknown to firms at the time that they choose flexible inputs—which we call "*ex post*" shocks. If not controlled for, the *ex ante* demand shocks generate transmission bias, because they both influence flexible input choices and directly affect revenues. The *ex post* shocks do not generate transmission bias, but rationalize variation in flexible input shares in revenues across firms that use the same quantities of inputs. *Ex post* firm-destination-period shocks also generate variation in prices and markups, both across firms and within firms across destinations.[4] In our main specification, we assume firms engage in monopolistic competition, but we explain how to extend our approach to Cournot competition.[5] In both cases we must assume that the demand curve is iso-elastic.

In each period firms choose simultaneously—conditional on quasi-fixed inputs—the destinations to serve (which we call their "export strategy"), flexible inputs for production, and quantity allocations across destinations to maximize expected profits. While the export strategy is, in principle, the solution to a combinatorial discrete choice problem, in practice our estimation strategy does not require us to solve the optimal export strategy of the firm. As long as the first-order conditions are satisfied conditional on the export strategy, we can characterize firm behavior without imposing additional restrictions on the profit function.[6]

Exploiting nonparametric identification results from Gandhi et al. (2020), we estimate the output elasticities of flexible and quasi-fixed inputs without imposing any assumptions on the production function—in particular, about returns to variable inputs—beyond multiplicative separability between the productivity shifter and the rest of the production function. Estimation proceeds in two steps. First, relying on the first order conditions of the optimization problem conditional on the export strategy and quasi-fixed inputs, we project the expenditure share of flexible inputs (e.g., materials) in revenues on all inputs. This "factor share" regression nonparametrically identifies the

[4]Hence, while we impose structure on demand and competition in order to circumvent missing price data, our model is flexible enough to allow for heterogeneous pricing behavior that is potentially important in real-world markets. While our model allows for these features, we cannot study strategic pricing behavior, given the data constraints.

[5]Adapting our estimator to Cournot competition requires dropping *ex post* firm-destination-period shocks, while allowing for an unanticipated (by the firm) shock to firm-level productivity. See Appendix B for details.

[6]See Arkolakis et al. (2023) for details on how to solve the combinatorial discrete choice problem under additional restrictions on the profit function.

"revenue elasticity" of flexible inputs, which combines output elasticities with demand elasticities.

In the second step we use the revenue elasticities of flexible inputs from the first step regression to compute the contribution of flexible inputs to domestic revenues. We subtract this contribution from domestic revenues, and then regress the resulting values on predetermined inputs (e.g., capital) and a firm-specific demand proxy constructed from firm-level export revenue shares and the residual from the first stage. Variation in this demand proxy allows us to estimate the curvature of demand without building aggregate demand shifters in each market from industry-wide price indices—an empirical effort plagued by thorny measurement problems that we discuss below—and without even knowing the set of destinations served by firms. The domestic demand elasticity is then used to recover output elasticities and retrieve the production function itself.

In Monte Carlo simulations, we compare the statistical properties of our estimator to (1) the estimator from Aw et al. (2011) that imposes constant returns to variable inputs, (2) the estimator from De Loecker (2011) that allows for different demand elasticities on different markets, (3) the standard practice of deflating firm-level revenues by industry-wide price indices and ignoring demand and price variation across firms, and (4) the estimator that controls for aggregate sales in the domestic market as in Klette & Griliches (1996).[7] Only our multi-destination factor share estimator recovers estimates that are close to the true parameter values in these experiments, whereas the other estimators deviate substantially when the data generating process allows for multiple markets and non-constant returns to variable inputs.

We use our estimator to study returns to scale, the elasticity of demand, and the effect of exporting on productivity in a panel of French manufacturing firms in 1994–2016. We find price elasticities of demand ranging between -19.42 and -3.56 across industries, which is consistent with estimates from the gravity literature (for example, Shapiro 2016; Fontagné et al. 2022). On the supply side, we estimate that returns to flexible inputs are less than 1, on average, which imply negative cross-market cost complementarities, as in Almunia et al. (2021). We find overall increasing returns to scale, on average around 1.15. We also find evidence of learning by exporting

[7] For the alternatives (3) and (4), we consider both the factor share and the control function approaches.

(LBE) between zero and 4 percent year-on-year, which imply cross-sectional differences in productivity between exporters and non-exporters of up to 40 percent. The model with no demand correction yields lower returns to scale, consistent with unaddressed transmission bias. The model with a single-market correction delivers unrealistic elasticities of demand (despite the relatively long period) and unrealistic returns to scale. Estimators based on the control function approach yield extremely low capital elasticities and imprecisely-estimated price elasticity of demand.

Finally, we compare the quantitative implications of these results in two partial equilibrium exercises. Our estimates imply that a 10% increase in tariffs to reach the U.S. (entire world) would lower sales of French exporters to the U.S. (entire world) by 44% (11%). On average across industries, our estimator implies 5 times (8 times) larger effects of the U.S. tariff increase compared to the constant-returns-to-variable-inputs estimator (single-market-correction estimator)—and 86% (140%) larger effects of the global tariff increase.

Our main contribution is to develop a production function estimator that exploits moment conditions that are consistent with a model in which firms potentially serve multiple destination markets where they face idiosyncratic demand shocks—without relying on quantity data. When quantity data are observed, output elasticities can be estimated without imposing structure on pricing behavior (as in, for example, Roberts et al. 2018 and Blum et al. 2023). But in this case, structural assumptions are required on the supply side with respect to how firms apportion inputs across multiple production lines or products. To balance high demands on the data stemming from the existence of multiple production lines, quantity-based multi-product productivity estimators often rely on restrictive functional form assumptions for the production function (e.g., Cobb-Douglas) (Blum et al., 2023; de Roux et al., 2021). Additionally, it may be difficult to compare quantities across firms and products in a meaningful way (De Loecker & Goldberg 2014).[8] Moreover, physical quantity data are only available in rare datasets, and even then only for particular industries, limiting the applicability of quantity-based multi-product estimation techniques.[9] In contrast, we

[8] Quoting directly from De Loecker & Goldberg (2014) referring to quantity data, "the introduction of additional data creates its own challenges; although more data may help alleviate some of the problems discussed above, they are not a panacea" (page 206).

[9] For example, Blum et al. (2023) focus on only ten 3-digit ISIC industries for which there is a standard and uniform

offer an approach that can be implemented in a broad range of differentiated-product markets with information that is widely available.[10]

Beyond the production function estimation literature discussed above, our paper relates to a small literature on cross-market cost complementarities. Berman et al. (2015), Barrows & Ollivier (2021) and Almunia et al. (2021) all estimate the effect of demand shocks in a given market on sales in a different market. If the returns to flexible inputs are decreasing, then more supply to one market increases the cost of serving other markets. Hence, positive *ex ante* demand shocks in one market should lower sales in another market. Neither Berman et al. (2015) nor Barrows & Ollivier (2021) estimate production function parameters, but rather focus on the reduced-form connection between demand shocks in one market and sales in another market. Almunia et al. (2021) specifies a similar model to the one we develop, but adopt a production function estimation that assumes constant marginal costs and myopia relative to *ex post* demand shocks. We show in Appendix E.6 that these assumptions are not consistent with the conditions necessary for cross-market cost complementarities.

Finally, our application is related to the literature on productivity-enhancing effects of exporting (Van Biesebroeck 2005, De Loecker 2007, Wagner 2007, Wagner 2012, De Loecker 2013, Garcia-Marin & Voigtländer 2019, Atkin et al. 2017, Buus et al. 2022). When quantities are observed, one can study learning by exporting (LBE) without explicitly modeling the export market entry decision and export market itself. However, when outputs are denominated in value it is essential to employ a multi-destination model that includes a correction for demand from multiple markets. Otherwise, an inconsistency arises between studying the impact of exporting and an estimation procedure that allows for only one—domestic—market. In fact, we do not know a paper on LBE—

measure of physical quantities of output across firms, which yields only 2,749 firms for Chilean manufacturing. Dhyne et al. (2022) analyze just firms that produce the same two 6-digit products, thereby excluding roughly 90% of firm-year observations. de Roux et al. (2021) focus only on firms producing rubber and plastic products in the Columbian manufacturing survey (covering 362 firms).

[10] CES demand and monopolistic competition are fairly standard assumptions for most manufacturing industries. In Monte Carlo experiments, we find that even when the true data-generating process features Nash competition in quantities, monopolistic competition remains a good approximation to firms' equilibrium behavior, similarly to Barrows et al. (2025) and Head & Mayer (2026).

including those cited just above—that addresses this issue.[11]

2 Model

We model the supply and demand for horizontally differentiated varieties that are produced by single-product firms and consumed in multiple destination markets. The model delivers an estimation equation that links observable outputs (revenues) to inputs and observable demand shifters, and also serves as the data generating process for Monte Carlo simulations in Section 5.

2.1 Demand

There are a fixed number of destination markets indexed by $d \in \{0,1,...,\mathscr{D}\}$, and industries indexed by $i \in \{1,...,\mathscr{I}\}$. Within an industry i, the quantity demanded for a given variety f at time t by consumers in destination market d takes the familiar constant elasticity of substitution (CES) form:

$$X^d_{fit} = \left(\frac{P^d_{fit}}{\Upsilon^d_{it}}\right)^{\frac{1}{\rho^d_i - 1}} B^d_{it} \exp\left(\frac{\varepsilon^d_{fit}}{1-\rho^d_i} + \frac{u^d_{fit}}{1-\rho^d_i}\right) \tag{1}$$

where P^d_{fit} is the price charged for variety f in market d in time t, Υ^d_{it} and B^d_{it} are, respectively, the CES aggregate price and quantity indices, ε^d_{fit} is an *ex ante* variety-specific demand shock (realized prior to production), u^d_{fit} is an *ex post* variety-specific demand shock (realized at the point of sales),[12] and $1/(\rho^d_i - 1) = \eta^d_i < -1$ is a constant industry-specific price elasticity of demand, which may vary across destinations with the demand curvature parameter ρ^d_i. Revenues on each destination market d are given by:

$$R^d_{fit} = \left(X^d_{fit}\right)^{\rho^d_i} \frac{\Upsilon^d_{it}}{\left(B^d_{it}\right)^{\rho^d_i - 1}} \exp(\varepsilon^d_{fit} + u^d_{fit}). \tag{2}$$

Since the estimation proceeds industry by industry, we drop the index i from here on.

[11] Van Biesebroeck (2005), De Loecker (2007) and De Loecker (2013) all deflate sales or value added by industry price indices—invariably, domestic price indices—in order to approximate quantities. This leaves firm-level variation in demand shocks a source of transmission bias. In addition, using domestic price indices implies that price conditions faced by exporters are identical in the domestic and foreign markets, which is at odds with the existence of variable trade barriers and different market conditions.

[12] The *ex post* demand shock, u^d_{ft}, also captures firm-destination-time measurement error in revenues.

2.2 Production

Each firm produces a single differentiated variety that may be shipped to any destination market.[13] Hence, we can use f interchangeably to denote varieties or firms. Firms produce outputs using materials M_{ft}, labor L_{ft}, and capital K_{ft}. We assume that materials are flexible inputs, i.e., they are chosen optimally each period, given material input prices W_t^M.[14] Capital is a quasi-fixed input that evolves each period according to the depreciation rate and an endogenous investment choice:

$$K_{ft} = (1-\varphi^K)K_{f,t-1} + \iota^K_{ft-1}, \tag{3}$$

where φ^K denotes the rate of depreciation and ι^K_{ft-1} is the investment choice in period $t-1$.

We can allow for different assumptions on how labor is chosen. For settings with flexible labor markets, we can treat labor as a flexible input—just like materials—with exogenous price W_t^L. But in most settings, labor can be viewed as a dynamic input. For expositional purposes, we treat labor as a quasi-fixed input with evolution:

$$L_{ft} = (1-\varphi^L)L_{f,t-1} + \iota^L_{ft-1}, \tag{4}$$

with depreciation rate φ^L and net hiring ι^L_{ft-1}. We point out below where the procedure needs to be adjusted to allow for alternative assumptions on labor.[15]

Output is a function of a Hicks-neutral productivity shock ω_{ft} and a twice continuously differentiable transformation of variable and quasi-fixed inputs:

$$Q_{ft} = \exp(\omega_{ft})F(m_{ft}, l_{ft}, k_{ft}) \Longleftrightarrow q_{ft} = \omega_{ft} + f(m_{ft}, l_{ft}, k_{ft}), \tag{5}$$

[13] In Appendix D, we extend the model to the case that firms produce multiple products with heterogeneous technologies, combining our multi-destination framework with the multi-product framework from Valmari (2023).

[14] We impose homogeneity in material input prices because firm-specific material input prices are rarely observed. Homogeneity in material prices implies that quantities of materials are proportional to expenditures on materials. We deflate materials expenditures with an industry-specific input price index. In the case that quantities of materials are observed, we can allow that material input prices vary by firm, but we still impose that firms are price takers. Allowing for manipulation of the input prices would require amendments to the first order conditions, and hence, the factor share regression.

[15] In particular, our French empirical context, characterized by a dual labor market (with both short, fixed-term contracts and long, indefinite-term contracts), may justify an intermediate approach: labor could be treated as a dynamic input, but not quasi-fixed. The stock of labor may evolve slowly through depreciation and investment, yet there could remain a margin that can be adjusted flexibly within a period t. Even though the French dual labor market is known for its rigidity, many French firms adjust their current labor stock to contemporaneous supply and demand shocks, even if not completely (Saint-Paul, 1996; Reshef et al., 2022).

with Q_{ft} (q_{ft}) denoting the quantity in levels (logs) of output produced by firm f in year t, and $F(\cdot)$ ($f(\cdot)$) is an industry-specific function expressed in levels (logs). Productivity ω_{ft} may depend on observable determinants such as export market participation or hiring techies, as in De Loecker (2013) or Harrigan et al. (2026), respectively. Estimating the elasticity of productivity to these determinants is one objective of this paper.

2.3 Optimization

Firms may ship their variety to any destination market, subject to fixed and variable trade costs. We denote the domestic market by $d = 0$ and assume that researchers observe only firms located in that market, as is typically the case. To serve a given market, firms must pay a firm-destination-time specific fixed cost C_{ft}^{d} and a destination-time specific ad valorem "iceberg" cost $\tau_t^d \geq 1$.[16] To sell X_{ft}^{d} units to destination market d, firm f must produce $Q_{ft}^{d} = \tau_t^d X_{ft}^{d}$ units. We assume that there are no domestic fixed costs ($C_{ft}^{0} = 0$), so all firms sell on the domestic market. In contrast, firms may serve any subset of foreign destinations. We denote the set of *foreign* destinations served by firm f in year t as Ω_{ft}, which we refer to as an "export strategy". We remain agnostic about how this set is chosen, as this choice does not affect our estimation procedure.[17]

At each period t, the sum of all units sold to all destination markets must equal total output: $\sum_{d \in 0 \cup \Omega_{ft}} Q_{ft}^{d} = Q_{ft}$. For a general production function, this constraint obliges the firm to simultaneously determine the quantity of output to ship to each destination. Only in the case of constant returns to variable inputs (and, hence, constant marginal costs) would it be optimal for the firm to solve for destination-specific quantities separately, i.e. treat the markets as segmented.[18]

[16] We could allow iceberg trade costs to vary across firms in foreign markets without requiring any modification to the estimation procedure. In contrast, if iceberg costs vary across firms in the domestic market, they would enter the second stage of the estimation. In that case, they would need to be parameterized as functions of observables and included accordingly in the second-stage specification. For simplicity, we maintain the assumption that τ_t^d is homogeneous across firms, with this caveat in mind.

[17] Presumably, firms choose the set of destinations they serve to maximize their expected profits in each period, net of the fixed costs of exporting. Solving for the optimal set of destinations to serve is a complex combinatorial discrete choice problem. Under certain conditions on the profit function, this problem can be solved with the algorithm from Arkolakis et al. (2023), for instance. Alternatively, with a relatively small number of potential destinations, this problem can be solved iteratively by comparing all potential strategies and their associated profits. This is what we do in the Monte Carlo experiments.

[18] Without *ex post* demand shocks, the solution to the firm's problem is the same whether it chooses quantities or prices. In contrast, with *ex post* demand shocks, only a quantity-setting formulation is consistent with the timing of

For a given export strategy Ω_{ft}, firms choose a vector of destination-specific output shares $\boldsymbol{\chi}_{ft} = (\chi^0_{ft}, \chi^1_{ft}, ..., \chi^{\mathscr{D}}_{ft})$, with $\chi^d_{ft} \equiv Q^d_{ft}/Q_{ft}$, and material inputs M_{ft} to maximize expected profits, given material input prices, quasi-fixed inputs, iceberg trade costs, and market-specific demand conditions. While firms observe *ex ante* variety-destination-specific taste shocks before making their quantity choices, they do not observe *ex post* demand shocks at the time of production. Hence, firms take expectations over *ex post* shocks u^d_{ft}, which are assumed to be i.i.d. with a constant mean u that is known to the firm.[19]

Using (2), we write the optimization problem for a given export strategy Ω_{ft} as

$$\begin{aligned} \max_{m_{ft}, \chi^0_{ft}, \{\chi^d_{ft}\}_{d \in \Omega_{ft}}} \quad & \mathbb{E}\Bigg[\Big(\chi^0_{ft} \exp\left(\omega_{ft}\right) F\left(m_{ft}, l_{ft}, k_{ft}\right) \Big)^{\rho^0} D^0_t \exp(\varepsilon^0_{ft}) \exp(u^0_{ft}) \\ + \quad & \sum_{d \in \Omega_{ft}} \Big(\chi^d_{ft} \exp\left(\omega_{ft}\right) F\left(m_{ft}, l_{ft}, k_{ft}\right) \Big)^{\rho^d} D^d_t \exp(\varepsilon^d_{ft}) \exp(u^d_{ft}) - W^M_t M_{ft} \Bigg] \\ + \quad & \lambda_{ft} \left(1 - \chi^0_{ft} - \sum_{d \in \Omega_{ft}} \chi^d_{ft} \right), \end{aligned} \tag{6}$$

where $D^d_t \equiv \Upsilon^d_t \left(B^d_t\right)^{1-\rho^d} \left(\tau^d_t\right)^{-\rho^d}$ is a destination-specific demand shifter for all $d \in \{0 \cup \Omega_{ft}\}$ for the specific industry, and λ_{ft} is the multiplier associated with the constraint $\chi^0_{ft} + \sum_d \chi^d_{ft} = 1$. Assuming monopolistic competition implies that firms take D^d_t as given, and we assume that these aggregate demand shifters are observed by firms before making their decisions.[20]

The first-order necessary condition (FONC) with respect to materials yields

$$\begin{aligned} W^M_t \quad = \quad & \mathscr{E}_u \rho^0 \exp\left(\rho^0 \omega_{ft}\right) F\left(m_{ft}, l_{ft}, k_{ft}\right)^{\rho^0 - 1} \frac{\partial F}{\partial M_{ft}} \left(\chi^0_{ft}\right)^{\rho^0} D^0_t \exp\left(\varepsilon^0_{ft}\right) \\ + \quad & \sum_{d \in \Omega_{ft}} \mathscr{E}_u \rho^d \exp\left(\rho^d \omega_{ft}\right) F\left(m_{ft}, l_{ft}, k_{ft}\right)^{\rho^d - 1} \frac{\partial F}{\partial M_{ft}} \left(\chi^d_{ft}\right)^{\rho^d} D^d_t \exp(\varepsilon^d_{ft}), \end{aligned} \tag{7}$$

the model. Firms choose quantities to maximize expected profits given the distribution of shocks, and then set prices *ex post* to clear demand. A price-setting formulation would be time-inconsistent: a firm could set prices *ex ante* based on expected demand, but would have an incentive to revise them once shocks are realized. Since production decisions are irreversible at the time of sale, the quantity-setting formulation avoids this time inconsistency.

[19] As in Gandhi et al. (2020), *ex post* shocks rationalize errors in the factor share equation, which will be defined below. We extend this notion to the multi-market context.

[20] We develop the case of Cournot competition in Appendix B. This requires dropping *ex post* firm-destination-period shocks, though in this case, we instead allow for an unanticipated (by the firm) shock to firm-level productivity.

with $\mathbb{E}\left[\exp(u_{ft}^d)\right] = \mathscr{E}_u$, a constant, for all firms and destinations. Rearranging yields:

$$\frac{W_t^M M_{ft}}{R_{ft}} = \mathscr{E}_u \rho^0 \sigma_{ft}^M \underbrace{\left[\frac{R_{ft}^0}{R_{ft}} \exp\left(-u_{ft}^0\right) + \sum_{d\in\Omega_{ft}} \frac{\rho^d}{\rho^0} \frac{R_{ft}^d}{R_{ft}} \exp\left(-u_{ft}^d\right)\right]}_{\equiv Z_{ft}}, \tag{8}$$

where $\sigma_{ft}^M \equiv \frac{\partial F\left(m_{ft},l_{ft},k_{ft}\right)}{\partial M_{ft}} \frac{M_{ft}}{F\left(m_{ft},l_{ft},k_{ft}\right)}$ denotes the output elasticity of materials, which is allowed to vary across firms and over time. The "factor shares" equation (8) serves as the basis for the first step in the estimation procedure.

We denote the term in large brackets in equation (8) by Z_{ft}, which is a weighted sum of the inverse *ex post* shocks, where the weights are a combination of destination-specific revenue shares and the ratio of curvature parameters relative to the domestic ρ^0. As we explain below, this term plays an important role in the estimation procedure.

First-order necessary conditions with respect to output shares χ_{ft}^d yield, for each $d \in \{0 \cup \Omega_{ft}\}$,

$$\mathscr{E}_u \rho^d \exp(\rho^d \omega_{ft}) F\left(m_{ft}, l_{ft}, k_{ft}\right)^{\rho^d} (\chi_{ft}^d)^{\rho^d - 1} D_t^d \exp(\varepsilon_{ft}^d) = \lambda_{ft}, \tag{9}$$

implying

$$(Q_{ft})^{\rho^d} (\chi_{ft}^d)^{\rho^d} = \frac{\rho^0}{\rho^d} \frac{\chi_{ft}^d}{\chi_{ft}^0} (Q_{ft})^{\rho^0} (\chi_{ft}^0)^{\rho^0} \frac{D_t^0 \exp(\varepsilon_{ft}^0)}{D_t^d \exp(\varepsilon_{ft}^d)}. \tag{10}$$

Substituting (10) into the expression for total revenues gives

$$R_{ft} = \left(Q_{ft}\right)^{\rho^0} \left(\chi_{ft}^0\right)^{\rho^0} D_t^0 \exp(\varepsilon_{ft}^0) \exp(u_{ft}^0) \left[1 + \frac{\rho^0 \psi_{ft}^x}{\chi_{ft}^0 \exp(u_{ft}^0)}\right], \tag{11}$$

where $\psi_{ft}^x \equiv \sum_{d\in\Omega_{ft}} \chi_{ft}^d \exp(u_{ft}^d)/\rho^d$ is a weighted sum of *ex post* shocks in foreign destinations, with weights equal to the ratio of destination-specific output shares over demand curvature.[21] For non-exporters, this expression of total (domestic) revenues simplifies with $\psi_{ft}^x = 0$ and $\chi_{ft}^0 = 1$. From this expression we also observe that when firms export, their domestic output share (χ_{ft}^0) and their domestic revenue share (R_{ft}^0/R_{ft}) are not equal, and the difference depends on relative *ex post* demand shocks and demand curvature for the domestic and foreign markets.

Using the FONCs with respect to quantity shares and the definition of Z_{ft}, we can write domestic

[21] The weights χ_{ft}^d/ρ^d for ψ_{ft}^x do not in general sum to 1 because $\sum_{d\in\Omega_{ft}} \chi_{ft}^d = 1 - \chi_{ft}^0 \leq 1$ as long as firms sell their products in the domestic market and because ρ^d is included.

revenues as

$$R^0_{ft} = \left(Q_{ft}\right)^{\rho^0} \left(\frac{R^0_{ft}}{R_{ft} Z_{ft}}\right)^{\rho^0} D^0_t \exp(\varepsilon^0_{ft}) \left(\exp(u^0_{ft})\right)^{1-\rho^0}, \tag{12}$$

which holds for both non-exporters and exporters.[22] Equation (12) links domestic revenues, R^0_{ft}, to total quantity of output, Q_{ft}, the domestic share of revenues, R^0_{ft}/R_{ft}, the weighted sum of the inverse of *ex post* shocks, Z_{ft}, and *ex ante* and *ex post* domestic demand shocks (i.e., ε^0_{ft}, u^0_{ft}).

A key feature of our estimation strategy is that neither aggregate nor firm-specific *ex ante* foreign demand conditions intervene directly in equation (12). Instead, the metric $R^0_{ft}/(R_{ft}Z_{ft})$ encodes this information. To see how, consider two firms that produce the same quantity of output and face the same domestic demand conditions. The firm with higher foreign demand—either because the firm serves better markets or because the firm sees higher demand shocks—will ship a higher share of output to, and earn a higher share of revenue from, foreign markets (lower value for $R^0_{ft}/(R_{ft}Z_{ft})$). The correlation between this term and domestic revenues identifies the curvature of domestic demand. While the domestic share of revenues is likely endogenous, driven by both unobserved shocks to supply and demand, the elasticity of domestic revenues to $R^0_{ft}/(R_{ft}Z_{ft})$ can be estimated without ever having to observe foreign demand conditions, as we show below.[23]

3 Empirical strategy

We base our estimator on the two-step factor shares approach of Gandhi et al. (2020). Specifically, we extend the "revenue production function" estimator in their Appendix O6-4 to the case where firms serve potentially multiple destination markets, wherein they face heterogeneous demand conditions.[24] Later, we compare the performance of our estimator with alternative estimation strategies, either based on different versions of the same factor shares approach, or on the control function approach.

[22] See Appendix A for the complete derivation.

[23] In Appendix C, we demonstrate that this result holds even if we relax the CES assumption for demand. However, without additional restrictions, we cannot recover the production function separately from the inverse demand function. Any change in revenues can be rationalized either by a movement along the supply curve, or a movement along the demand curve. The CES assumption generates separability between the demand terms and the supply terms, enabling joint identification of the demand and supply elasticities.

[24] In their main text, Gandhi et al. (2020) treat only the case in which outputs are denominated in quantity.

3.1 Multi-destination estimator: first step

In the first step, we derive the estimation equation from the first-order conditions for material inputs.[25] As implied by the term "factor share approach", output elasticities with respect to material inputs are identified from projecting factor expenditure shares on logs of input use. When outputs are denominated in value, these elasticities include the demand-side parameters.[26]

Conditioning on non-exporters and re-writing equation (8) yields:

$$\ln s_{ft}^{M} = \ln\left[\mathscr{E}_u \beta_{ft}^{M}(m_{ft}, l_{ft}, k_{ft})\right] - u_{ft}^{0} \tag{13}$$

where we define $s_{ft}^{M} \equiv M_{ft} W_t^M / R_{ft}$ and where $\beta_{ft}^{M}\left(m_{ft}, l_{ft}, k_{ft}\right) \equiv \rho^0 \sigma_{ft}^{M}$ denotes the output elasticity of materials multiplied by ρ^0, or the "revenue elasticity" of material inputs. It is important to condition on non-exporters for the first step to ensure that the residual ($-u_{ft}^{0}$) is orthogonal to inputs choices.[27] However, we emphasize that this does not introduce sample selection bias when estimating (13) because u_{ft}^{0} is unknown to the firm at the time it chooses an export strategy.[28]

Following Gandhi et al. (2020), we approximate $\beta_{ft}^{M}(\cdot)$ with a complete polynomial function of

[25]It is standard in production function applications to treat capital as a quasi-fixed input and to treat materials as a flexible input. The treatment of labor varies by application: sometimes, labor is assumed to be quasi-fixed (as we do), whereas other applications, especially in the developing world, treat labor like a flexible input. In the latter case, researchers must amend our procedure by following the same steps for labor as for materials.

[26]In general, the "revenue elasticities" of flexible inputs are identified from the factor share regressions as long as the markup does not depend on input levels (and as long as the orthogonality condition discussed below is met). But this does not require that markups are fixed. Markups may vary over time and across firms in our model because of *ex post* demand shocks.

[27]When the sample includes exporters the residual of the factor share regression depends on a weighted sum of *ex-post* demand shocks. While these *ex-post* demand shocks are by definition orthogonal to input choices, the weighted sum of these *ex-post* shocks is not necessarily orthogonal to input choices, because the weights are influenced both by the shocks and quasi-fixed inputs.

[28]Moreover, this restriction does not require that exporters and non-exporters have similar input intensities. Nothing about our estimation strategy precludes exporters from having higher capital stocks, for example, which would lead to systematically different input intensities, given the flexible production function. However, our estimation strategy requires that the parameters of the $\beta_{ft}^{M}\left(m_{ft}, l_{ft}, k_{ft}\right)$ function are the same for exporters and non-exporters. While this assumption is implicit in the specification of a common production function across firms in the industry, it rules out phenomena of interest, such as differential processing for export versus domestic markets (e.g., Verhoogen 2008).

degree two in all inputs. We estimate $\beta_{ft}^{M}(\cdot)$ by non-linear least squares (NLLS):

$$\begin{aligned}\min_{\{g_j^M\}} \quad & \sum_f \sum_t \Bigg\{ \ln s_{ft}^M - \ln \Bigg(g_0^M + g_m^M m_{ft} + g_l^M l_{ft} + g_k^M k_{ft} \\ + \quad & g_{mm}^M m_{ft} m_{ft} + g_{ll}^M l_{ft} l_{ft} + g_{kk}^M k_{ft} k_{ft} + g_{mk}^M m_{ft} k_{ft} + g_{ml}^M m_{ft} l_{ft} + g_{lk}^M l_{ft} k_{ft} \Bigg) \Bigg\}^2 \end{aligned} \quad (14)$$

where all the g^M coefficients include the constant $\mathscr{E}_u$. To purge this constant from the g^M coefficients, we compute

$$\widehat{\mathscr{E}}_u = \frac{1}{N} \sum_f \sum_t \exp(\widehat{u}_{ft}^0), \quad (15)$$

where N is the number of firm-year observations and $\widehat{u}_{ft}^0$ is minus the residual from (14).[29] We then divide all g^M coefficients by this constant and compute

$$\begin{aligned}\widehat{\beta}_{ft}^M(m_{ft}, l_{ft}, k_{ft}) \quad = \quad & \widehat{g}_0^M + \widehat{g}_m^M m_{ft} + \widehat{g}_l^M l_{ft} + \widehat{g}_k^M k_{ft} + \widehat{g}_{mm}^M m_{ft} m_{ft} + \widehat{g}_{ll}^M l_{ft} l_{ft} \\ + \quad & \widehat{g}_{kk}^M k_{ft} k_{ft} + \widehat{g}_{mk}^M m_{ft} k_{ft} + \widehat{g}_{ml}^M m_{ft} l_{ft} + \widehat{g}_{lk}^M l_{ft} k_{ft} \end{aligned} \quad (16)$$

where, in a slight abuse of notation, we denote by $\widehat{g}_j^M$ the deflated estimated coefficients from (14). Because $\widehat{g}_j^M$ characterize the production function for all firms, we compute the estimates of $\widehat{\beta}_{ft}^M(m_{ft}, l_{ft}, k_{ft})$ for every firm, including exporters. Consequently, we can estimate Z_{ft} for all firms: $\widehat{Z}_{ft} \equiv (\frac{W_t^M M_{ft}}{R_{ft}}) / (\widehat{\beta}_{ft}^M(\cdot) \widehat{\mathscr{E}}_u)$. We use this residual in the second stage of the estimation.

3.2 Multi-destination estimator: second step

In the second step, we use the information from the first step to recover the rest of the production function. The basic insight from Gandhi et al. (2020) is that the material input elasticity defines a partial differential equation that can be integrated to compute the part of the production function related to material inputs.

By the fundamental theorem of calculus, we have

$$\int_{\underline{\mathrm{m}}}^{m_{ft}} \beta_{ft}^M(m_{ft}, l_{ft} k_{ft}) dm_{ft} = \rho^0 f(m_{ft}, l_{ft}, k_{ft}) + \rho^0 \mathscr{C}^M \left(l_{ft}, k_{ft} \right) \quad (17)$$

where $\underline{\mathrm{m}}$ is the minimum possible value for material inputs and $\mathscr{C}^M(\cdot)$ is a constant of integration

[29] Since (15) calls for the use of the exponential function, the estimate of $E\,\widehat{[\exp(u)]}$ may be sensitive to the presence of extreme outliers. In the French data, we exclude any firm that ever has a material expenditure share or a labor expenditure share greater than 20 or less than 0.001. This restriction excludes less than 0.1% of the data.

that depends only on quasi-fixed inputs (here, labor and capital). Therefore, we compute log domestic revenues for *all* firms net of the contribution of flexible inputs:

$$\widetilde{r}^0_{ft} \equiv r^0_{ft} \quad - \quad \int_{\underline{\mathrm{m}}}^{m_{ft}} \widehat{\beta}^M_{ft}(m_{ft}, l_{ft}k_{ft})dm_{ft} \tag{18}$$

and substitute into (12) to derive

$$\begin{aligned}\widetilde{r}^0_{ft} &= b_l l_{ft} + b_k k_{ft} + b_{ll} l_{ft} l_{ft} + b_{kk} k_{ft} k_{ft} + b_{lk} l_{ft} k_{ft} \\ &+ \ln D^0_t + \rho^0 \ln\left(\frac{R^0_{ft}}{R_{ft}\widehat{Z}_{ft}}\right) + \varepsilon^0_{ft} + \left(1-\rho^0\right) u^0_{ft} + \rho^0 \omega_{ft}\end{aligned} \tag{19}$$

where the term $-\rho^0 \mathscr{C}^M \left(l_{ft}, k_{ft}\right)$ is approximated by a complete polynomial function of degree two in quasi-fixed factors—i.e., first line of the right hand side of (19).

In equation (19), ω_{ft}, ε^0_{ft}, and u^0_{ft} are all endogenous to $\ln[R^0_{ft}/(R_{ft}\widehat{Z}_{ft}])$, both through the endogenous choice of destinations in the export strategy and through realized sales in the domestic market. Additionally, if ω_{ft}, ε^0_{ft}, and u^0_{ft} are persistent, then they correlate with all quasi-fixed inputs through the investment rule. As in many production function estimation routines, we exploit assumptions on the evolution of these unobserved shocks for identification. In particular, we assume that productivity shocks ω_{ft} and domestic demand shocks ε^0_{ft} evolve according to first order Markov processes with the same persistence parameter h.[30] In addition, we allow ω_{ft} to depend on a lagged firm decision, here an export participation indicator $e_{f,t-1}$.[31]

Combining all unobservables into a composite shock, $\nu_{ft} \equiv \varepsilon^0_{ft} + \left(1-\rho^0\right) u^0_{ft} + \rho^0 \omega_{ft}$, yields

$$\nu_{ft} = h\nu_{f,t-1} + \rho^0 \mu e_{f,t-1} + \xi_{ft}, \tag{20}$$

where $\omega_{ft} = h\omega_{f,t-1} + \mu e_{f,t-1} + \widetilde{\omega}_{ft}$ and $\varepsilon^0_{ft} = h\varepsilon^0_{f,t-1} + \widetilde{\varepsilon}^0_{ft}$, with $\widetilde{\omega}_{ft}$ and $\widetilde{\varepsilon}^0_{ft}$ indicating i.i.d. innovations to productivity and domestic demand shock. The parameter μ indicates the effect of "learning by exporting" (LBE), i.e., the potential gains in productivity from selling to foreign markets. Note that we make no assumptions on the evolution of ε^d_{ft} for $d > 0$, i.e., on any other

[30] Allowing for non-unit-root persistence in both productivity and domestic demand shocks entails that both ω_{ft} and ε^0_{ft} must follow the same linear AR(1), as explained by Melitz & Levinsohn (2006); De Loecker (2011). Alternatively, one could impose that ε^0_{ft} is i.i.d., conditional on observables, in which case one could allow for a more flexible first-order Markov process in ω_{ft}.

[31] Note that the estimation sample in the second stage includes all firms, not just exporters. As a result, identification does not rely on moments that condition on exporting status. Therefore, even if contemporaneous productivity influences the export decision, this does not generate a sample selection problem in the second stage.

market that is not the domestic one.[32] The term $\xi_{ft} \equiv \widetilde{\varepsilon}^0_{ft} + \rho^0 \widetilde{\omega}_{ft} + (1-\rho^0)u^0_{ft} - h(1-\rho^0)u^0_{ft-1}$ is an MA(1) error term (due to the presence of u^0_{ft-1}).

Substituting ν_{ft} into (19) yields

$$\widetilde{r}^0_{ft} = b_l l_{ft} + b_k k_{ft} + b_{ll} l_{ft} l_{ft} + b_{kk} k_{ft} k_{ft} + b_{lk} l_{ft} k_{ft} + \rho^0 \ln\left(\frac{R^0_{ft}}{R_{ft}\widehat{Z}_{ft}}\right) + \alpha_t + \nu_{ft} \tag{21}$$

where α_t is a time fixed effect that absorbs the domestic demand shifter, $\ln D^0_t$.[33]

For any candidate vector $(\rho^{0^*}, b^*_l, b^*_k, b^*_{ll}, b^*_{kk}, b^*_{lk})$, we can compute

$$\widehat{\nu_{ft} + \alpha_t} \quad = \quad \widetilde{r}^0_{ft} - \rho^{0^*} \ln\left(\frac{R^0_{ft}}{R_{ft}\widehat{Z}_{ft}}\right) - b^*_l l_{ft} - b^*_k k_{ft} - b^*_{ll} l_{ft} l_{ft} - b^*_{kk} k_{ft} k_{ft} - b^*_{lk} l_{ft} k_{ft}, \tag{22}$$

and then regress $\widehat{\nu_{ft} + \alpha_t}$ on $\widehat{\nu_{f,t-1} + \alpha_{t-1}}$, the past exporting decision $e_{f,t-1}$ and time fixed effects, and compute the residual $\widehat{\xi}_{ft}\left(\rho^{0^*}, b^*_l, b^*_k, b^*_{ll}, b^*_{kk}, b^*_{lk}\right)$. We then build the moment conditions:

$$E\left[\widehat{\xi}_{ft}\left(\rho^{0*}, b^*_l, b^*_k, b^*_{ll}, b^*_{kk}, b^*_{lk}\right) \cdot \left(\ln \frac{R^0_{f,t-2}}{R_{f,t-2}\widehat{Z}_{f,t-2}}, l_{ft}, k_{ft}, ll_{ft}, kk_{ft}, lk_{ft}\right)'\right] = 0. \tag{23}$$

and minimize deviations from these moments by GMM.

At the true parameter values, $\widehat{\xi}_{ft}$ is orthogonal to all quasi-fixed inputs in period t. This follows from the fact that $\widehat{\xi}_{ft}$ contains only period t innovations to productivity $\tilde{\omega}_{ft}$ and domestic demand $\tilde{\varepsilon}^0_{ft}$, and *ex post* domestic demand shocks u^0_{ft} and $u^0_{f,t-1}$, none of which influence the investment decision in period $t-1$. However, even at the true parameter values, $\widehat{\xi}_{ft}$ correlates with $\ln[R^0_{ft}/(R_{ft}\widehat{Z}_{ft})]$ and $\ln[R^0_{f,t-1}/(R_{f,t-1}\widehat{Z}_{f,t-1})]$ through the endogenous set of destinations and through sales on the domestic market.[34] Thus, we use $\ln[R^0_{f,t-2}/(R_{f,t-2}\widehat{Z}_{f,t-2})]$, which is orthogonal to $\widehat{\xi}_{ft}$, to ensure that the moment condition hold.

Although this moment condition should hold (given the model), one may wonder whether it is relevant for identifying ρ^0 in (23). We offer two explanations for the correlation between

[32] The LBE coefficient μ is mediated by the curvature of domestic demand ρ^0. The reason is that (20) is identified by domestic revenues. An increase in (log) output increases domestic revenues by ρ^0. LBE increases output by μ, so domestic revenues by $\rho^0 \times \mu$. Had we been using revenues from a different market to identify LBE, say, market $d > 0$, then the coefficient to $e_{f,t-1}$ would have been $\rho^d \times \mu$.

[33] By absorbing the (industry-specific) time-varying demand shifter into a fixed effect, we exploit different variation than estimators that rely on time series variation in a domestic aggregate quantity index, such as Klette & Griliches (1996).

[34] Recall that in order to build $\ln[R^0_{f,t-1}/(R_{f,t-1}\widehat{Z}_{f,t-1})]$ we use realized domestic sales, which are directly affected by $u^0_{f,t-1}$.

$\ln[R^0_{f,t-2}/(R_{f,t-2}\widehat{Z}_{f,t-2})]$ and $\ln[R^0_{ft}/(R_{ft}\widehat{Z}_{ft})]$, conditional on quasi-fixed inputs, time fixed effects, and $\widehat{\nu_{f,t-1}+\alpha_{t-1}}$. First, persistence in the *ex ante* foreign demand shocks ε^d_{ft}, for $d \neq 0$, yields correlation between $\ln[R^0_{f,t-2}/(R_{f,t-2}\widehat{Z}_{f,t-2})]$ and $\ln[R^0_{ft}/(R_{ft}\widehat{Z}_{ft})]$. To see this, consider two firms with the same evolution of ν_{ft} (which includes only domestic demand shocks ε^0_{ft}) and quasi-fixed inputs, serving the same set of destinations Ω_{ft}. Suppose that the first firm has persistently higher draws for ε^d_{ft} for some $d > 0$ than the second firm. The former firm will tend to earn a higher share of revenues from the export market than the latter, and hence will tend to have lower $\ln[R^0_{ft}/(R_{ft}\widehat{Z}_{ft})]$ in all periods.

The second mechanism that generates correlation between $\ln[R^0_{f,t-2}/(R_{f,t-2}\widehat{Z}_{f,t-2})]$ and $\ln[R^0_{ft}/(R_{ft}\widehat{Z}_{ft})]$ relies on persistent firm-specific fixed costs of market entry. To see this, consider two firms with the same evolution of ν_{ft} and quasi-fixed inputs, but different fixed costs of reaching different markets. In this case, the two firms will likely serve different markets. If these fixed costs are persistent, then the two firms will be exposed to different aggregate shocks. Suppose that the first firm has lower fixed costs compared to the second firm for serving a particular large foreign market. Then the former firm will tend to earn more from exporting than the latter, all else equal, and thus will tend to have a lower $\ln[R^0_{ft}/(R_{ft}\widehat{Z}_{ft})]$ in all periods.[35] While both mechanisms give rise to a "relevant" moment condition, we remain agnostic about their relative importance.[36]

[35]Alternatively, we could exploit a shift-share instrument instead of $\ln[R^0_{f,t-2}/(R_{f,t-2}\widehat{Z}_{f,t-2})]$ in (23), where the weights would be pre-period market shares for each firm and the shocks would reflect industry-destination-period demand B_t. However, this would require knowledge of the entire destination network of each firm and measures of aggregate demand. We prefer to use $\ln[R^0_{f,t-2}/(R_{f,t-2}\widehat{Z}_{f,t-2})]$ as the instrument because it requires only knowledge of the domestic share in revenues and it allows for persistence in the ε^d_{ft} draws to contribute to the relevance of the instrument.

[36]While our approach to identifying the price elasticity of demand is non-standard, similar approaches have been used in the literature. Fundamentally, identification in our framework comes from variation in the domestic revenue *share*. Holding total output fixed, reallocating sales across markets changes domestic revenues and, given equation (12), we recover the elasticity of domestic revenues with respect to this reallocation. This mechanism is closely related to standard results in models of oligopoly with CES demand, where demand elasticities and markups can be expressed as functions of market shares (Alviarez et al., 2025). In those settings, shares reflect the allocation of demand across firms within a market. In contrast, in our setting, shares reflect the allocation of a firm's sales across destinations. Despite this difference, the underlying logic is the same: shares summarize how demand is allocated across alternatives, and curvature parameters govern how outcomes respond to changes in those shares.

Finally, we compute the output elasticity for each quasi-fixed input $j \in \{K, L\}$:[37]

$$\widehat{\sigma}^{j}_{ft} = \frac{1}{\widehat{\rho}^{0}}\left(\frac{\partial \widehat{r}^{0}_{ft}}{\partial \ln j_{ft}} + \frac{\partial}{\partial \ln j_{ft}}\left[\int \widehat{\beta}^{M}_{ft}(m_{ft}, l_{ft}k_{ft})dm_{ft}\right]\right) \tag{24}$$

and for flexible inputs:

$$\widehat{\sigma}^{M}_{ft} = \widehat{\beta}^{M}_{ft}(m_{ft}, l_{ft}k_{ft})/\widehat{\rho}^{0}. \tag{25}$$

We compute returns to scale as the sum of output elasticities from variable and quasi-fixed inputs, and LBE as the point estimate on the lagged export indicator from the regression of the Markov process, all deflated by $\widehat{\rho}^{0}$. As a result, for our estimation procedure, knowledge of just the domestic elasticity of demand is sufficient to estimate factor output elasticities, returns to scale, and elasticities of productivity to observables, such as learning-by-exporting.

Since the second step uses estimated objects from the first step, we bootstrap the entire two-step procedure to compute standard errors. The bootstrap procedure samples firms rather than individual observations, which is akin to clustering standard errors by firm.

4 Existing Estimators

In this section we present existing approaches to estimating production functions when output is denominated in value, and discuss the biases that arise from ignoring the multi-destination dimension. See Appendix E for further details.

Multi-destination estimator with constant returns to flexible inputs. Previous works in the international trade literature exploit the ratio of revenues to total variable costs in order to identify the elasticity of demand (Aw et al. 2011, Almunia et al. 2021). This is a valid approach only when the output elasticity to variable inputs is equal to 1 (i.e., constant marginal costs). We can derive the estimating equations for this strategy by imposing this restriction in our model.

To illustrate this, consider Aw et al. (2011) as a special case of our framework. Starting from the

[37] Specifically, assuming a second degree polynomial function for both the first step and the second step yields

$$\begin{aligned}
\widehat{\sigma}^{K}_{ft} &= \left(\widehat{b}_{k} + 2\widehat{b}_{kk}k_{ft} + \widehat{b}_{kl}l_{ft} + \widehat{g}^{M}_{k}m_{ft} + 2\widehat{g}^{M}_{kk}m_{ft}k_{ft} + \widehat{g}^{M}_{lk}l_{ft}m_{ft} + \frac{1}{2}\widehat{g}^{M}_{mk}m_{ft}m_{ft}\right)/\widehat{\rho}^{0} \\
\widehat{\sigma}^{L}_{ft} &= \left(\widehat{b}_{l} + 2\widehat{b}_{ll}l_{ft} + \widehat{b}_{kl}k_{ft} + \widehat{g}^{M}_{l}m_{ft} + 2\widehat{g}^{M}_{ll}m_{ft}l_{ft} + \widehat{g}^{M}_{lk}k_{ft}m_{ft} + \frac{1}{2}\widehat{g}^{M}_{ml}m_{ft}m_{ft}\right)/\widehat{\rho}^{0}.
\end{aligned}$$

model in Section 2, if we impose: (i) constant returns to materials, $\sigma_{ft}^{M}=1$, (ii) no *ex post* demand shocks, $u_{ft}^{d}=0$ for all d, and (iii) a common demand curvature across foreign destinations, $\rho^{d}=\rho^{x}$ for all $d=1,\ldots,\mathscr{D}$, then the factor-share equation (8) simplifies to

$$W_{t}^{M}M_{ft}=\rho^{0}R_{ft}^{0}+\rho^{x}R_{ft}^{x}, \tag{26}$$

where $R_{ft}^{x}\equiv\sum_{d\in\Omega_{ft}}R_{ft}^{d}$. Adding a regression error term, equation (26) can be estimated by OLS to recover ρ^{0} and ρ^{x}, which is precisely what Aw et al. (2011) do.

Identification of ρ^{0} and ρ^{x} relies critically on these restrictions. In particular, without constant returns to materials, the function $\beta_{ft}^{M}(m_{ft},l_{ft},k_{ft})$ enters multiplicatively on the right hand side of (26), implying that the demand elasticities are not identified. Moreover, if *ex post* demand shocks are destination-specific, they enter each additive component separately, preventing the factorization that underlies (26) and rendering the equation infeasible to estimate using standard methods. Finally, if destination-level foreign sales were observed, restriction (iii) could be relaxed; however, given the maintained assumptions on data availability, it is required.

Next, to estimate returns to capital and learning-by-exporting, Aw et al. (2011) derive an expression for R_{ft}^{0} that depends only on K_{ft}, ω_{ft}, W_{t}^{M}, and the domestic demand shifter D_{t}^{0}. This yields

$$R_{ft}^{0} \quad=\quad (K_{ft})^{\frac{\gamma^{K}\rho^{0}}{1-\rho^{0}}}(W_{t}^{M})^{\frac{-\rho^{0}}{1-\rho^{0}}}\left(\rho^{0}\right)^{\frac{\rho^{0}}{1-\rho^{0}}}\left(\exp(\omega_{ft})\right)^{\frac{\rho^{0}}{1-\rho^{0}}}\left(D_{t}^{0}\exp(\varepsilon_{ft}^{0})\right)^{\frac{1}{1-\rho^{0}}}, \tag{27}$$

where γ^{K} represents the returns to capital. This expression can be used to identify γ^{K} and learning-by-exporting via GMM, exploiting the AR(1) structure of productivity.

The key to identification in (27) is again the assumption of constant returns to materials, together with Cobb-Douglas production function, which allows materials to drop out of the expression (see Appendix E.4).

Taken together, this shows that the estimation strategy in Aw et al. (2011)—estimating (26) followed by (27)—is a special case of our multi-destination estimator under restrictions (i)–(iii). When these restrictions are violated we would not expect this procedure to deliver consistent estimates. Nevertheless, we implement this estimator in Monte Carlo experiments and in the French data to assess its performance as an approximation to our more general data-generating process.

Factor share method with no demand correction. When estimating production function parameters with data denominated in value, researchers typically deflate firm-level revenues by the domestic price deflator and treat the resulting series as if they were quantities. Given our model, this would convert firm-level revenues into firm-level quantities only if firms serve only the domestic market, where they face perfect competition $\rho^0 = 1$. In this case, the conditions discussed in the main text of Gandhi et al. (2020) would be met, and thus their factor shares estimator could be applied. By contrast, when goods are not perfect substitutes ($\rho^0 < 1$), or when firms sell to multiple markets, deflating revenues by the domestic price index and implementing the estimation procedure from Gandhi et al. (2020) will bias the estimator.

The factor shares estimator from Gandhi et al. (2020) follows two steps. The first step proceeds exactly as we describe in section 3.1, except that the entire sample of firms is used to estimate the factor share equation. In the second step, the contribution from materials and the residual are computed exactly as described in section 3.2. Both terms are subtracted off from deflated revenues, and the resulting value, $\tilde{r}_{ft}^{NoD}$ (indexed by NoD for "no demand correction"), is projected on polynomial functions of the quasi-fixed inputs

$$\tilde{r}_{ft}^{NoD} \quad = \quad b_l^{NoD} l_{ft} + b_k^{NoD} k_{ft} + b_{ll}^{NoD} l_{ft} l_{ft} + b_{kk}^{NoD} k_{ft} k_{ft} + b_{lk}^{NoD} l_{ft} k_{ft} + \nu_{ft}^{NoD}, \tag{28}$$

where ν_{ft}^{NoD} collects all unobserved shocks. For a candidate vector of parameters, the structural residual ξ_{ft}^{NoD} can be computed, moments evaluated, and parameters chosen to minimize the usual GMM objective function. Compared with the moments described in (23), only the moment combining the structural residual with $\ln[R_{f,t-2}^0/(R_{f,t-2}\widehat{Z}_{f,t-2})]$ is omitted. See Appendix E.1 for discussion of bias in this estimator.

Factor share method with a single-market correction. The few papers that explicitly address the value-versus-quantity distinction in the context of production function estimation typically implement some versions of Klette & Griliches (1996)'s procedure that controls for the domestic aggregate CES quantity index (e.g., De Loecker 2011 and Grieco et al. 2016). The factor share approach to this estimation follows from Appendix O6-4 in Gandhi et al. (2020).

The first step is exactly the same as in the no demand approach just above. The second step also proceeds exactly as in the no demand approach, except that an additional regressor denoted by $\ln B_t^{proxy}$ is included on the right hand side of the deflated revenues equation:

$$\widetilde{r}_{ft}^{SMC} = b_l^{SMC} l_{ft} + b_k^{SMC} k_{ft} + b_{ll}^{SMC} l_{ft} l_{ft} + b_{kk}^{SMC} k_{ft} k_{ft} + b_{lk}^{SMC} l_{ft} k_{ft} + \left(1-\rho^0\right) \ln B_t^{proxy} + \nu_{ft}^{SMC}, \quad (29)$$

where variables and parameters are indexed by SMC for "single-market correction". Construction of $\widehat{\xi}_{ft}^{SMC}$ proceeds as in section 3.2, accounting for the extra regressor, moment conditions are constructed, and parameters chosen to minimize the objective function.

This additional regressor, B_t^{proxy}, captures the aggregate domestic demand conditions. If firms serve only one destination market—the domestic one—and if researchers could compute the empirical price index and B_t^{proxy} in a theory-consistent way (see Appendix E.7 for details), then we would have $B_t^{proxy} = B_t / \Upsilon_0$, where Υ_0 captures the price index normalization. In this case, the constructed residual $\widehat{\xi}_{ft}^{SMC}$ would be orthogonal to quasi-fixed inputs in period t because $\xi_{ft}^{SMC} \equiv \widetilde{\varepsilon}_{ft} + \rho \widetilde{\omega}_{ft}$ at the true parameter values. Indeed, when all firms serve a single market, the *ex post* demand shock u_{ft} is identified by the factor share regression in the first step, and hence does not appear in the second step. Moreover, the aggregate demand shifter $\ln B_t^{proxy}$ is orthogonal to $\widehat{\xi}_{ft}^{SMC}$ by assumption. Hence, the parameter ρ^0 would be identified by time series variation in industry-wide demand aggregates, and this estimation procedure would also identify all output elasticities.

When firms select endogenously into multiple destination markets, several sources of bias arise. We discuss the bias in detail in Appendix E.2.

Control function versions of the standard approaches. The two preceding estimation models could also be estimated by the control function method.

First, following the gross output control function estimator from Gandhi et al. (2020), we invert the material demand function to substitute for the unobserved shocks to supply and demand. This substitution generates the estimation equation

$$r_{ft}^{CF} = \Phi(m_{ft}, l_{ft}, k_{ft}) + \delta_t + \vartheta_{ft}^{CF} \quad (30)$$

where $\Phi(m_{ft}, l_{ft}, k_{ft})$ is an unknown function that combines $f(\cdot)$ and the inverted material demand function and δ_t is a time fixed effect that controls for material input prices. When controlling for a single-market demand shifter, we simply include B_t^{proxy} in $\Phi(\cdot)$. We approximate $\Phi(\cdot)$ with a second-degree polynomial function, estimate by OLS, and extract the residual $\widehat{\vartheta}_{ft}^{CF}$.

Next, we subtract $\widehat{\vartheta}_{ft}^{CF}$ from deflated revenues. In the case that the demand-side correction is ignored we obtain

$$\widetilde{r}_{ft}^{CF-NoD} = f(m_{ft}, l_{ft}, k_{ft}) + \nu_{ft}. \tag{31}$$

For the case in which a single-market demand correction is employed we obtain

$$\widetilde{r}_{ft}^{CF-SMC} = \rho^0 f(m_{ft}, l_{ft}, k_{ft}) + \left(1-\rho^0\right) \ln B_t^{proxy} + \nu_{ft}, \tag{32}$$

where ν_{ft} collects unobserved shocks to supply and demand. We adopt a complete polynomial function of degree 2 to approximate $f(\cdot)$ and assume that ν_{ft} follows an AR(1) process. For any candidate vector of parameters, we can compute $\widehat{\nu}_{ft}$, then regress $\widehat{\nu}_{ft}$ on $\widehat{\nu}_{f,t-1}$ and $e_{f,t-1}$ (the indicator for exporting) and compute the residual $\widehat{\xi}_{ft}^{CF}$. We build moment conditions by multiplying $\widehat{\xi}_{ft}^{CF}$ by the levels of all quasi-fixed inputs, lags of flexible inputs, along with the appropriate interaction and square terms, as well as $\ln B_t^{proxy}$ in the case of the single-market correction model. We then minimize the objective function to estimate parameters.

Beyond the model misspecification issues mentioned above, these control function approaches are unlikely to generate consistent estimates of model parameters because of weak instruments (Gandhi et al., 2020) and multiplicity of solutions to the GMM optimization Ackerberg et al. (2023). See Appendix E.3 for further details.

Estimator with multiple aggregate quantity indices. Finally, De Loecker (2011) provides an alternative extension of Klette & Griliches (1996)'s model and (similarly to us) proposes an estimation strategy for the case in which outputs are observed only in value. There is just a single destination (Europe) and a single industry (Textile & Apparel) in De Loecker (2011), but there are multiple product markets (e.g., yarns, clothing, interior fabrics, etc.)—which De Loecker (2011) calls "segments"—so the model has a similar structure to the multi-destination model we propose.

Hence, *modulo* some adjustments to translate the multi-segment dimension into a multi-destination dimension, and some additional restrictions on the production function and the demand shocks, we could also use the approach from De Loecker (2011) to estimate returns to scale, curvature of demand, and LBE in our setting. See Appendix E.5 for details.

However, there are two significant advantages to the procedure we propose. First, to implement De Loecker (2011)'s procedure, we would need to build analogues to the aggregate CES quantity indices on all destination markets. This would require computing theory-consistent price indices for every industry-destination cell in the world, which is, to our knowledge, not possible with currently available data. Second, when firms serve multiple markets, the estimation equation from De Loecker (2011) only holds as a first-order approximation. If higher order terms of this approximation are important in this nonlinear setting, then the values estimated by this procedure may be far from the truth, even if the data generating process coincides with the structural model.

5 Monte Carlo simulations

In this section, we study the finite sample properties of the multi-destination estimator presented in Section 3, and the alternative estimators presented in Section 4 using Monte Carlo simulations. We use the multi-destination model from Section 2 as the data generating process.

The parameters of the Monte Carlo simulations are chosen to balance two objectives. First, we draw on parameter values used in prior Monte Carlo studies (Grieco et al., 2016; Gandhi et al., 2020) for comparability and to ensure plausibility of the economic environment. Second, we calibrate key parameters governing demand and trade frictions to generate sufficient (yet plausible) variation in firms' export behavior. This variation is central for identification, as both the domestic demand curvature and LBE effects are pinned down by variation across firms and over time.

5.1 Data Generating Process and Estimators

For these experiments, we impose that firms produce with a Cobb-Douglas production function with one flexible input, materials (M), and one quasi-fixed input, capital (K):

$$Q_{ft} = \exp\left(\omega_{ft}\right) M_{ft}^{\gamma^M} K_{ft}^{\gamma^K}, \tag{33}$$

with $\gamma^M = 0.8$ and $\gamma^K = 0.3$. We draw initial capital stocks $K_{f1} \sim U(1,201)$, initial productivity shocks $\omega_{f1} \sim N(0,0.0025)$, and initial *ex ante* demand shocks $\varepsilon^d_{f1} \sim N(0,0.0025)$. We let ω and *ex ante* demand shocks for the domestic market ($d = 0$) update according to an AR(1) process with persistence parameter $h = 0.8$ and where the innovations follow a normal distribution: $\widetilde{\omega}_{ft} \sim N(0,0.0025)$ and $\widetilde{\varepsilon}^0_{ft} \sim N(0,0.0025)$. Foreign *ex ante* demand shocks are unconstrained in their evolution. We draw *ex post* demand shocks using $u^d_{ft} \sim N(0,0.0025)$. We set $\rho^0 = 0.8$ and all foreign $\rho^d = 0.6$ for $d \neq 0$.

We simulate 500 samples of a single industry with 200 firms over 10 periods. In order to keep the computational burden manageable, we allow for only four destination markets. Destination-specific industry-wide expenditures, material input prices, and fixed costs of entry into foreign markets are calibrated to ensure plausible rates of export revenues shares. Foreign destination quantity indices are treated as exogenous objects and are drawn directly each period, consistent with the assumption that firms take these foreign demand conditions as given. Absent any fixed costs to serve the domestic market, all firms serve that market.

Taking expectations over the *ex post* demand shocks u^d_{ft}, firms evaluate the expected profitability of all feasible export strategies conditional on conjectures about destination-specific quantity indices B^d_t.[38] Conditional on the optimal export strategy, firms choose material inputs and destination-specific quantity allocations to maximize expected profits under monopolistic competition, according to (6). After firms solve their optimization problems, domestic sales are aggregated across firms and the domestic quantity index is updated to satisfy the CES adding-up constraint.[39] Finally, given u^d_{ft} draws, we compute optimal prices and revenues.

We simulate the model period by period. In the first period, we solve for the set of destinations that maximizes expected profits for each firm. From these values, we determine which firms are

[38]For foreign destinations, firms correctly treat the quantity indices as fixed and equal to their equilibrium values. For the domestic market, however, firms form conjectures about the domestic quantity index when choosing export participation and production plans—conjectures that turn out to be incorrect, since the domestic quantity index will be solved in equilibrium by combining all quantities sold by operating firms in a CES form.

[39]This procedure defines a fixed-point problem. Following the update to the domestic quantity index, firms re-compute optimal quantities conditional on the revised domestic market conditions, and the algorithm iterates until convergence of firm-level quantities and the domestic quantity index. See Appendix F.1.2 for more details.

active on the export market. We then update firm productivity for period 2, which includes the LBE effect, setting $\mu = 0.05$. We also update the capital stock using a fixed investment rule, which corresponds to $\iota^K_{ft} = \exp\left[0.8 * (\rho^0 * \omega_{ft} + \varepsilon^0_{ft})\right] K^{0.2}_{ft}$, and a depreciation rate of 10 percent.[40] Given ω_{f2} and K_{f2}, we then solve the combinatorial problem for each firm in period 2. We again determine which firms are active on the export market in period 2, and update firm productivity accordingly. We continue in this fashion until the final period.

For each simulated sample, we implement (1) our multi-destination factor shares procedure denoted "BOR", along with procedures for (2) the estimator from Aw et al. (2011) that imposes constant returns to variable inputs (denoted "ARX"), (3) the estimator from De Loecker (2011) (denoted by "DL") that allows for different CES parameters on different markets, (4) both factor shares and control function versions of the standard practice of deflating firm-level revenues by industry-wide price indices and ignoring demand-side corrections (denoted by "NoD"), and (5) both factor shares and control function versions of the estimator that controls for aggregate sales in the domestic market as in Klette & Griliches (1996) (denoted by "SMC").[41] In each case, we assume researchers observe R_{ft}, M_{ft}, K_{ft}, W^M_t, B_t, Υ_t and domestic revenue shares R^0_{ft}/R_{ft}.

5.2 Results

Table 1 reports the median and standard deviation of parameter estimates across 500 simulated data samples by estimator.[42] The first row of the table reports the true value of the parameters across all samples. The second row presents the results from our multi-destination factor shares BOR procedure. Subsequent rows present results from alternative estimators, grouped by whether they rely on factor shares or control function approaches.

As expected, we find in the second row that our BOR procedure recovers median estimates of

[40] This investment rule is similar to the rule used to generate simulation data in Grieco et al. (2016). We could alternatively solve for investment as the solution to a dynamic problem, but this would not change anything about the first order necessary conditions with respect to materials and quantity shares, which are the only conditions we need to generate our estimation equation.

[41] For these estimators, we use a common practitioners' approach to choosing initial conditions for the nonlinear optimizations based on OLS. See Appendix F.1.5.

[42] Results using average values instead of medians are qualitatively similar, though some extreme outliers can produce non-trivial deviations in the average values from medians, so we focus on medians. Estimators using the control function approach suffer the most from this issue.

population parameters that are very close to the true underlying model parameters used to simulate the data. Median estimates for all parameter estimates are within 2 percentage points of the true parameter values, and standard deviations of the distribution of estimates are relatively tight. Investigating the distributions visually (see Appendix Figure G.1) reveals that the distributions are approximately centered on the true parameter values. Our multi-destination estimator hence performs well even in relatively small samples.

In contrast, we find that all other estimators are biased. In the third row of Table 1, we present results from the estimator of Aw et al. (2011), which imposes constant returns to variable inputs ($\sigma^M = 1$) and sets $u^d_{ft} = 0$. Both restrictions are inconsistent with the assumed DGP, so it is unsurprising that this estimator fails to recover the model parameters. Indeed, we find that the median estimates of σ^K, ρ^0, and μ are 160% higher, 21% lower, and 414% higher than the true underlying values, respectively. For each parameter, almost the entire distribution of estimates lies to one side of the true value (see Appendix Figure G.2). Having omitted materials from the second-step GMM estimation, the effect of materials loads onto the estimated revenue elasticity of capital, leading to the reported overestimate. Additionally, underestimating ρ^0 further inflates the estimated capital elasticity, as well as the estimate on LBE. These results indicate that the procedure from Aw et al. (2011) could lead to severely biased estimates when returns to variable inputs are non-constant, or when there are *ex post* demand shocks.

In rows 4 and 5 we report the results from the factor shares versions of the single-market demand correction estimator (SMC) and the no-demand-correction estimator (NoD), respectively. Given that firms can potentially serve multiple destination markets, these procedures are biased and yield inaccurate estimates.

The SMC estimator adapted to the factor shares approach tends to overestimate ρ^0 (by 4% for the median estimate), and thus underestimate both returns to materials and capital (by 15% and 82%, respectively). The learning by exporting effect is also underestimated with this procedure, with a median estimate 34% lower than the true value. By allowing $\rho^0 \neq 1$, the SMC factor shares estimator gets closer to the true parameter values than the NoD factor shares estimator (except for the capital elasticity). But by ignoring the multi-destination dimension of the problem, transmission bias leads to an underestimate of ρ^0, which leads to overestimates of factor returns.

The NoD estimator tends to underestimate the median returns to materials by 29% and capital by 82%. This underestimation is in part due to the fact that the NoD implicitly imposes $\rho^0 = 1$, and thus, neglects to deflate parameter values (while the true value of ρ^0 is less than 1). The learning by exporting effect is also underestimated with this procedure, with a median estimate 42% lower than the true value.

In the lower panel of Table 1 we present estimates from three different versions of control function estimators. The first control function estimator (DL) is the procedure adapted from De Loecker (2011) for multiple destinations and presented in Appendix Section E.5. Even though we assume that quantity indices B^ds can be computed from the data and that destination-specific revenue shares are observed (which is typically not the case in the data), the 6th row of Table 1 shows that this procedure is biased. The median estimates for σ^M and ρ^0 are biased up by 25% and 43% respectively. The median estimates for σ^K and μ are biased downwards and have the opposite signs from the true values. For each parameter, the distribution of estimated coefficients is centered well away from the true values (see Appendix Figure G.3). Furthermore, in several replications, the estimated parameters are orders of magnitude away from the true values, generating extremely large standard deviations. Extreme outliers arise when the estimate of ρ^0 approaches zero because all other parameters are deflated by $\widehat{\rho}^0$.

In rows 7 and 8 of Table 1 we present results from control function versions of the single-market demand correction (SMC) and the no-demand-correction (NoD) estimators, respectively. With a DGP in which firms charge different prices and can potentially serve multiple destination markets, these procedures are biased, and indeed the estimates are off the mark. For the SMC estimator the median estimate of ρ^0 is 40% higher than the true value. Median returns to materials are overestimated by 20%, and returns to capital and the LBE effect are underestimated. Median estimates for the NoD estimator look quite similar to those from the SMC.[43]

[43] In Appendix F.2, we explore further the performance of the control function. Gandhi et al. (2020) show, in a single-market homogeneous-good model, that substantial time series variation in material input prices is required for

As robustness exercises, we consider two adversarial data-generating processes in which firms (i) compete strategically in quantities (Cournot) on the domestic market, or (ii) face non-CES demand (Kimball, 1995) on the domestic market. Export markets, however, continue to operate under monopolistic competition and foreign quantity indices remain exogenous from the perspective of domestic firms. These exercises are intentionally demanding for the proposed estimator. The multi-destination estimator developed in Section 3 is derived under monopolistic competition and CES demand, where firms treat destination-level quantity indices as fixed scalars. Under Nash competition or Kimball demand, however, the first-order conditions and equilibrium mapping differ from the maintained assumptions of the estimator. Therefore, these experiments provide stress tests for departures from the assumptions under which the BOR estimator is derived. See Appendices F.1.3 and F.1.4 for details.

The results of these adversarial tests are reported in Appendix Tables G.1, G.2 and G.3. Despite the departure from monopolistic competition (Appendix Tables G.1-G.2 for 200 firms and 40 firms, respectively) or CES demand (Appendix Table G.3 for Kimball demand), the multi-destination estimator continues to perform very well and substantially outperforms the alternative estimators. The main qualitative patterns from the baseline simulations remain unchanged: estimators that ignore the multi-destination structure continue to exhibit large biases in estimated returns to scale, demand elasticities, and learning-by-exporting effects.

The strong performance of the estimator in these environments is perhaps not surprising. Although firm-level elasticities depend on market share in these two experiments, even for relatively small industries, individual firms remain sufficiently small relative to the aggregate market that their influence on the domestic quantity index and equilibrium prices is limited.[44] As a result, monopolistic competition and CES remains a good approximation to firms' equilibrium behavior,

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the control function to recover the true values of the output elasticities. We replicate this result in Appendix F.2 using a single-market differentiated-good model. Additionally, we show that this result holds only if the nonlinear optimization in the second stage starts from the *true* underlying structural parameters. If the latter condition does not hold and the nonlinear optimization starts from the OLS values—as would be the natural initial conditions when estimating in the real data—the estimator is biased, as we can see in Appendix F.2.

[44] For the oligopolistic DGP, even when we drop the number of firms to 40 (Appendix Table G.2), the multi-market estimator generates small errors, on average.

even though the true data-generating process features Cournot competition or Kimball demand.[45]

In summary, given the assumed data generating process and available data, our multi-destination factor shares BOR estimator recovers estimates of the output elasticities, the curvature of the domestic demand function, and LBE that are close to the true values in the Monte Carlo experiments. Alternative estimators can be severely biased, leading to incorrect inference.

6 Application to French manufacturing

In this section we describe our data sources, report descriptive statistics, and then report estimates of returns to scale, the elasticity of demand, the output elasticities of inputs, and learning-by-exporting effects for French manufacturing firms. We report results for our multi-market estimator, the estimator from Aw et al. (2011) that imposes constant returns to variable inputs, the single-market correction estimator, and the standard estimator that makes no correction for demand—all based on the factor share approach. For the main results we assume that labor is predetermined in the previous period, like capital, and, as a robustness check, we report additional results that assume a partial adjustment process for labor in Appendix J.2. We also report results using the control function method in Appendix J.3. The latter are just for comparison, as we expect this estimator to be biased, as demonstrated in the previous section.

6.1 Data and descriptive statistics

We use administrative data sources to build a quasi-exhaustive panel of the universe of French manufacturing firms in 1994–2016. Most of the data comes from firm balance sheets from the FICUS and FARE datasets, which originate in firms' tax declarations. We use total revenues, material expenditures, employment, and book-value of capital stocks. We obtain information on firms' exports from the French Customs. It is straightforward to merge the customs data to FICUS/FARE because they use the same firm-level SIREN identifier. We deflate expenditures on materials by industry-level input price indices that we obtain from the EU KLEMS dataset. We build firm-level

[45] Barrows et al. (2025) and Head & Mayer (2026) similarly evaluate estimators derived under monopolistic competition in environments where firms actually compete oligopolistically, and likewise find that monopolistic-competition-based estimators perform comparatively well.

capital stocks using the methodology of Bonleu et al. (2013) and Cette et al. (2015). Appendix H provides further details on the data sets and explains how we construct firm-level capital stocks.

We report descriptive statistics in Appendix Table H.1. The skewed firm size distribution is apparent from the difference between means and medians, for example, in revenue and employment. This feature is common in many manufacturing datasets. The high percentages of exporting firms is typical of European economies, who trade intensively within Europe. On average, 25.1% of firms in our data export at least once, but this varies considerably across industries, with a low of 6.7% in "Food, beverage, tobacco", and a high of 71.1% in "Chemical products".

In Appendix Table H.2 we report descriptive statistics for the export intensity among firms that export. Within exporting firms, the export share also varies considerably, both across industries and across firms within industries. While the median exporter obtains 4.3% of revenue from exporting, the 90^{th} percentile firm obtains almost 40% of revenue from foreign markets.

Tables H.1 and H.2 make two important points. First, the fact that many firms export in addition to serving the domestic market implies that estimation methods that assume that all sales are on a single, domestic market ignore important information. In particular, controlling for just the *domestic* aggregate quantity index does not control for aggregate demand conditions faced by exporting firms. Second, variation in the extensive exporting margin and the high variation in export intensity among exporting firms jointly indicate that there is sufficient variation to identify ρ^0 in our setting, coming from both the cross section of firms and firm-level changes over time. This is in contrast to methods that assume only one market, where only time series variation in an aggregate demand shifter identifies the demand curvature.

6.2 Main results

We present our main results graphically in Figures 1 and 2, which facilitates comparison of estimated parameters across estimators. The figures show estimated total returns to scale, variable returns to scale, the domestic price elasticity of demand, and the learning by exporting parameter. The corresponding detailed estimates are provided in Appendix Tables J.1–J.5, where we also report the persistence parameter h, the demand curvature parameter ρ^0, and the long run effect of exporting $\mu/(1-h)$, as well as bootstrapped standard errors for all estimates.[46]

Total Returns to Scale. We present estimates of total returns to scale (RTS) in the top left panels of Figures 1 and 2. Our multi-market estimator, denoted "BOR" (depicted by blue circles) yields estimated total returns to scale that range from 1.05 (Electrical equipment) to 1.24 (Wood, paper products), and for one industry up to 1.35 (Food, beverage, tobacco). Bars represent 95% bootstrapped confidence intervals. The mean (median) estimate across the 11 industries is 1.15 (1.13), which is in line with estimates in Antweiler & Trefler (2002).

In contrast, the estimator that makes no demand correction, depicted in Figure 1 with red triangles, yields estimated total returns to scale slightly below 1 for most industries. The mean and median of the industry-specific average returns to scale are both equal to 0.96, with tight confidence intervals. These estimates are close to what researchers tend to find with this approach. For example, after deflating revenues by industry-wide price indices and interpreting them as quantities, Gandhi et al. (2020) find in their factor shares approach average returns to scale in Colombia between 0.99 and 1.06, and between 1.04 and 1.15 for Chile. Comparing these estimates with the ones obtained with the BOR estimator indicates that the constant returns to scale estimated by the no demand correction model masks returns that are actually increasing (in our notation, $\sigma^M_{ft} + \sigma^L_{ft} + \sigma^k_{ft} > 1$), as hypothesized by Klette & Griliches (1996).[47] The reason is simple: estimators that fail to adjust for the demand curvature report "revenue elasticities" that are, in our notation, a factor $1/\rho^0$ smaller than output elasticities.

Next, in Figure 2, we compare our BOR estimates (blue circles) with the single market correction (black squares) and the multi-market estimator with constant returns to variable inputs (orange crosses). Because the scale of the y-axis must be expanded to accommodate the larger magnitude

[46]While we treat labor as a quasi-fixed input in our main specification, which is often assumed in the French context (Harrigan et al., 2026), we investigate the sensitivity of the results to the possibility that firms partially adjust labor to contemporaneous shocks. Appendix J.2 shows that this does not materially alter the main results.

[47]Klette & Griliches (1996) argue that ignoring unobserved firm-specific prices would tend to lead to a downward bias in estimated returns to scale, *ceteris paribus*. As we discuss in section E.1, there are, in fact, several forces that bias the estimator with no demand correction, and the overall sign cannot be determined in general. Nevertheless, the evidence in the upper left panel of Figure 1 is consistent with the central hypothesis from Klette & Griliches (1996). For comparison, estimates of average returns to scale using the control function approach with no demand correction are all close to 1 (except for Wood), with very low estimates of the capital output elasticity; see Appendix J.3.

of the estimates from these two alternatives, we split the presentation of the results into two figures.

If the true DGP corresponds to the multi-destination model from Section 2, the SMC estimator only partially addresses the transmission bias from missing output prices. As shown in the top left panel of Figure 2, the single-market correction estimator delivers highly dispersed estimates across industries. We find that the average returns to scale range from -3 (Rubbers and plastics) to 7.3 (Chemicals, omitted from the figure to ease visibility of other results). We find only three industries with plausible estimates for returns to scale: Auto and transportation (1.09), Communication electronics (1.29), and Electrical equipment (1.34).[48]

The large range of estimates for returns to scale can be explained by the range of estimates of ρ^0, which are used to "deflate" the revenue elasticities. With the SMC estimator, we estimate a range for ρ^0 from -0.61 to 1.49 across industries. When ρ^0 is estimated to be close to zero, then the returns to scale become very large in absolute value, as they do for Chemicals. When the estimate of ρ^0 is negative, this leads to negative estimates of returns to scale (Rubbers and plastics and Other manufacturing). These extreme estimates of ρ^0 are not merely due to estimation uncertainty; the estimates are quite precise (see Appendix Table J.3 for bootstrapped standard errors).

The wide range of SMC estimates of returns to scale is notable because we find a much smaller range of estimated returns to scale in the Monte Carlo experiments when using this estimator. A likely explanation for the difference is that in the Monte Carlo simulations we assume the researcher observes the true domestic quantity index B_t^0. In the application, we follow the common practice to approximate B_t^0 using all of the firms' revenues (not distinguishing exports and domestic sales, nor adding imports) and use price indices built from producer prices in the domestic market (as discussed in Appendix E.2). These discrepancies highlight an additional, practical advantage for our multi-market estimator: it does not require making such data compromises, as it does not require building B_t^0, nor does it require to deflate firm revenues.[49]

[48]The high and low estimates of average returns to scale is not just a matter of outlier observations. Medians within the industry are very close to the means.

[49]Given the theoretical drawbacks of applying the SMC estimator to a sample of multi-destination firms, a natural alternative would be to estimate the factor shares approach with a single-market correction for a set of non-exporters. By conditioning on non-exporters, the endogeneity bias stemming from multiple destinations cannot influence the estimates. However, in this case, measurement error in B_t^{proxy} and selection bias would still lead to biased estimates,

Figure 2 also shows that estimates of returns to scale using the estimator of Aw et al. (2011) (orange $\times$'s) are implausibly high, ranging from 2.27 to 4.21. This is driven not only by the output elasticity for materials, which is fixed to unity by construction, but also by the output elasticity of predetermined labor being estimated above 1 for all industries but one (see Appendix Table J.5).

In our preferred specification (BOR, blue circles) average returns to scale are greater than 1 for all industries. This indicates that there are efficiency gains from size embedded in the technology used by firms, regardless of how total factor productivity evolves. We estimate increasing returns to scale in virtually all firm-year observations, not just on average (see Appendix Figure J.1).[50] From a welfare perspective, increasing returns imply a cost to diversification that weighs against love of variety, as hypothesized by Krugman (1979). In addition, increasing returns imply larger business cycle fluctuations, and may provide a rationale for targeted interventions during downturns.

Returns to flexible inputs. In our main specification, only materials are treated as flexible inputs, hence the variable returns to scale (VRTS) are just the output elasticity with respect to materials. The top right panel of Figure 1 reports the average VRTS by industry and by estimator. The mean (median) of the average estimates across industries is 0.34 (0.35) with our multi-market estimator and 0.30 (0.30) with the no demand correction estimator. Both estimators share the same first step. But the first step without demand correction identifies the revenue elasticity with respect to materials, which must be divided by ρ^0 to recover the output elasticity—as the BOR multi-market correction does. Since $\rho^0 < 1$ in our estimates, the implied output elasticity of materials is larger under our BOR approach.

Returns to flexible inputs well below 1 imply negative cross-market complementarities in the short-to-medium run. For example, a positive demand shock in one market leads to more sales to that market, an increase in marginal costs, lower sales to other markets and a reduction in the

as discussed in Appendix E.2. Nevertheless, for comparison, we present results from this strategy in Appendix Table J.4. The estimates are very similar to those obtained when the single-market correction is applied to the full sample, including exporters (Appendix Table J.3).

[50]Several mechanisms could explain this phenomenon. The simplest explanation for increasing returns to scale is the presence of fixed costs of operation. Alternatively, complementarities within the firm could generate increasing returns at any point along the firm-size distribution. For example, externalities across workers could lead to increasing returns (e.g., learning by doing), as in, for example Kellogg (2011) and Hjort (2014).

likelihood of selling to other markets. This is consistent with findings in Almunia et al. (2021), who argue that the massive negative domestic demand shock in Spain during the financial crisis caused an increase in exporting, presumably due to a reduction in scale and in marginal costs.[51]

The top right panel of Figure 2 reports the implicit variable returns to scale from the estimator of Aw et al. (2011) (with $\sigma^M = 1$) and the estimated VRTS from the single-market correction method. While the former is likely too high, the latter approximately coincides with our BOR estimates for 6 industries out of 11, with implausibly large or even negative estimates in the remaining industries. The SMC estimates range from -0.83 (Rubber and plastic) to 2.54 (Chemical products, omitted from the Figure), with an average and median around 0.45. As noted above, this wide variation is mostly due to variation in the estimates of ρ^0.

Elasticity of Demand. In the bottom left panel of Figures 1 and 2, we report estimates of the domestic price elasticity of demand $\eta^0 = 1/(\rho^0 - 1)$. With our multi-market estimator, we estimate a range of demand elasticities from -19.42 (Chemicals) to -3.56 (Food, beverage and tobacco), with no outlier estimates. The mean (median) estimate across industries is -9.9 (-5.8), which is comparable to the mean and median estimates typically obtained in gravity regressions (e.g., Shapiro 2016; Fontagné et al. 2022).[52] As ρ^0 approaches 1 (e.g., in electrical equipment), standard errors for η^0 become extremely large, making it more informative to examine the standard errors for ρ^0. The bootstrapped confidence intervals for ρ^0 are narrow, and we can reject $\rho^0 = 1$ in all industries, supporting our key assumption that manufacturing products are differentiated.[53]

We compare these BOR estimates with the ones resulting from the SMC estimator and the estimator with constant returns to variable inputs in Figure 2. With the SMC estimator, most of the demand elasticities fall within the range -4.23 to -0.62, which is significantly lower in magnitude than our BOR estimates. Additionally, with the single market correction, we estimate a positive

[51] Almunia et al. (2021) perform production function and productivity estimation, but when developing their estimator they ignore cross-market complementarities. We explain how their estimator differs fundamentally from ours in Appendix E.6.

[52] For example, using data on trade flows and trade costs, Shapiro (2016) estimates an average trade elasticity across industries of -8.16, which translates into a price elasticity of demand of -9.16. This estimate can be interpreted as the demand elasticity under the assumption of an Armington trade model.

[53] Bootstrapped standard errors are reported in Appendix Table J.1.

demand elasticity for Textile and Apparel ($\eta = 2.03$), and a very high (in magnitude) demand elasticity for Autos and transport equipment (-29.8).

The demand elasticities obtained using Aw et al. (2011)'s approach (constant VRTS) are all much smaller in absolute value than those estimated with BOR. The domestic demand elasticities range from -1.37 to -3.89, while the demand elasticities on foreign markets ($\eta^x = 1/(\rho^x - 1)$) range from -1.68 to -2.57, both with a median of about -2 (see Appendix Table J.5). Both these ranges are much lower in magnitude than what is typically found in the literature.[54]

Learning by Exporting. Finally, in the bottom right panels of Figures 1 and 2 we report estimates of LBE effects by industry and estimator. With our multi-market estimator, we estimate LBE effects in the range of -0.005 (Food, beverage and tobacco) to 0.040 (Textile and apparel). We can reject the null hypothesis of zero LBE for all but two industries with very low estimated LBE effects (Food, beverage and tobacco; Rubbers and plastics). The average (median) estimate of LBE across all eleven industries is 0.017 (0.018). These estimates are lower than the estimates with the no demand correction estimator (average = 0.04 and median = 0.038), the SMC estimator (average = 0.035 and median = 0.030), and the estimator with constant VRTS (average = 0.067 and median = 0.035, with Food, beverage, tobacco an outlier at 0.34). The comparison suggests that estimated LBE effects are biased upward in the other estimators, possibly because at least the NoD and the SMC estimators mistakenly attribute the effect of foreign demand shocks to LBE.

Overall, we find robust evidence of significant LBE effects in French manufacturing data. Estimates using our multi-market procedure imply as much as 40% long run cross-sectional differences in productivity between exporters and non-exporters (Appendix Table J.1, last column).[55] Compared to previous work, these effects are smaller than those estimated via RCT with Egyptian firms (Atkin et al., 2017) and via structural approaches with Chilean, Colombian, and Mexican firms, e.g., (Garcia-Marin & Voigtländer, 2019), but larger than estimates from Danish firms using a

[54] Estimates of the demand elasticity using the control function approach with no demand correction or the single market correction are very noisy; see Appendix J.3.

[55] The long run, cross-sectional difference is computed as $\mu/(1-h)$, where μ is the effect of exporting today on productivity tomorrow and h is the persistence parameter in the AR(1) process for productivity.

quasi-natural experiment (Buus et al., 2022).[56]

6.3 Partial Equilibrium Quantification

To illustrate how differences in estimated parameters translate into economically meaningful outcomes, we perform two partial equilibrium quantification exercises under hypothetical shocks. These experiments aggregate over estimated firm-level responses for the year 2015. We outline these experiments here and relegate all derivations to Appendix I.

In the first exercise we consider the effects of a 10 percent increase in variable trade costs τ_t^d to serve destination $d = U.S.$ on firm-destination exports. This exercise approximates a realistic open-economy environment in which firms serve multiple destinations—a natural setting in which to gauge the performance of the multi-market estimator. In this setting, we must take caution when comparing results using the multi-destination estimators (BOR or the constant VRTS) to those using the single-market correction (SMC). The reason is that the SMC estimator is derived from economic structure that is inconsistent with the underlying experiment of changes in trade barriers towards a *specific* market. Nevertheless, we use the point estimates from the SMC estimator to quantify the experiment, which is embedded in a multi-market model.[57]

For this reason, we consider a second quantification exercise where variable trade costs increase across all markets symmetrically, rather than a single market. For homogeneous η across markets, this exercise is equivalent to a change in production costs in a single-market model. While this environment is less realistic economically, it provides a common benchmark for comparing estimators because the underlying economic structure is common across specifications.

In both exercises, we keep labor, capital, and export strategy fixed, allowing only variable inputs

[56] The other notable comparisons in the literature is De Loecker (2013), who estimates LBE effects in the range of 0.017 to 0.066 across Slovenian manufacturing industries. Since De Loecker (2013) does not correct for demand conditions, these estimates are best compared to the results from our no-demand-correction estimator, which we find in a similar range of 0.016 to 0.092.

[57] In the single-market environment, changing τ_t changes trade costs for the firm's entire single market rather than for a single destination. Consequently, an identical change in trade costs represents a substantially larger and qualitatively different shock in the SMC framework than in the multi-market model. For this exercise, we nevertheless substitute the SMC estimates of σ_{ft}^M and η into the elasticity formulas derived in Appendix I in order to evaluate the implied counterfactual responses under the single-market approximation. The single tariff experiment is not well-defined under perfect competition (the DGP consistent with maintained assumptions of the NoD estimator)—so we omit it from this exercise. Under perfect competition, firms sell to the market offering the highest net-of-trade-cost price. If the U.S. raised tariffs, French exporters to the U.S. would reallocate all sales away from the U.S. and toward other markets.

(i.e., materials) and exports (or total sales) to adjust to the shock. In all cases we use a single demand elasticity per industry, although we derive the expressions for heterogeneous elasticities across destinations for the multi-market model (see Appendix I).

Figure I.1 in Appendix I reports the implied effects of the two counterfactual tariff shocks. In the U.S. tariff experiment, our BOR estimator generally predicts substantially larger declines in exports to the U.S. than the constant-VRTS estimator (44% on average for the BOR, vs 11% for the constant-VRTS estimator). The SMC estimator predicts small negative responses for 6 industries, very large responses for one industry (Autos and transport equipments), and *positive* responses for one industry (Textiles and apparel). With the SMC estimator, the responses are *not defined* for three industries ($\widehat{\eta}_i > -1$) . Taking the average of the industry-by-industry comparisons, the BOR estimator implies 5 times larger effects of the U.S. tariff increase than the constant-VRTS, and 8 times larger than the SMC estimator.

In the global tariff experiment, the BOR estimator predicts larger declines in firm-level revenues (11% on average) than either the SMC (5% on average) or the constant-VRTS estimator (7% on average). Again, the responses are not defined for the SMC for three industries. Taking the average of the industry-by-industry comparisons, the BOR estimator implies 86% larger effects of the global tariff increase than the constant-VRTS, and 140% larger than the SMC estimator.

7 Conclusion

Production function estimation is central to many economic analyses, yet the conditions assumed in theory rarely match those faced by applied researchers (De Loecker & Goldberg, 2014; De Loecker & Syverson, 2021). In particular, most datasets report output only in values, not in quantities. In addition, many firms serve multiple destination markets wherein they face heterogeneous demand conditions, which is inconsistent with estimators that control for a single market demand shifter. These discrepancies matter for estimation, even when researchers are not interested in exporting or the effect of exporting *per se*.

In this paper, we show how to estimate output elasticities, the domestic price elasticity of demand, the elasticity of productivity to observable determinants, and productivity itself when firms serve multiple destination markets and when outputs are denominated only in monetary terms. We show that existing production function estimators that either do not address the missing price issue or do so using restrictive assumptions (e.g., single market or constant returns to variable inputs) yield biased inference in this case. By contrast, our estimator recovers estimates that are close to the true parameters values in Monte Carlo simulations.

Our estimator is no more difficult to implement than existing methods and requires only one additional piece of information: firms' export shares. This novel source of firm-level variation identifies the demand curvature faced by firms, and is advantageous compared to other methods that rely solely on time-series variation in aggregate demand shifters. Moreover, we do not need to construct measures of these aggregate demand shifters, which would be very demanding if required for all destination-industry-years. Finally, our estimator does not rely on functional form assumptions for the production function, and although it assumes destination-industry-specific constant elasticities of demand, it allows for firm-destination-year variation in prices and markups.

We estimate in the French data demand elasticities between -19.42 and -3.56, which are in a range that is consistent with much of the literature. We estimate average returns to scale ranging from 1.05 to 1.24 with one industry at 1.35, and average returns to flexible inputs uniformly below 1. The latter result implies cross-market complementarities: additional sales for a given market raises the cost of serving all other markets in the short run. Alternative approaches yield estimates that substantially deviate from our BOR estimates. These differences in estimated parameters translate into substantially different evaluations of the effect of tariff increases.

Finally, our approach allows us to study the effect of learning by exporting on productivity using an estimation framework that is consistent with heterogeneous firms' export decisions and pricing to market. We estimate learning-by-exporting effects ranging from 0 to 4% per year, which imply cross-sectional differences in productivity between exporters and non-exporters up to 40%.

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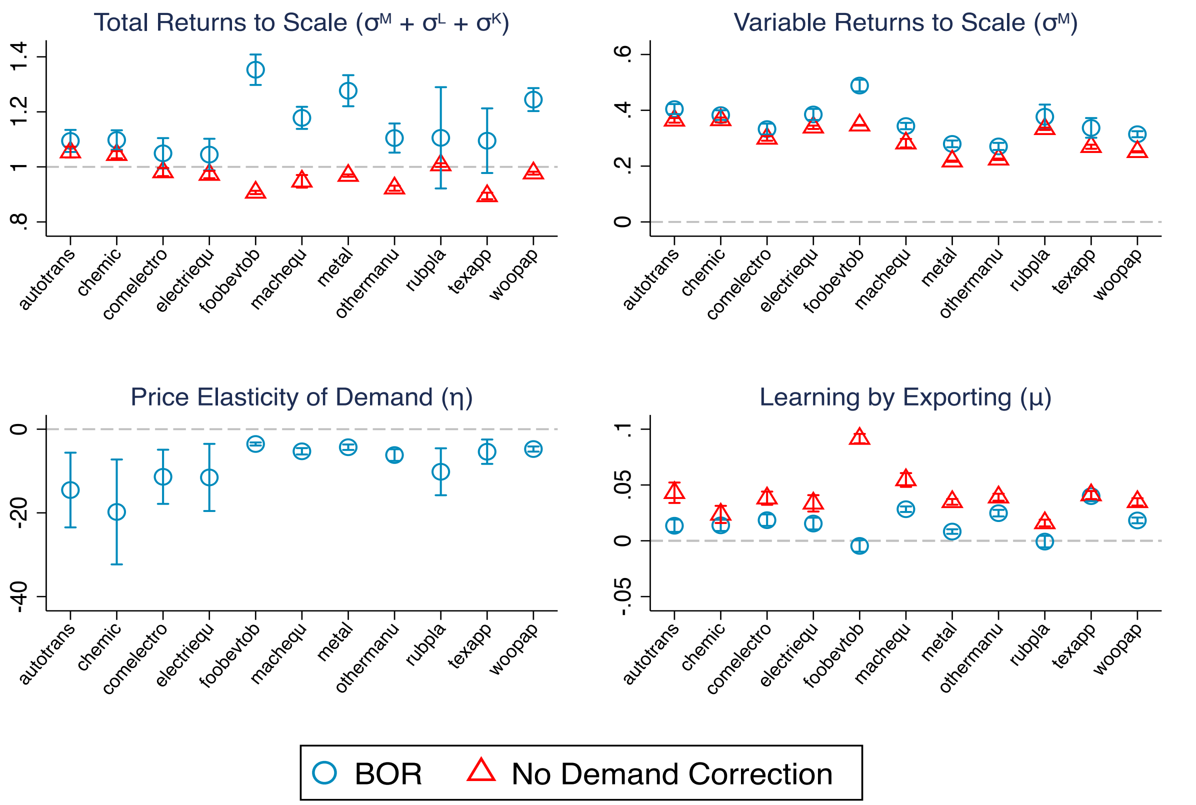
Table 1: Monte Carlo Experiments

	σ^M	σ^K	ρ^0	μ
True Value	0.8000	0.3000	0.8000	0.0500
Factor Shares				
Multi-Market Demand Correction (BOR)	0.7948	0.2916	0.8053	0.0483
	(0.0968)	(0.4197)	(0.0997)	(0.0209)
Multi-Market, Constant Variable Returns to Scale (ARX)	-	0.7788	0.6336	0.2570
		(0.1034)	(0.0026)	(0.0362)
Single Market Demand Correction (SMC)	0.6803	0.0559	0.8351	0.0333
	(1.4131)	(0.8020)	(0.2193)	(0.1022)
No Demand Correction (NoD)	0.5704	0.0523	-	0.0294
	(0.0073)	(0.5696)		(0.0153)
Control Function				
Multi-Market Demand Correction (DL)	1.1416	-0.0525	0.9986	-0.0040
	(0.37)	(0.09)	(0.182)	(0.005)
Single Market Demand Correction (CF-SMC)	0.9591	-0.0251	1.1211	0.0047
	(0.24)	(0.0731)	(0.1789)	(0.0078)
No Demand Correction (CF-NoD)	1.0850	-0.0308	-	0.0041
	(0.1079)	(0.2647)		(0.0109)

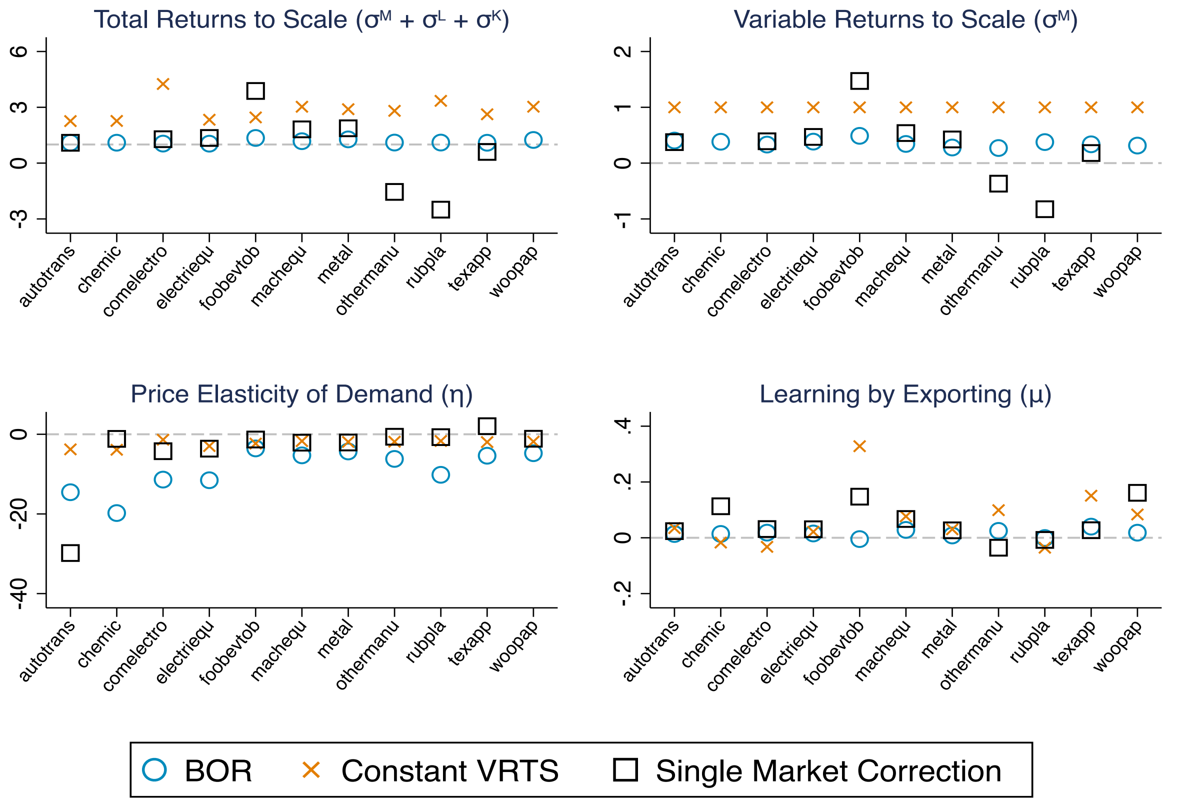
Notes: Table reports median and standard deviation of parameter estimates across 500 simulated data samples, by estimator. Data generating process includes 4 destination markets, 200 firms, and 10 time periods. True parameter values are reported in the first row. Additionally, all foreign $\rho^d = 0.6$ for $d \neq 0$.

Figure 1: Factor Share Estimates by Industry, BOR vs. No Demand Correction



Notes. The figure reports factor share estimates of average Total Returns to Scale, Variable Returns to Scale, the Price Elasticity of Demand on the domestic market $\eta^0 = 1/\left(\rho^0 - 1\right)$, and the LBE learning by exporting (LBE) parameter μ by industry and estimator. Bars represent 95% bootstrap confidence intervals. Detailed estimates are reported in Tables J.1 and J.2.

Figure 2: Factor Share Estimates by Industry, BOR vs. Alternative Estimators



Notes. The figure reports factor share estimates of average Total Returns to Scale, Variable Returns to Scale, the Price Elasticity of Demand on the domestic market $\eta^0 = 1/\left(\rho^0 - 1\right)$, and the LBE learning by exporting (LBE) parameter μ by industry and estimator. Confidence intervals have been suppressed for ease of viewing. Detailed estimates are reported in Table J.3. Results for Chemicals and Wood and Paper for Single Market Correction in top row have been suppressed for ease of viewing.

Online Appendix to
Production Function Estimation with Multi-Destination Firms

A Derivation of equation (12)

The FONCs w.r.t. quantity shares (10) imply

$$R_{ft}^{d}\exp(-u_{ft}^{d}) = \frac{\rho^{0}}{\rho^{d}}\frac{\chi_{ft}^{d}}{\chi_{ft}^{0}}R_{ft}^{0}\exp(-u_{ft}^{0}).$$

Use this in the definition of Z_{ft} and exploit $\sum_{d\in\Omega_{ft}}\chi_{ft}^{d} = 1-\chi_{ft}^{0}$ to get

$$\begin{aligned} Z_{ft} &= \frac{R_{ft}^{0}}{R_{ft}}\exp\left(-u_{ft}^{0}\right) + \sum_{d\in\Omega_{ft}}\frac{\rho^{d}}{\rho^{0}}\frac{R_{ft}^{d}}{R_{ft}}\exp\left(-u_{ft}^{d}\right) \\ &= \frac{1}{\chi_{ft}^{0}}\frac{R_{ft}^{0}}{R_{ft}}\exp(-u_{ft}^{0}), \end{aligned}$$

which implies

$$\chi_{ft}^{0} = \frac{R_{ft}^{0}}{R_{ft}Z_{ft}\exp(u_{ft}^{0})}.$$

Now use this in the equation for domestic revenues

$$\begin{aligned} R_{ft}^{0} &= (Q_{ft})^{\rho^{0}}\left(\chi_{ft}^{0}\right)^{\rho^{0}}D_{t}^{0}\exp(\varepsilon_{ft}^{0})\exp(u_{ft}^{0}) \\ &= (Q_{ft})^{\rho^{0}}\left(\frac{R_{ft}^{0}}{R_{ft}Z_{ft}}\right)^{\rho^{0}}D_{t}^{0}\exp(\varepsilon_{ft}^{0})\left[\exp(u_{ft}^{0})\right]^{1-\rho^{0}}, \end{aligned}$$

which is equation (12) in the text.

B Cournot competition with CES demand

In this appendix we characterize the model and estimation equations if firms compete à la Cournot and demand is CES. We show how to derive similar estimation equations to the ones we present in the main text and discuss estimation. There are two key differences with respect to the monopolistic competition case.

First, there is an additional data requirement. We must observe the share of each firm's sales in the domestic market's total absorption (for the given industry). If the sample of firms is exhaustive (as is our case, using French administrative data), then building domestic absorption requires only adding total imports of products that fall within the scope of the industry (e.g., using an HS6-to-industry code crosswalk) to total firms' domestic sales.

Second, we must assume away destination-specific demand shocks u^d_{ft}. If revenues are affected by *ex post* destination-specific demand shocks u^d_{ft}, then we cannot estimate the model. The reason is that these shocks interfere with the simple characterization of the firm-specific demand elasticity. Therefore, we drop u^d_{ft} demand shocks, which would make the estimation infeasible. Instead, we introduce an unforeseen (by the firm) productivity shock u_{ft}, as in, for example Gandhi et al. (2020).

The model yields an estimation strategy almost identical to what we present in the main text. While we explain how to implement the estimator, we do not actually apply this estimator in the data for two reasons. First, we believe that *ex post* demand shocks are much more important source of uncertainty for firms versus unanticipated productivity shocks. Second, in our new Monte-Carlo experiments with Cournot competition (discussed below) we find that our monopolistic competition-based estimator performs extremely well. This is because the main difference in estimators is due to market shares, which are, in broad industries, never far from zero for the vast majority of firms.

Model Set up. There are a fixed number of destination markets indexed by $d \in \{0, 1, ..., \mathscr{D}\}$ for a generic industry (we omit industry indices from the get go). Preferences are CES as in the main text, and demand (quantity sold) is given by (1) *without* the *ex post* destination-specific demand shocks,

$$X^d_{ft} = \left(\frac{P^d_{ft}}{\Upsilon^d_t} \right)^{\frac{1}{\rho^d_i - 1}} B^d_t e^{\frac{\varepsilon^d_{ft}}{1-\rho^d}} \tag{B.1}$$

where P^d_{ft} is the price charged for variety f in market d in time t, Υ^d_t and B^d_t are, respectively, the CES aggregate price and quantity indices,

$$B^d_t = \left[\sum_{j \in \Phi^d_t} (X^d_{ft})^{\rho^d} e^{\varepsilon^d_{ft}} \right]^{1/\rho^d} , \tag{B.2}$$

Φ_t^d is the set of firms serving/varieties sold to destination d in time t, ε_{ft}^d is an *ex ante* variety-specific demand shock (realized prior to production), and $\eta^d = 1/(\rho^d - 1) < -1$ is a constant price elasticity of demand, which may vary across destinations with the demand curvature parameter ρ^d. Revenues on each destination market d are given by:

$$R_{ft}^d = \left(X_{ft}^d\right)^{\rho^d} Y_t^d \left(B_t^d\right)^{-\rho^d} e^{\varepsilon_{ft}^d}, \tag{B.3}$$

where $Y_t^d = \Upsilon_t^d B_t^d$ is total expenditures in market d, assumed fixed. In order to sell X_{ft}^d the firm must produce $Q_{ft}^d = \tau_t^d X_{ft}^d$, because τ_t^d "melts" along the way, so that $X_{ft}^d = Q_{ft}^d/\tau_t^d$. Thus

$$R_{ft}^d = (Q_{ft}^d)^{\rho^d} Y_t^d (B_t^d)^{-\rho^d} (\tau_t^d)^{-\rho^d} e^{\varepsilon_{ft}^d}, \tag{B.4}$$

Output is given by

$$Q_{ft} = \exp(\omega_{ft} + u_{ft}) F(m_{ft}, l_{ft}, k_{ft}) \Longleftrightarrow q_{ft} = \omega_{ft} + u_{ft} + f(m_{ft}, l_{ft}, k_{ft}). \tag{B.5}$$

where u_{ft} is an unforeseen (by the firm) productivity shock.

In order to avoid clutter, we suspend for now using f and t subscripts, because the following derivations are done for any firm in any period, and do not involve dynamics. We will re-introduce these subscripts in the end. We keep only the t subscripts for aggregates.

Optimization Start with the optimization problem for a given export strategy Ω

$$\begin{aligned}\max_{m,\chi^0,\{\chi^d\}_{d\in\Omega}} \quad & E\Big\{ \left[\chi^0 e^{\omega+u} F(m,l,k)\right]^{\rho^0} Y_t^0 (B_t^0)^{-\rho^d} (\tau_t^0)^{-\rho^0} e^{\varepsilon^0} \\ & + \textstyle\sum_{d\in\Omega} \left[\chi^d e^{\omega+u} F(m,l,k)\right]^{\rho^d} Y_t^d (B_t^d)^{-\rho^d} (\tau_t^d)^{-\rho^d} e^{\varepsilon^d} \Big\} \\ & - W_t M + \lambda \left(1 - \chi^0 - \textstyle\sum_{d\in\Omega} \chi^d\right).\end{aligned}$$

Firms take expectations over u draws. These expectations appear in a nonlinear fashion that varies by destination, $\mathscr{E}_u^d \equiv E[e^{\rho^d u}]$, which is the same for all firms serving d. With this, we can re-write the optimization problem

$$\begin{aligned}\max_{m,\chi^0,\{\chi^d\}_{d\in\Omega}} \quad & \mathscr{E}_u^0 \left[\chi^0 e^{\omega} F(m,l,k)\right]^{\rho^0} Y_t^0 (B_t^0)^{-\rho^d} (\tau_t^0)^{-\rho^0} e^{\varepsilon^0} \\ & + \textstyle\sum_{d\in\Omega} \mathscr{E}_u^d \left[\chi^d e^{\omega} F(m,l,k)\right]^{\rho^d} Y_t^d (B_t^d)^{-\rho^d} (\tau_t^d)^{-\rho^d} e^{\varepsilon^d} \\ & - W_t M + \lambda \left(1 - \chi^0 - \textstyle\sum_{d\in\Omega} \chi^d\right).\end{aligned}$$

Firms internalize their effect on the quantity index. This will introduce derivatives of the quantity index with respect to firm-level quantities into the FONCs. We can nest monopolistic competition by setting these derivatives to zero.

Factor Share Equation The FONC w.r.t. materials yields

$$\begin{aligned} W_t &= \sum_{d\in 0\cup\Omega} \mathscr{E}_u^d Y_t^d (\tau_t^d)^{-\rho^d} e^{\varepsilon^d} (\chi^d)^{\rho^d} e^{\rho^d\omega} \left\{ (B_t^d)^{-\rho^d} \rho^d \left[F(m,l,k)\right]^{\rho^d-1} \frac{\partial F}{\partial M} \right. \\ &\left. - \left[F(m,l,k)\right]^{\rho^d} \frac{\partial\left[(B_t^d)^{-\rho^d}\right]}{\partial Q^d} \frac{\partial Q^d}{\partial F} \frac{\partial F}{\partial M} \right\}. \end{aligned}$$

Multiply by M and use (B.2) and

$$\frac{\partial Q^d}{\partial F} = \frac{\partial\left[\chi^d e^{\omega+u} F(m,l,k)\right]}{\partial F} = \chi^d e^{\omega+u} = \frac{Q^d}{F} \tag{B.6}$$

to write

$$\begin{aligned} W_t M &= \sum_{d\in 0\cup\Omega} \mathscr{E}_u^d Y_t^d (\tau_t^d)^{-\rho^d} e^{\varepsilon^d} (\chi^d)^{\rho^d} e^{\rho^d\omega} \left[F(m,l,k)\right]^{\rho^d} \left\{ (B_t^d)^{-\rho^d} \rho^d \frac{\partial F}{\partial M} \frac{M}{F} \right. \\ &\left. + \frac{\partial\left[\sum_j (X^d)^{\rho^d} e^{\varepsilon^d}\right]^{-1}}{\partial Q^d} Q^d \frac{\partial F}{\partial M} \frac{M}{F} \right\}. \end{aligned} \tag{B.7}$$

Now

$$\begin{aligned} \frac{\partial\left[\sum_j (X^d)^{\rho^d} e^{\varepsilon^d}\right]^{-1}}{\partial Q^d} &= -\left[\sum_j (X^d)^{\rho^d} e^{\varepsilon^d}\right]^{-2} \rho^d (X^d)^{\rho^d-1} e^{\varepsilon^d} \frac{\partial X^d}{\partial Q^d} \\ &= -\rho^d \left[\sum_j (X^d)^{\rho^d} e^{\varepsilon^d}\right]^{-1} \frac{(X^d)^{\rho^d} e^{\varepsilon^d}}{\sum_j (X^d)^{\rho^d} e^{\varepsilon^d}} (X^d)^{-1} (\tau^d)^{-1} \\ &= -\rho^d (B_t^d)^{-\rho^d} s^d (Q^d)^{-1} \end{aligned} \tag{B.8}$$

since $X^d = Q^d/\tau^d$, and where

$$s^d = \frac{(X^d)^{\rho^d} e^{\varepsilon^d}}{\sum_j (X^d)^{\rho^d} e^{\varepsilon^d}}$$

is the revenue share of firm f in market d (in time t). Using (B.8) in (B.7) and collecting terms yields

$$\begin{aligned} W_t M &= \sum_{d\in 0\cup\Omega} \rho^d \sigma^M \mathscr{E}_u^d \left[(\chi^d e^{\omega} F(m,l,k)\right]^{\rho^d} Y_t^d (B_t^d)^{-\rho^d} (\tau_t^d)^{-\rho^d} e^{\varepsilon^d} (1-s^d) \\ &= \sigma^M \sum_{d\in 0\cup\Omega} \rho^d \mathscr{E}_u^d R^d (1-s^d) e^{-\rho^d u} \end{aligned}$$

where we use (B.3) and $\sigma^M = (\partial F/\partial M)/(M/F)$.

Now divide by R to get the M-share equation

$$\frac{W_t M}{R} = \sigma^M \sum_{d\in 0\cup\Omega} \rho^d \mathscr{E}_u^d \frac{R^d}{R} (1-s^d) e^{-\rho^d u}. \tag{B.9}$$

We can also write this as

$$\frac{W_t M}{R} = \rho^0 \mathscr{E}_u^0 \sigma^M \left[\frac{R^0}{R}(1-s^0)e^{-\rho^0 u} + \sum_{d\in\Omega} \frac{\rho^d}{\rho^0} \frac{\mathscr{E}_u^d}{\mathscr{E}_u^0} \frac{R^d}{R}(1-s^d)e^{-\rho^d u} \right], \tag{B.10}$$

where we will use later

$$Z \equiv \frac{R^0}{R}(1-s^0)e^{-\rho^0 u} + \sum_{d\in\Omega} \frac{\rho^d}{\rho^0} \frac{\mathscr{E}_u^d}{\mathscr{E}_u^0} \frac{R^d}{R}(1-s^d)e^{-\rho^d u}. \tag{B.11}$$

Conditioning on non-exporters, (B.10) becomes

$$\frac{W_t M}{R} = \rho^0 \mathscr{E}_u^0 \sigma^M (1-s^0)e^{-\rho^0 u} \tag{B.12}$$

because for these firms $R^0 = R$ and $R^d = 0$. Taking logs, rearranging and reintroducing the necessary subscripts, we have

$$\ln \frac{W_t M_{ft}}{R_{ft}(1-s_{ft}^0)} = \ln[\rho^0 \mathscr{E}_u^0 \sigma^M (m_{ft}, l_{ft}, k_{ft})] - \rho^0 u_{ft}. \tag{B.13}$$

The composite $\rho^0 \mathscr{E}_u^0 \sigma^M$ can be estimated using a polynomial in inputs as described in the main text. The term $\widehat{\mathscr{E}_u^0}$ can then be constructed $\widehat{\mathscr{E}_u^0} = (1/N^0)\sum_t \sum_{f\in\Phi_t^0} e^{\rho^0 u_{ft}}$ (and remember to flip signs). Then $\widehat{\mathscr{E}_u^0}$ can be used to build an estimate of $\rho^0 \sigma^M$ and to build Z from (B.10) on a sample of all firms.

Revenue Equation First-order necessary conditions with respect to output shares χ^d yield, for each $d \in 0 \cup \Omega$,

$$\lambda = \mathscr{E}_u^d \left[e^{\omega} F(m,l,k)\right]^{\rho^d} Y_t^d (\tau_t^d)^{-\rho^d} e^{\varepsilon^d} \left\{ \rho^d (\chi^d)^{\rho^d - 1} (B_t^d)^{-\rho^d} + \left[\chi^d\right]^{\rho^d} \frac{\partial \left[(B_t^d)^{-\rho^d}\right]}{\partial Q^d} \frac{\partial Q^d}{\partial \chi^d} \right\}.$$

Using (B.8) and

$$\frac{\partial Q^d}{\partial \chi^d} = \frac{\partial \chi^d Q}{\partial \chi^d} = Q$$

we can write, after some manipulation,

$$\lambda \quad = \quad \rho^d \mathscr{E}_u^d (Q^d)^{\rho^d} Y_t^d (B_t^d)^{-\rho^d} (\tau_t^d)^{-\rho^d} e^{\varepsilon^d} (\chi^d)^{-1} (1-s^d) e^{-\rho^d u}$$

This implies

$$\rho^d \mathscr{E}_u^d R^d (\chi^d)^{-1} (1-s^d) e^{-\rho^d u} = \lambda.$$

For $d = 0$ this gives

$$\rho^0 \mathscr{E}_u^0 R^0 (\chi^0)^{-1} (1-s^0) e^{-\rho^0 u} = \lambda.$$

Taking the ratio yields

$$\frac{\rho^d \mathscr{E}_u^d R^d (\chi^d)^{-1} (1-s^d) e^{-\rho^d u}}{\rho^0 \mathscr{E}_u^0 R^0 (\chi^0)^{-1} (1-s^0) e^{-\rho^0 u}} = 1.$$

Manipulating this to isolate R^d gives

$$R^d = \frac{\rho^0 \chi^d}{\rho^d \chi^0} R^0 \frac{\mathscr{E}_u^0}{\mathscr{E}_u^d} \frac{(1-s^0)e^{\rho^d u}}{(1-s^d)e^{\rho^0 u}}.$$

Use this in the definition of Z (B.11) to get, after some manipulation,

$$\begin{aligned} Z &\equiv \frac{R^0}{R}(1-s^0)e^{-\rho^0 u} + \sum_{d \in \Omega} \frac{\rho^d}{\rho^0} \frac{\mathscr{E}_u^d}{\mathscr{E}_u^0} \frac{R^d}{R}(1-s^d)e^{-\rho^d u} \\ &= \frac{1}{\chi^0} \frac{R^0}{R}(1-s^0)e^{-\rho^0 u}. \end{aligned}$$

This implies that

$$\chi^0 = \frac{R^0(1-s^0)}{RZ} e^{-\rho^0 u} \tag{B.14}$$

Write domestic revenues as

$$R^0 = (Q^0)^{\rho^0} Y_t^0 (B_t^0)^{-\rho^0} (\tau_t^0)^{-\rho^0} e^{\varepsilon^0} = (\chi^0 Q)^{\rho^0} Y_t^0 (B_t^0)^{-\rho^0} (\tau^0)^{-\rho^0} e^{\varepsilon^0}$$

and using (B.14) gives

$$R^0 = \left(\frac{R^0(1-s^0)}{RZ} \right)^{\rho^0} Q^{\rho^0} Y_t^0 (B_t^0)^{-\rho^0} (\tau^0)^{-\rho^0} e^{\varepsilon^0} (e^{-\rho^0 u})^{\rho^0}.$$

After re-introducing subscripts added, the revenue equation is thus

$$R_{ft}^0 = \left(\frac{R_{ft}^0(1-s_{ft}^0)}{R_{ft} Z_{ft}} \right)^{\rho^0} Q_{ft}^{\rho^0} Y_t^0 (B_t^0)^{-\rho^d} (\tau_t^0)^{-\rho^0} e^{\varepsilon_{ft}^0} (e^{-\rho^0 u_{ft}})^{\rho^0}.$$

Taking logs and rearranging yields

$$\ln R_{ft}^0 = \rho^0 \ln \left(\frac{R_{ft}^0(1-s_{ft}^0)}{R_{ft} Z_{ft}} \right) + \rho^0 f(m_{ft}, l_{ft}, k_{ft}) + \mathrm{FE}_t + \varepsilon_{ft}^0 + (1-\rho^0)\rho^0 u_{ft} + \rho^0 \omega_{ft}$$

where

$$\mathrm{FE}_t = \ln Y_t^0 (B_t^0)^{-\rho^d} (\tau_t^0)^{-\rho^0}$$

is a fixed effect. This applies to all firms, but using only domestic revenues.

Using the same method as in the main text, the component of $\rho f(m_{ft}, l_{ft}, k_{ft})$ that is due to materials use can be subtracted from both sides of this equation. Moment conditions for the GMM estimator are the same as in the main text.

C Model with Kimball Demand function

In this appendix, we characterize the model if demand is derived from Kimball preferences (Kimball, 1995). We demonstrate that for a general Kimball demand function, it is possible to express revenues on the domestic market as a function of inputs, domestic revenues shares, domestic demand conditions, and a computable residual Z_{ft}, which encodes information about the foreign demand conditions, as in equation (12) in the main text.

However, though foreign demand conditions can be eliminated from the expression, supply and demand parameters cannot be estimated without more structure. Without additional restrictions, there is no way to recover the production function separately from the inverse demand function. Any change in revenues induced by an input can be rationalized either by a movement along the supply curve, or a movement along the demand curve. This demonstrates the role of CES demand in our estimation strategy.

Model Set up There are a fixed number of destination markets indexed by $d \in \{0,1,...,\mathscr{D}\}$ for a generic industry (we omit industry indices). In what follows we omit *ex post* u^d_{ft} demand shocks, because including them would necessitate computing expectations of u^d_{ft}s inside a nonlinear function.

Within an industry i, demanded for a given variety f at time t by consumers in destination market d is governed by a general Kimball form

$$\sum_j \Psi\left(\frac{e^{\frac{\varepsilon^d_{ft}}{\rho^d}} Q^d_{ft}}{B^d_t}\right) = 1, \tag{C.1}$$

where $\Psi(\cdot)$ is some aggregator function and B^d_t is the quantity index implicitly defined by $\Psi(\cdot)$.

The solution to the consumer's problem implies that revenues on each destination market d are given by:

$$R^d_{ft} = Y^d_t H^d\left(\frac{e^{\frac{\varepsilon^d_{ft}}{\rho^d}} Q^d_{ft}}{B^d_t}\right) \tag{C.2}$$

where $Y^d_t \equiv B^d_t \Upsilon^d_t$ is expenditures, assumed fixed. The function $H^d\left(\frac{e^{\frac{\varepsilon^d_{ft}}{\rho^d}} Q^d_{ft}}{B^d_t}\right) = \frac{e^{\frac{\varepsilon^d_{ft}}{\rho^d}} Q^d_{ft}}{B^d_t} \times \Psi'\left(\frac{e^{\frac{\varepsilon^d_{ft}}{\rho^d}} Q^d_{ft}}{B^d_t}\right)$, and is allowed to vary by d. In general, there is no closed-form for $\Psi'(\cdot)$.

Output is a function of a Hicks-neutral productivity shock ω_{ft} and a twice continuously differentiable transformation of variable and quasi-fixed inputs:

$$Q_{ft} = \exp(\omega_{ft}) F(m_{ft}, l_{ft}, k_{ft}) \Longleftrightarrow q_{ft} = \omega_{ft} + f(m_{ft}, l_{ft}, k_{ft}), \tag{C.3}$$

Optimization Using (2), we write the optimization problem for a given export strategy Ω_{ft} as

$$\max_{m_{ft},\chi^0_{ft},\{\chi^d_{ft}\}_{d\in\Omega_{ft}}} \sum_{d\in 0\cup\Omega_{ft}} Y^d_t H^d\left(\frac{\exp\left(\frac{\varepsilon^d_{ft}}{\rho^d}\right)\chi^d_{ft}\exp\left(\omega_{ft}\right)F\left(m_{ft},l_{ft},k_{ft}\right)}{B^d_t}\right)-W^M_t M_{ft}$$
$$+\lambda_{ft}\left(1-\chi^0_{ft}-\sum_{d\in\Omega_{ft}}\chi^d_{ft}\right), \tag{C.4}$$

where λ_{ft} is the Lagrangian associated to the constraint $\chi^0_{ft}+\sum_d \chi^d_{ft}=1$.

We allow that firms internalize their effect on the quantity index. This will introduce derivatives of the quantity index with respect to firm-level quantities into the FONCs. We can nest monopolistic competition by simply setting these derivatives to 0.

First-order necessary conditions with respect to output shares χ^d_{ft} yield, for each $d\in\{0\cup\Omega_{ft}\}$,

$$\lambda_{ft}=Y^d_t\frac{\partial H^d(\cdot)}{\partial\Xi^d_{ft}}\left[\frac{\exp\left(\frac{\varepsilon^d_{ft}}{\rho^d}\right)\exp\left(\omega_{ft}\right)F\left(m_{ft},l_{ft},k_{ft}\right)}{B^d_t}-\frac{\exp\left(\frac{\varepsilon^d_{ft}}{\rho^d}\right)\chi^d_{ft}\exp\left(\omega_{ft}\right)F\left(m_{ft},l_{ft},k_{ft}\right)}{B^d_t}\frac{\partial B^d_t}{\partial\chi^d_{ft}}\frac{1}{B^d_t}\right] \tag{C.5}$$

where Ξ^d_{ft} is the argument of $H^d(\cdot)$. With some rearranging

$$\lambda_{ft}=Y^d_t\mu^d\left(\Xi^d_{ft}\right)\frac{H^d\left(\Xi^d_{ft}\right)}{\chi^d_{ft}}\left(1-s^d_{ft}\right) \tag{C.6}$$

where $s^d_{ft}\equiv\frac{\partial B^d_t}{\partial Q^d_{ft}}\frac{Q^d_{ft}}{B^d_t}=\frac{R^d_{ft}}{\sum_j R^d_{jt}}$, $\mu^d\left(\Xi^d_{ft}\right)\equiv\frac{\partial H^d(\cdot)}{\partial\Xi^d_{ft}}\frac{\Xi^d_{ft}}{H^d(\cdot)}$.

These FONCs imply

$$Y^d_t H^d\left(\Xi^d_{ft}\right)=Y^0_t H^0\left(\Xi^0_{ft}\right)\frac{\mu^0\left(\Xi^0_{ft}\right)}{\mu^d\left(\Xi^d_{ft}\right)}\frac{\chi^d_{ft}}{\chi^0_{ft}}\frac{\left(1-s^0_{ft}\right)}{\left(1-s^d_{ft}\right)} \tag{C.7}$$

Substituting (C.7) into the expression for total revenues gives

$$\begin{aligned}R_{ft} &= \sum_{d\in 0\cup\Omega_{ft}} Y^d_t H^d\left(\Xi^d_{ft}\right)\\ &= R^0_{ft}\left(1+\frac{\mu^0\left(\Xi^0_{ft}\right)}{\chi^0_{ft}}\left(1-s^0_{ft}\right)\psi^x_{ft}\right)\end{aligned} \tag{C.8}$$

where $\psi^x_{ft}\equiv\sum_{d\in\Omega_{ft}}\frac{\chi^d_{ft}}{\mu^d\left(\Xi^d_{ft}\right)\left(1-s^d_{ft}\right)}$

The first-order necessary condition with respect to materials yields

$$W_t^M = \sum_{d\in 0\cup\Omega_{ft}} Y_t^d \frac{\partial H^d(\cdot)}{\partial \Xi_{ft}^d}\left(\frac{\exp\left(\frac{\varepsilon_{ft}^d}{\rho^d}\right)\chi_{ft}^d \exp(\omega_{ft})}{B_t^d}\frac{\partial F}{\partial M_{ft}}\right.$$
$$\left. - \frac{\exp\left(\frac{\varepsilon_{ft}^d}{\rho^d}\right)\chi_{ft}^d \exp(\omega_{ft}) F(m_{ft},l_{ft},k_{ft})}{B_t^d}\frac{\partial B_t^d}{\partial Q_{ft}^d}\frac{\partial Q_{ft}^d}{\partial Q_{ft}}\frac{\partial F_{ft}}{\partial M_{ft}}\right) \tag{C.9}$$

Factor Share Equation Multiplying both sides by M_{ft} and rearranging

$$\begin{aligned} W_t^M M_{ft} &= \sum_{d\in 0\cup\Omega_{ft}} Y_t^d \frac{\partial H^d(\cdot)}{\partial \Xi_{ft}^d}\frac{\exp\left(\frac{\varepsilon_{ft}^d}{\rho^d}\right)\chi_{ft}^d \exp(\omega_{ft}) F(m_{ft},l_{ft},k_{ft})}{B_t^d}\sigma_{ft}^M\left(1-s_{ft}^d\right) \\ &= \sum_{d\in 0\cup\Omega_{ft}} Y_t^d \frac{\partial H^d(\cdot)}{\partial \Xi_{ft}^d}\frac{\Xi_{ft}^d}{H^d(\cdot)}\frac{H^d(\cdot)}{\Xi_{ft}^d}\frac{\exp\left(\frac{\varepsilon_{ft}^d}{\rho^d}\right)\chi_{ft}^d \exp(\omega_{ft}) F(m_{ft},l_{ft},k_{ft})}{B_t^d}\sigma_{ft}^M\left(1-s_{ft}^d\right) \\ &= \sum_{d\in 0\cup\Omega_{ft}} Y_t^d \mu^d\left(\Xi_{ft}^d\right) H^d(\cdot)\,\sigma_{ft}^M\left(1-s_{ft}^d\right) \end{aligned} \tag{C.10}$$

And substituting with revenues

$$W_t^M M_{ft} = \sum_{d\in 0\cup\Omega_{ft}} \sigma_{ft}^M R_{ft}^d \mu^d\left(\Xi_{ft}^d\right)\left(1-s_{ft}^d\right) \tag{C.11}$$

Rearranging yields:

$$\frac{W_t^M M_{ft}}{R_{ft}} = \sigma_{ft}^M\left[\frac{R_{ft}^0}{R_{ft}}\mu^0\left(\Xi_{ft}^0\right)\left(1-s_{ft}^0\right) + \sum_{d\in\Omega_{ft}}\frac{R_{ft}^d}{R_{ft}}\mu^d\left(\Xi_{ft}^d\right)\left(1-s_{ft}^d\right)\right] \tag{C.12}$$

Here, we see that the extension presents a problem for estimation of the factor shares regression. The elasticity of demand $\mu^d\left(\Xi_{ft}^d\right) \equiv \frac{\partial H^d(\cdot)}{\partial \Xi_{ft}^d}\frac{\Xi_{ft}^d}{H^d(\cdot)}$ intervenes in the brackets. In general, this object depends on Ξ_{ft}^d, which includes Q_{ft}^d, which itself depends on input choices. So we cannot recover σ_{ft}^M from the factor share regression, because σ_{ft}^M depends on input choices too, in general.

However, we could identify a composite function $\beta_{ft}^M = \sigma_{ft}^M \times \mu^0\left(\Xi_{ft}^0\right)$. Rewrite (C.12) as

$$\frac{W_t^M M_{ft}}{R_{ft}\left(1-s_{ft}^0\right)} = \mu^0\left(\Xi_{ft}^0\right)\sigma_{ft}^M\left[\frac{R_{ft}^0}{R_{ft}} + \sum_{d\in\Omega_{ft}}\frac{R_{ft}^d}{R_{ft}}\frac{\mu^d\left(\Xi_{ft}^d\right)}{\mu^0\left(\Xi_{ft}^0\right)}\frac{\left(1-s_{ft}^d\right)}{\left(1-s_{ft}^0\right)}\right] \tag{C.13}$$

where the term in brackets is Z_{ft}. Condition on non-exporters,

$$\frac{W_t^M M_{ft}}{R_{ft}\left(1-s_{ft}^0\right)} = \mu^0\left(\Xi_{ft}^0\right)\sigma_{ft}^M. \tag{C.14}$$

We can add an error term to (C.14) that reflects measurement error in revenues. This would introduce the expected value of this shock into the equation, and we can deal with this as in the main text. Then we can regress the material share on polynomials in inputs to estimate $\mu^0\left(\Xi^0_{ft}\right)\sigma^M_{ft}$.

Revenue Equation Now to derive the second-step estimation equation. Starting from domestic revenues

$$R^0_{ft} = Y^0_t H^0\left(\frac{\exp\left(\frac{\varepsilon^0_{ft}}{\rho^0}\right)\chi^0_{ft}\exp\left(\omega_{ft}\right)F\left(m_{ft},l_{ft},k_{ft}\right)}{B^0_t}\right) \tag{C.15}$$

Now to substitute for χ^0_{ft}, use that

$$Y^d_t H^d\left(\Xi^d_{ft}\right) = Y^0_t H^0\left(\Xi^0_{ft}\right)\frac{\mu^0\left(\Xi^0_{ft}\right)}{\mu^d\left(\Xi^d_{ft}\right)}\frac{\chi^d_{ft}}{\chi^0_{ft}}\frac{\left(1-s^0_{ft}\right)}{\left(1-s^d_{ft}\right)} \tag{C.16}$$

Now substitute into Z_{ft}

$$\begin{aligned}
Z_{ft} &= \frac{R^0_{ft}}{R_{ft}} + \sum_{d\in\Omega_{ft}}\frac{R^d_{ft}}{R_{ft}}\frac{\mu^d\left(\Xi^d_{ft}\right)}{\mu^0\left(\Xi^0_{ft}\right)}\frac{\left(1-s^d_{ft}\right)}{\left(1-s^0_{ft}\right)} \\
&= \frac{R^0_{ft}}{R_{ft}} + \sum_{d\in\Omega_{ft}}\frac{R^0_{ft}}{R_{ft}}\frac{\chi^d_{ft}}{\chi^0_{ft}} \\
&= \frac{R^0_{ft}}{R_{ft}}\left(1+\sum_{d\in\Omega_{ft}}\frac{\chi^d_{ft}}{\chi^0_{ft}}\right) \\
&= \frac{R^0_{ft}}{R_{ft}\chi^0_{ft}} \qquad \text{(C.17)}
\end{aligned}$$

which implies

$$\chi^0_{ft} = \frac{R^0_{ft}}{R_{ft}Z_{ft}} \tag{C.18}$$

Now substituting in

$$R^0_{ft} = Y^0_t H^0\left(\frac{\exp\left(\frac{\varepsilon^0_{ft}}{\rho^0}\right)\frac{R^0_{ft}}{R_{ft}Z_{ft}}\exp\left(\omega_{ft}\right)F\left(m_{ft},l_{ft},k_{ft}\right)}{B^0_t}\right) \tag{C.19}$$

This expression generalizes the result from equation (12): for a general Kimball demand function, it is possible to express revenues on the domestic market as a function of inputs, domestic revenues shares, domestic demand conditions, and a computable residual Z_{ft}, which encodes information about the foreign demand conditions, as in equation (12) in the main text.

However, the equation reveals the main obstacle to implementation. Though foreign demand conditions can be eliminated from the expression, supply and demand parameters cannot be estimated without more structure. Without additional restrictions, there is no way to recover the production function separately from the inverse demand function. Any change in revenues induced by an input can be rationalized either by a movement along the supply curve, or a movement along the demand curve. This demonstrates the role of CES demand in our estimation strategy.

D Extension to Multi-Product Firms

In this appendix, we characterize a model and estimation strategy for the case in which firms produce multiple products with different technologies. The model combines our multi-destination framework with the multi-product framework from Valmari (2023). As in Valmari (2023), we assume that inputs are observed only at the firm level, while revenues are observed separately for each product. Crucially, however, we continue to assume that outputs are observed only in value terms; prices and quantities are never separately observed.

Relative to our baseline single-product model, identification requires three additional restrictions. First, we assume away *ex-post* demand shocks, but allow for unanticipated productivity shocks, u_{ft}, as in Appendix B. Second, we assume that demand curvature is common and constant across both products and destinations. Third, we simplify the production function to a Cobb-Douglas specification, as in Valmari (2023).

Although ruling out *ex-post* demand shocks is more restrictive than our baseline specification, the resulting framework remains more general than Valmari (2023), who assumes that there are no unanticipated shocks of any kind, either to production or to demand. Moreover, we show that both supply and demand parameters remain identifiable without observing prices or quantities separately, despite the fact that Valmari (2023) makes use of price data.

With respect to the production function, in our baseline single-product model, input elasticities can be recovered non-parametrically because all inputs are observed at the level at which production takes place. In the multi-product setting, however, the allocation of inputs across products is unobserved. As in Valmari (2023), our estimation strategy for this case conditions on candidate values of the input elasticities to recover the product-specific input allocations from the observed data. Under a fully general production function, these elasticities would themselves depend on the unobserved product-specific input allocations, making the inversion impossible. For this reason, we impose a Cobb-Douglas specification, as in Valmari (2023).

While these assumptions deliver an estimation strategy for firms producing multiple products with heterogeneous technologies (but common, constant demand elasticities)—we cannot implement it using the French data because firm-product revenue information is unavailable. Accordingly, we present the identification and estimation strategy for settings in which such data are available, but we do not estimate the extended model empirically.

Model Set up. There are a fixed number of destination markets indexed by $d \in \{0, 1, ..., \mathscr{D}\}$ and a fixed number of products $j \in \{0, 1, ..., \mathscr{J}\}$. Preferences are CES as in the main text, and demand (quantity sold) is given by

$$X_{fjt}^{d} = \left(\frac{P_{fjt}^{d}}{\Upsilon_{jt}^{d}}\right)^{\frac{1}{\rho-1}} B_{jt}^{d} e^{\frac{\varepsilon_{fjt}^{d}}{1-\rho}} \tag{D.1}$$

where P^d_{fjt} is the price charged for variety f for product j in market d in time t, Υ^d_{jt} and B^d_{jt} are, respectively, the CES aggregate price and quantity indices,

$$B^d_{jt} = \left[\sum_{k \in \Phi^d_{jt}} (X^d_{fjt})^\rho e^{\varepsilon^d_{fjt}} \right]^{1/\rho}, \tag{D.2}$$

Φ^d_{jt} is the set of firms serving (varieties sold to) destination d for product j in time t, ε^d_{fjt} is an *ex ante* variety-specific demand shock (realized prior to production), and $\eta = 1/(\rho - 1) < -1$ is a constant price elasticity of demand. Revenues on each destination-product market d are given by:

$$R^d_{fjt} = \left(X^d_{fjt} \right)^\rho Y^d_{jt} \left(B^d_{jt} \right)^{-\rho} e^{\varepsilon^d_{fjt}}, \tag{D.3}$$

where $Y^d_{jt} = \Upsilon^d_{jt} B^d_{jt}$ is total expenditures in market d, assumed fixed. In order to sell X^d_{fjt} the firm must produce $Q^d_{fjt} = \tau^d_t X^d_{fjt}$, because τ^d_t "melts" along the way, so that $X^d_{fjt} = Q^d_{fjt}/\tau^d_t$. Thus

$$R^d_{fjt} = (Q^d_{fjt})^\rho Y^d_{jt} (B^d_{jt})^{-\rho} (\tau^d_t)^{-\rho} e^{\varepsilon^d_{fjt}}, \tag{D.4}$$

Products are produced with product-specific productions functions. Firms produce a set of products Ω^J_{ft}, which, like the export strategy Ω_{ft}, we need not specify how it is chosen.

Firms have heterogeneous (anticipated) productivity levels ω_{fjt}, and unanticipated productivity shocks u_{ft}. Assuming Cobb-Douglas production yields output of product j by firm f at time t is

$$Q_{fjt} = \exp(\omega_{fjt} + u_{ft}) M_{fjt}^{\gamma^M_j} L_{fjt}^{\gamma^L_j} K_{fjt}^{\gamma^K_j}, \tag{D.5}$$

Using (D.3) we can write revenues for firm f from product j shipped to destination d as

$$R^d_{fjt} = \left(\chi^d_{fjt} Q_{fjt} \right)^\rho D^d_{jt} \exp\left(\varepsilon^d_{fjt} \right), \tag{D.6}$$

where $\chi^d_{fjt} \equiv Q^d_{fjt}/Q_{fjt}$ is the share of product j's total output allocated to destination d (with $\sum_d \chi^d_{fjt} = 1$), $D^d_{jt} \equiv \Upsilon^d_{jt} B^d_{jt} (\tau^d_t)^{-\rho}$ is a product-destination-time aggregate demand shifter.

Optimization. At the beginning of period t the firm observes ω_{fjt}, $\left\{ \varepsilon^d_{fjt} \right\}_{j,d}$, and $\left\{ D^d_{jt} \right\}_{j,d}$. It then chooses materials $\{M_{fjt}\}_j$, output shares $\{\chi^d_{fjt}\}_{j,d}$, and the allocation of quasi-fixed inputs L_{ft} and K_{ft} across product lines, with totals for quasi-fixed inputs L_{ft} and K_{ft} are pre-determined. The unanticipated productivity shock u_{ft} is realized only after production and trade decisions are made; firms take expectations over its distribution, which has a constant mean known to the firm and is independent across periods.

For a given export strategy $\left\{\Omega_{fjt}\right\}_j$, firm f solves

$$
\begin{aligned}
\max_{\substack{M_{fjt},\chi^0_{fjt},\{\chi^d_{fjt}\}_{d\in\Omega_{fjt}}\\ L_{fjt},K_{fjt}}} \quad & E_u\left[\sum_{j\in\Omega^J_{ft}}\left((\chi^0_{fjt}Q_{fjt})^\rho D^0_{jt}\exp(\varepsilon^0_{fjt})+\sum_{d\in\Omega_{fjt}}(\chi^d_{fjt}Q_{fjt})^\rho D^d_{jt}\exp(\varepsilon^d_{fjt})\right)\right] \\
& - \; W^M_t\sum_{j\in\Omega^J_{ft}}M_{fjt}+\sum_{j\in\Omega^J_{ft}}\lambda^{EX}_{fjt}\left(1-\chi^0_{fjt}-\sum_{d\in\Omega_{fjt}}\chi^d_{fjt}\right) \\
& + \; \lambda^L_{ft}\left(L_{ft}-\sum_{j\in\Omega^J_{ft}}L_{fjt}\right)+\lambda^K_{ft}\left(K_{ft}-\sum_{j\in\Omega^J_{ft}}K_{fjt}\right),
\end{aligned}
\tag{D.7}
$$

taking expectations over u_{ft}, where λ^{EX}_{fjt} is the Lagrange multiplier for the output-share constraint for product j, and λ^L_{ft}, λ^K_{ft} are the shadow prices of the quasi-fixed input constraints. Define $\mathscr{E}_u \equiv E[\exp(\rho u_{ft})]$, which is the same for all firms.

The first-order necessary condition (FONC) for materials, using $\partial Q_{fjt}/\partial M_{fjt}=\gamma^M_j Q_{fjt}/M_{fjt}$, gives

$$
W^M_t M_{fjt}=\gamma^M_j\sum_{d\in\{0\}\cup\Omega_{fjt}}\rho\,\mathscr{E}_u R^d_{fjt}\exp\left(-\rho u_{ft}\right). \tag{D.8}
$$

Summing (D.8) over all products of the firm and dividing by total revenues $R_{ft}\equiv\sum_j\sum_d R^d_{fjt}$ yields

$$
s^M_{ft} \;\equiv\; \frac{W^M_t M_{ft}}{R_{ft}} \;=\; \sum_{j\in\Omega^J_{ft}}\gamma^M_j\sum_{d\in\{0\}\cup\Omega_{fjt}}\rho\,\mathscr{E}_u\exp\left(-\rho u_{ft}\right)\frac{R^d_{fjt}}{R_{ft}},
$$

thus

$$
s^M_{ft}=\sum_{j\in\Omega^J_{ft}}\gamma^M_j\rho\,\mathscr{E}_u\exp\left(-\rho u_{ft}\right)\sum_{d\in\{0\}\cup\Omega_{fjt}}\frac{R^d_{fjt}}{R_{ft}}. \tag{D.9}
$$

Defining $R_{fjt}\equiv\sum_d R^d_{fjt}$ the total revenue from product j of firm f across all destinations yields

$$
s^M_{ft}=\rho\,\mathscr{E}_u\exp\left(-\rho u_{ft}\right)\sum_{j\in\Omega^J_{ft}}\gamma^M_j\frac{R_{fjt}}{R_{ft}}, \tag{D.10}
$$

where the product-specific revenue shares R_{fjt}/R_{ft} are directly observable for all firms. This equation is the direct multi-product analog of equation (8) of the main text.

The first-order necessary condition for output shares χ^d_{fjt} for each $d\in\{0\}\cup\Omega_{fjt}$ and each product j gives (under monopolistic competition, firms take D^d_{jt} as given)

$$
\rho\,\mathscr{E}_u(\chi^d_{fjt})^{\rho-1}Q^\rho_{fjt}D^d_{jt}\exp(\varepsilon^d_{fjt})=\lambda^{EX}_{fjt}, \qquad d\in\{0\}\cup\Omega_{fjt}, \quad j\in\Omega^J_{ft}. \tag{D.11}
$$

Setting the conditions for destinations d and d' equal,

$$
\frac{(\chi^{d'}_{fjt})^{\rho-1}}{(\chi^d_{fjt})^{\rho-1}}=\frac{D^d_{jt}\exp(\varepsilon^d_{fjt})}{D^{d'}_{jt}\exp(\varepsilon^{d'}_{fjt})}. \tag{D.12}
$$

These measure-Ω^J_{ft} systems (one per product) determine the optimal output allocation across destinations. In this setting, without *ex post* demand shocks, revenue shares are equal to quantity shares since

$$\frac{R^d_{fjt}}{\chi^d_{fjt}} = \frac{R^0_{fjt}}{\chi^0_{fjt}}, \qquad d \in \Omega_{fjt}, \quad j \in \Omega^J_{ft}. \tag{D.13}$$

The FONCs for quasi-fixed input allocation yield (using $\partial Q_{fjt}/\partial L_{fjt} = \gamma^L_j Q_{flt}/L_{fjt}$)

$$\lambda^L_{ft} = \frac{\gamma^L_j}{L_{fjt}} \sum_{d\in\{0\}\cup\Omega_{fjt}} \rho\, \mathscr{E}_u R^d_{fjt} \exp\left(-\rho u_{ft}\right), \qquad j \in \Omega^J_{ft}, \tag{D.14}$$

with an analogous condition for K_{fjt}. Equating the conditions for products j and j',

$$L_{fjt} = \frac{\gamma^L_j \sum_d R^d_{fjt}}{\sum_{j'} \gamma^L_{j'} \sum_d R^d_{fj't}} L_{ft}, \tag{D.15}$$

and analogously for K_{fjt}.

Factor-share equation. The first-stage factor regression is estimated on all firms in this extension to multi-product firms since both the u shocks and ρ are homogeneous across destinations and products. Taking logarithms of (D.10) yields

$$\ln s^M_{ft} = \ln\left(\sum_{j\in\Omega^J_{ft}} \mathscr{E}_u \beta^M_j \frac{R_{fjt}}{R_{ft}}\right) - \rho u_{ft}, \tag{D.16}$$

with $\beta^M_j \equiv \rho\gamma^M_j$. The residual ρu_{ft} is orthogonal to $\{R_{fjt}/R_{ft}\}$, hence the composite parameters $\mathscr{E}_u \beta^M_j, \quad j = 1, ...J$, are identified and can be estimated by NLLS on all firms, following equation (14) of the main text. $\mathscr{E}_u$ is then recovered by averaging $\exp(\widehat{\rho u}_{ft})$ over the sample. Identification exploits cross-firm variation in the product revenue mix: firms with different product compositions trace out the relative materials intensities $\{\beta^M_j\}$.

Revenue equation. Because u_{ft} enters through the production function rather than demand, the realized domestic output share equals the domestic revenue share exactly: $\chi^d_{fjt} = \frac{R^d_{fjt}}{R_{fjt}}$ for all d. Substituting it into the demand equation for the domestic market gives, for all firms including exporters:

$$R^0_{fjt} = \left(\frac{R^0_{fjt}}{R_{fjt}} Q_{fjt}\right)^\rho D^0_{jt} \exp\left(\varepsilon^0_{fjt}\right), \tag{D.17}$$

the product-level analog of equation (12) in the main text. Taking logarithms and substituting the production function (D.5) yields

$$r^0_{fjt} = \rho\gamma^M_j m_{fjt} + \rho\gamma^L_j l_{fjt} + \rho\gamma^K_j k_{fjt} + \rho \ln\left(\frac{R^0_{fjt}}{R_{fjt}}\right) + \ln D^0_{jt} + \varepsilon^0_{fjt} + \rho\omega_{fjt} + \rho u_{ft}. \tag{D.18}$$

The ratio R^0_{fjt}/R_{fjt} encodes foreign demand conditions without requiring observation of destination-specific prices or knowledge of the full export strategy: a firm facing stronger foreign demand ships a larger output share abroad, lowering R^0_{fjt}/R_{fjt} and, through the coefficient ρ, reducing domestic revenues.

The FONC for materials gives product-level materials for all firms:

$$\widehat{M}_{fjt} = \frac{\widehat{\mathscr{E}_u \beta^M_j}\exp(-\widehat{\rho u}_{ft})R_{fjt}}{W^M_t}, \tag{D.19}$$

where $\widehat{\rho u_{ft}}$ is the first-stage residual, which allows to compute $\widehat{\mathscr{E}}_u$. Define the net domestic log-revenue

$$\tilde{r}^0_{fjt} \equiv r^0_{fjt} - \widehat{\beta^M_j}\hat{m}_{fjt}, \tag{D.20}$$

where $\hat{m}_{fjt} = \ln \hat{M}_{fjt}$. Absorbing $\ln D^0_{jt}$ into a product-time fixed effect α_{jt} and collecting $\nu_{fjt} \equiv \varepsilon^0_{fjt} + \rho\omega_{fjt} + \rho u_{ft}$, equation (D.18) becomes

$$\tilde{r}^0_{fjt} = b^L_j l_{fjt} + b^K_j k_{fjt} + \rho \ln\left(\frac{R^0_{fjt}}{R_{fjt}}\right) + \alpha_{jt} + \nu_{fjt}, \tag{D.21}$$

the product-level analog of equation (19) in the main text. The unknowns are the demand curvature ρ and $\{b^L_j, b^K_j\}_{j\in\Omega^J_{ft}}$. The quasi-fixed input allocations l_{fjt} and k_{fjt} depend on $\{b^L_j, b^K_j\}$ through (D.15) and are therefore estimated jointly with the production function parameters in the GMM.

Following the main text, assume ω_{fjt} and ε^0_{fjt} each follow a first-order Markov process with common persistence h and allow productivity to depend on lagged export participation $e_{f,j,t-1}$:

$$\nu_{fjt} = h\,\nu_{fj,t-1} + \rho\mu\, e_{fj,t-1} + \rho\xi_{fjt}, \tag{D.22}$$

where μ is the learning-by-exporting coefficient and $\xi_{fjt} = \tilde{\omega}_{fjt} + \tilde{\varepsilon}^0_{fjt}/\rho + u_{ft} - hu_{jj,t-1}$ is MA(1).

For any candidate $\theta = (h, \mu, \{b^L_j, b^K_j\}_j)$, compute $\hat{\nu}_{fjt}(\theta)$ from (D.21), form residuals $\hat{\xi}_{fjt}(\theta)$ by regressing $\hat{\nu}_{fjt}$ on $\hat{\nu}_{fj,t-1}$, $e_{fj,t-1}$, and product-time fixed effects, and minimise deviations from the moment conditions

$$E\left[\hat{\xi}_{fjt}(\theta)\cdot\left(l_{ft},\, k_{ft},\, \ln\frac{R^0_{fj,t-2}}{R_{fj,t-2}}\right)_{j\in\Omega^J_{ft}}\right] = 0 \tag{D.23}$$

by GMM, where l_{ft} and k_{ft} are total firm-level log-labor and log-capital. The lagged domestic revenue share $\ln(R^0_{fj,t-2}/R_{fj,t-2})$ instruments for the contemporaneous demand proxy $\ln(R^0_{fjt}/R_{fjt})$, paralleling the use of $\ln[R^0_{f,t-2}/(R_{f,t-2}\hat{Z}_{f,t-2})]$ in equation (23) of the main text. Validity follows from the MA(1) structure of ξ_{fjt}: period-t innovations are uncorrelated with instruments dated $t-2$ and earlier.

E Existing Estimators

In this section we present existing approaches to estimating production functions when output is denominated in value and discuss the biases that arise from ignoring the multi-destination dimension.

E.1 Factor share method with no demand correction

When estimating production function parameters with data denominated in value, researchers typically deflate firm-level revenues by the domestic price deflator and treat the resulting series as if they were quantities. We present this estimator in Section 4 of the main text.

If the true data generating process (DGP) coincides with the model from Section 2, but the conditions ensuring that firms sell at the same price are not met, then the moment conditions for the "no demand correction" estimator fail at two junctures. First, if firms serve multiple destination markets with heterogeneous demand curvatures (ρ^ds), and all firm-year observations are included in the first-stage NLLS, then correlation between Z_{ft} and inputs leads to a violation of the moment condition. Our procedure circumvents this problem by conditioning on non-exporters in the first stage.

Second, even if the contribution of materials could be accurately estimated and subtracted off from deflated revenues, the endogenous choice of destination-specific quantity shares leads to a violation the moment conditions in the second step GMM. This can be seen from (11), where, after moving the contribution of materials to the left-hand side, both χ^0_{ft} and ψ^x_{ft} directly affect revenues, and are included in the residual ξ^{NoD}_{ft} in (20). Since destination-specific output shares depend on quasi-fixed input levels, failure to control for χ^0_{ft} and ψ^x_{ft} implies a violation of the second step moment conditions.[1]

The violation causes biases in ways that are hard to assess and likely depend on parameter values. For example, χ^0_{ft} depends negatively on quasi-fixed inputs, since higher quasi-fixed input levels lead to lower marginal costs, higher marginal revenues, higher likelihood of exporting to any given destination, and hence higher export share. But it is not clear whether leaving χ^0_{ft} for the error term leads to upward or downward bias in the b coefficients, because the residual in (11) may depend either positively or negatively on χ^0_{ft}.[2]

E.2 Factor share method with a single-market correction

The few papers that explicitly address the value-versus-quantity distinction in the context of production function estimation typically implement some versions of Klette & Griliches (1996)'s

[1]In addition, χ^0_{ft} cannot follow the same AR(1) as ε^0_{ft} since χ^0_{ft} depends directly on ε^0_{ft}. Thus, χ^0_{ft} cannot be absorbed into ν_{ft}.

[2]If there is only one quasi-fixed input that enters linearly in (28), and the residual depends positively on χ^0_{ft}, then the bias is clearly negative. With multiple quasi-fixed inputs and higher order terms and interactions, it is not clear that omitting χ^0_{ft} leads to downward bias in all estimated b terms.

procedure that controls for the domestic aggregate CES quantity index (e.g., De Loecker 2011 and Grieco et al. 2016). We present this estimator in Section 4 of the main text.

This additional regressor, B_t^{proxy}, captures the aggregate domestic demand conditions. If firms serve only one destination market—the domestic one—and if researchers could compute the empirical price index and B_t^{proxy} in a theory-consistent way (see Appendix E.7 for details), then we would have $B_t^{proxy} = B_t/\Upsilon_0$, where Υ_0 captures the price index normalization. In this case, the constructed residual $\widehat{\xi}_{ft}^{SMC}$ would be orthogonal to quasi-fixed inputs in period t because $\xi_{ft}^{SMC} \equiv \widetilde{\varepsilon}_{ft} + \rho\,\widetilde{\omega}_{ft}$ at the true parameter values. Indeed, when all firms serve a single market, the *ex post* demand shock u_{ft} is identified by the factor share regression in the first step, and hence does not appear in the second step. Moreover, the aggregate demand shifter $\ln B_t^{proxy}$ is orthogonal to $\widehat{\xi}_{ft}^{SMC}$ by assumption. Hence, the parameter ρ would be identified by time series variation in industry-wide demand aggregates, and this estimation procedure would also identify all output elasticities.

However, when firms select endogenously into multiple destination markets, several sources of bias arise. First, unless χ_{ft}^0 follows exactly the same AR(1) process as ε_{ft}^0 and ω_{ft}—which is not possible, given the model—then $\widehat{\xi}_{ft}^{SMC}$ includes χ_{ft}^0. Since χ_{ft}^0 depends on quasi-fixed inputs, this implies a violation of the moment conditions, which results in a biased estimator.

Second, the demand shifter B_t^{proxy} is likely measured with error. Gandhi et al. (2020) cite De Loecker (2011) for how to construct B_t^{proxy} from the data as a weighted sum of deflated *total* revenues of domestic firms. We show in Appendix E.7 that if the price deflator is constructed in a theory-consistent way, then the domestic quantity index B_t^0 can be computed up to a normalization from price deflators and *total domestic absorption*, i.e. total domestic sales of domestic firms plus total imports from foreign firms. Incorrectly building B_t^{proxy} from total revenues of domestic firms (whether using weights or not) results in a discrepancy with B_t^0, that corresponds to the trade deficit, which would be relegated to the error term, multiplied by $1-\rho^0$. The trade deficit may be positively or negatively correlated with B_t^{proxy}, depending on whether local demand shocks or foreign supply shocks dominate. This, too, leads to a violation of the moment conditions and a biased estimator.

In light of these concerns, a tempting strategy would be to estimate the factor shares approach with a single-market correction for a set of non-exporters. For non-exporters, the first stage factor share regression identifies the revenue elasticity of flexible inputs as well as u_{ft}^0, the residual from this regression. Additionally, for all non-exporting firms, $\chi_{ft}^0 = 1$ and $Z_{ft} = \exp(-u_{ft}^0)$. In this case, equation (21) becomes

$$\begin{aligned} \widetilde{r}_{ft}^{SMC} &= b_l^{SMC} l_{ft} + b_k^{SMC} k_{ft} + b_{ll}^{SMC} l_{ft} l_{ft} + b_{kk}^{SMC} k_{ft} k_{ft} + b_{lk}^{SMC} l_{ft} k_{ft} \\ &+ \left(1-\rho^0\right) \ln B_t^{proxy} + \varepsilon_{ft}^0 + \rho^0 \omega_{ft}, \end{aligned} \tag{E.1}$$

where $\varepsilon_{ft}^0 + \rho^0 \omega_{ft}$ combine as an error term that follows an AR(1) process. However, bias persists for two reasons. First, B_t^{proxy} is still likely measured with error. Second, sample selection bias violates the orthogonality conditions in the second step: the residual from the AR(1) process does not have a zero mean conditional on quasi-fixed inputs. If higher levels of quasi-fixed inputs are associated with a greater probability to export—for instance due to increasing returns to scale—then the conditional mean of the residual will be negatively correlated with them because the sample never admits exporters. The direction of the bias will vary by type of cross-market complementarities and by the degree of returns to variable inputs.

Whether estimating the single-market demand-side correction model in the full sample or in a sub-sample of non-exporters, two additional problems arise. First, identification in this model relies on time-series variation in aggregate demand, which may not be sufficient in short panels. Second, given our application—the productivity effects of learning by exporting—using a method with a single-market correction entails an additional conceptual issue: there is no exporting in a single market model. There is a logical inconsistency in relying on an estimator that assumes only one, domestic destination to study the effect of serving different, foreign markets.

E.3 Control function versions of the standard approaches

The two preceding estimation models could also be estimated by the control function method, wherein unobserved productivity shocks are proxied by a model-inversion procedure. There are many ways to implement the control function approach. We follow the gross output control function estimator from Gandhi et al. (2020). Using B_t^{proxy} to identify the demand curvature parameter yields an estimator that is very similar to the homogeneous-ρ version of De Loecker (2011) or to the original demand-side correction model from Klette & Griliches (1996).

To fix ideas, we first invert the material demand function to substitute for the unobserved shocks to supply and demand. This substitution generates the estimation equation

$$r_{ft}^{CF} = \Phi(m_{ft}, l_{ft}, k_{ft}) + \delta_t + \vartheta_{ft}^{CF} \tag{E.2}$$

where $\Phi(m_{ft}, l_{ft}, k_{ft})$ is an unknown function that combines $f(\cdot)$ and the inverted material demand function and δ_t is a time fixed effect that controls for material input prices. When controlling for a single-market demand shifter, we simply include B_t^{proxy} in $\Phi(\cdot)$. We approximate $\Phi(\cdot)$ with a second-degree polynomial function, estimate by OLS, and extract the residual $\widehat{\vartheta}_{ft}^{CF}$.

Next, we subtract $\widehat{\vartheta}_{ft}^{CF}$ from deflated revenues. In the case that the demand-side correction is ignored we obtain

$$\widetilde{r}_{ft}^{CF-NoD} = f(m_{ft}, l_{ft}, k_{ft}) + \nu_{ft}. \tag{E.3}$$

For the case in which a single-market demand correction is employed we obtain

$$\widetilde{r}_{ft}^{CF-SMC} = \rho^0 f(m_{ft}, l_{ft}, k_{ft}) + \left(1-\rho^0\right) \ln B_t^{proxy} + \nu_{ft}, \tag{E.4}$$

where ν_{ft} collects unobserved shocks to supply and demand. We adopt a complete polynomial function of degree 2 to approximate $f(\cdot)$ and assume that ν_{ft} follows an AR(1) process. For any candidate vector of parameters, we can compute $\widehat{\nu}_{ft}$, then regress $\widehat{\nu}_{ft}$ on $\widehat{\nu}_{f,t-1}$ and $e_{f,t-1}$ (the indicator for exporting) and compute the residual $\widehat{\xi}_{ft}^{CF}$. We build moment conditions by multiplying $\widehat{\xi}_{ft}^{CF}$ by the levels of all quasi-fixed inputs, lags of flexible inputs, along with the appropriate interaction and square terms, as well as $\ln B_t^{proxy}$ in the case of the single-market correction model. We then minimize the objective function to estimate parameters.

Beyond the model misspecification issues mentioned above, there are two problems highlighted by the literature that suggest that neither of these control function estimators are likely to generate consistent estimates of model parameters. First, the control function method relies on time series variation in material input prices for identification. Gandhi et al. (2020) show that the control function approach generates inconsistent estimates when material price variation is low, even if the moment conditions hold. This is because low material price variation renders its lag to be a weak instrument for contemporaneous materials demand, conditional on productivity and capital. We replicate this finding in Appendix F.2 in our Monte Carlo simulations (extending the Monte Carlo experiments from Gandhi et al. (2020) to the case of imperfect competition) for a single-market version of the model.

Second, even when the sample size goes to infinity, the GMM objective function admits multiple solutions in the standard control function framework, as demonstrated by Ackerberg et al. (2023). Ackerberg et al. (2023) argue that choosing among these candidate solutions is not as simple as just choosing the parameter combination that yields the lowest objective function value. This is because there are, in fact, multiple parameter vectors for which the moment conditions are satisfied and the objective equals zero—among them the OLS parameter vector. Hence, the optimization problem is under identified.[3] Despite these issues, we estimate control function versions of the no-demand-correction and the single-market correction estimators both in Monte Carlo experiments and in the French data, for comparison.

E.4 Multi-destination estimator with constant returns to flexible inputs

Researchers in the international trade literature often estimate the elasticity of demand from the ratio of revenues to total variable costs (Aw et al. 2011, Almunia et al. 2021). We can derive the estimating equations for this strategy from our general model by imposing particular parametric restrictions.

To illustrate, we consider the estimation strategy from Aw et al. (2011). Starting from the model in section 2, if we impose (i) the same demand curvature for all foreign destinations: $\rho^d = \rho^x$ for all $d = 1, \dots \mathscr{D}$, (ii) a Cobb-Douglas production function in materials and capital, (iii) constant returns to materials $\sigma_{ft}^M = 1$, and (iv) the absence of *ex post* demand shocks: $u_{ft}^d = 0$ for all d, then

[3]It is well known that nonlinear estimation like GMM can be sensitive to initial values as well as searching algorithms (Knittel et al., 2014). As shown by Ackerberg et al. (2023), the problem with the control function method is more severe than mere numerical challenges.

the factor-share equation (8) simplifies to

$$W_t^M M_{ft} \quad = \quad \rho^0 R_{ft}^0 + \rho^x R_{ft}^x, \tag{E.5}$$

with $R_{ft}^x \equiv \sum_{d \in \Omega_{ft}} R_{ft}^d$. Adding a regression error term, equation (E.5) can be estimated by OLS to recover ρ^0 and ρ^x, which is precisely what Aw et al. (2011) do.

Next, Aw et al. (2011) estimate returns to capital and LBE. To do so, they derive an expression for R_{ft}^0 which depends on just K_{ft}, ω_{ft}, W_t^M, and the domestic aggregate demand shifter D_t^0. The key to identification is the assumption of constant returns to materials, under which materials drops out of the expression. To derive this equation from our general model, we impose no *ex post* shocks and $\rho^d = \rho^x$ for all $d > 1$ and combine (10) with (8) to obtain

$$W_t^M M_{ft} \quad = \quad \sigma_{ft}^M \rho^0 \left(Q_{ft}\right)^{\rho^0} \left(\chi_{ft}^0\right)^{\rho^0} D_t^0 \exp\left(\varepsilon_{ft}^0\right) \left[1 + \frac{1-\chi_{ft}^0}{\chi_{ft}^0}\right], \tag{E.6}$$

which implies $\chi_{ft}^0 = \rho^0 R_{ft}^0 / (W_t^M M_{ft})$ given the assumption $\sigma_{ft}^M = 1$. Next, we substitute this expression into the definition of domestic revenues using the Cobb-Douglas assumption with $\sigma_{ft}^M = \gamma_{ft}^M = 1$:

$$R_{ft}^0 \quad = \quad \left(M_{ft} K_{ft}^{\gamma^K} \exp\left(\omega_{ft}\right)\right)^{\rho^0} \left(\frac{\rho^0 R_{ft}^0}{M_{ft} W_t^M}\right)^{\rho^0} D_t^0 \exp\left(\varepsilon_{ft}^0\right), \tag{E.7}$$

where γ^j indicates the output elasticity of factor $j \in \{M, K\}$, and where materials M_{ft}—present both in Q_{ft} and in χ_{ft}^0—cancel out. Rearranging yields

$$R_{ft}^0 \quad = \quad \left(K_{ft}\right)^{\frac{\gamma^K \rho^0}{1-\rho^0}} \left(W_t^M\right)^{\frac{-\rho^0}{1-\rho^0}} \left(\rho^0\right)^{\frac{\rho^0}{1-\rho^0}} \left(\exp\left(\omega_{ft}\right)\right)^{\frac{\rho^0}{1-\rho^0}} \left(D_t^0 \exp\left(\varepsilon_{ft}^0\right)\right)^{\frac{1}{1-\rho^0}}, \tag{E.8}$$

which can be log-linearized. Aw et al. (2011) proceed to identify γ^K and LBE via GMM from this equation, exploiting the AR(1) assumption in the usual way.

This illustrates that the estimation strategy from Aw et al. (2011) (i.e., estimating (E.5) and then (E.8)) is a special case of our multi-destination estimator, subject to restrictions (i)-(iv) just above. If these restrictions do not hold, we would not expect this procedure to deliver consistent estimates of model parameters, but we implement it in Monte Carlo experiments and in the French data, for comparison.

E.5 Estimator with multiple aggregate quantity indices

De Loecker (2011) provides an alternative extension of Klette & Griliches (1996)'s model and (similarly to us) proposes an estimation strategy for the case in which outputs are observed only in value. When firms serve multiple markets, the estimation equation from De Loecker (2011) only holds as a first-order approximation. If higher order terms of this approximation are important in this nonlinear setting, then the values estimated by this procedure may be far from the truth, even if the data generating process coincides with the structural model.

To see this point we re-write the relevant equations from De Loecker (2011) in our notation.

We start from our general multi-destination model in Section 2, and impose (i) $\varepsilon_{ft}^d = \varepsilon_{ft}$ for all $d \in \{0 \cup \Omega_{ft}\}$, (ii) $u_{ft}^d = u_{ft}$ for all $d \in \{0 \cup \Omega_{ft}\}$, and (iii) a Cobb-Douglas structure for $F(\cdot)$, as in De Loecker (2011). Total revenues can then be written as

$$R_{ft} = \sum_{d \in \{0\} \cup \Omega_{ft}} \left(\exp(\omega_{ft}) M_{ft}^{\gamma^M} L_{ft}^{\gamma^L} K_{ft}^{\gamma^K} \right)^{\rho^d} (\chi_{ft}^d)^{\rho^d} D_t^d \exp(\varepsilon_{ft}) \exp(u_{ft}), \tag{E.9}$$

where γ^j indicates the output elasticity of factor $j \in \{M, L, K\}$.

De Loecker (2011) estimates the following equation (corresponding to his equation (8), written in our notation) via the control function approach:

$$\widetilde{r}_{ft} = \beta^M m_{ft} + \beta^L l_{ft} + \beta^K k_{ft} + \beta_N N_f + \sum_{d \in \{0\} \cup \Omega_{ft}} s_f^d \left(1 - \rho^d\right) \ln B_t^d + \omega_{ft} + \varepsilon_{ft} + u_{ft}, \tag{E.10}$$

where $\widetilde{r}_{ft}$ denotes deflated log revenues, β^j indicates the revenue elasticity of factor $j \in \{M, L, K\}$, and s_f^ds is a time-invariant measure of the importance of market d for firm f (e.g., base-year market shares). The term N_f denotes the number of products in the context of De Loecker (2011), but could represent the number of destinations in our multi-destination setting.

Careful examination shows that (E.10) does not follow from (E.9). With heterogeneous values of ρ^d, inputs do not pull out of the sum in (E.9), nor does ω_{ft}. Hence, the estimation equation in De Loecker (2011) is, strictly speaking, inconsistent with our model, even after imposing the restrictions specified above.

Nevertheless, by taking a first-order Taylor series approximation of (E.9) around a base-year equilibrium, it is possible to recover an estimation equation that is very close to (E.10). De Loecker (2011) does not derive this approximation, nor does he describe this equation as an approximation; nevertheless, this equation can be motivated by an approximation, as we will now show.

Starting from the model in Section 2, we impose: i) $\varepsilon_{ft}^d = \varepsilon_{ft}$ for all $d \in \{0 \cup \Omega_{ft}\}$, (ii) $u_{ft}^d = u_{ft}$ for all $d \in \{0 \cup \Omega_{ft}\}$, and (iii) a Cobb-Douglas structure for $F(\cdot)$, as in De Loecker (2011). Then from (2) and (5) we have

$$r_{ft}^d \equiv \ln R_{ft}^d = \rho^d \omega_{ft} + \rho^d \gamma^M m_{ft} + \rho^d \gamma^L \ell_{ft} + \rho^d \gamma^K k_{ft} + \rho^d \ln \chi_{ft}^d + \ln D_t^d + \varepsilon_{ft} + u_{ft}, \tag{E.11}$$

Define $\bar{\Omega}_f$ the export strategy of firm f in some baseline year. Then by definition, we have

$$\begin{aligned} \ln R_{ft} &= \ln \left(\sum_{d \in \{0\} \cup \Omega_{ft}} \exp(r_{ft}^d) \right) \\ &= \ln \left(\sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \exp(r_{ft}^d) + \sum_{d \in \cup \Omega_{ft} \setminus \bar{\Omega}_f} \exp(r_{ft}^d) \right) \\ &= \underbrace{\ln \left(\sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \exp(r_{ft}^d) \right)}_{\equiv G(\mathbf{r_{ft}})} + \underbrace{\ln \left(1 + \frac{\sum_{d \in \left(\Omega_{ft} \setminus \bar{\Omega}_f\right)} \exp\left(r_{ft}^d\right)}{\sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \exp\left(r_{ft}^d\right)} \right)}_{\equiv E_{ft}} \end{aligned} \tag{E.12}$$

where $\mathbf{r_{ft}}$ is the vector $\{r^d_{ft}\}, d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)$, i.e. the destination-specific log revenues for the set of destinations common between the export strategies $\bar{\Omega}_f$ and Ω_{ft}. The term $G(\mathbf{r_{ft}})$ represents the component of log revenues for firm f in year t from destinations in this set. The term E_{ft} indicates the component of log revenues from destinations in Ω_{ft}, but not in $\bar{\Omega}_f$.

For a fixed export strategy, $\Omega_{ft} = \bar{\Omega}_f$ for all f and t, and $E_{ft} = 0$ for all f and t. In this case, a first-order Taylor approximation of $G(\mathbf{r_{ft}})$ around the baseline $\bar{\mathbf{r}}_\mathbf{f}$ will generate the estimation equation. If the export strategy varies from the baseline, then E_{ft} introduces potentially higher-order bias. Hence, the approximation still holds, but not necessarily to a first order.

Taking a first-order Taylor approximation of $G(\mathbf{r_{ft}})$ around the baseline $\bar{\mathbf{r}}_\mathbf{f}$ we have

$$\begin{aligned} G(\mathbf{r_{ft}}) &= G(\bar{\mathbf{r}}_\mathbf{f}) + \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \frac{\partial G}{\partial \vartheta^d_{ft}}\Big|_{\vartheta^d_{ft} = \bar{r}^d_f} \left(r^d_{ft} - \bar{r}^d_f\right) \\ &= G(\bar{\mathbf{r}}_\mathbf{f}) + \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \bar{s}^d_f \left(r^d_{ft} - \bar{r}^d_f\right) \\ &= \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \bar{s}^d_f r^d_{ft} + \underbrace{G(\bar{\mathbf{r}}_\mathbf{f}) - \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \bar{s}^d_f \bar{r}^d_f}_{\equiv Cons_f} \end{aligned} \quad \text{(E.13)}$$

where $\bar{s}^d_f \equiv \frac{\bar{R}^d_f}{\sum_{z \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \bar{R}^z_f}$ is the revenues share of destination d in the common set $\{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)$, and $Cons_f$ is a firm-specific constant that does not depend on t.

Hence, to an approximation,

$$\ln R_{ft} \approx \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} \bar{s}^d_f r^d_{ft} + Cons_f + E_{ft} \quad \text{(E.14)}$$

A natural choice for baseline $\bar{\mathbf{r}}_\mathbf{f}$ would be initial period outcomes $\bar{\mathbf{r}}_\mathbf{f} = \mathbf{r_{f0}}$. Substituting in (E.11) and deflating the log of total revenues by the weighted average of price indices in each destination, we have

$$\begin{aligned} \ln R_{ft} - \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \ln \Gamma^d_t &= \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d \gamma^M m_{ft} + \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d \gamma^L \ell_{ft} \\ &+ \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d \gamma^K k_{ft} \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} (1 - \rho^d) \ln B^d_t \\ &+ \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d \ln \chi^d_{ft} - \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d \ln \tau^d_t \\ &+ Cons_f + E_{ft} + \left(\sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d \right) \omega_{ft} + \varepsilon_{ft} + u_{ft}. \end{aligned}$$

If we denote $\beta^j \equiv \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d \gamma^j$ and $\beta^\omega \equiv \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s^d_{f0} \rho^d$, this expression simplifies to

$$\begin{aligned}\widetilde{r}_{ft} &= \beta^M m_{ft} + \beta^L l_{ft} + \beta^K k_{ft} + \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s_{f0}^d \left(1 - \rho^d\right) \ln B_t^d + \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s_{f0}^d \rho^d \ln \chi_{ft}^d \\ &- \sum_{d \in \{0\} \cup \left(\bar{\Omega}_f \cap \Omega_{ft}\right)} s_{f0}^d \rho^d \ln \tau_t^d + Cons_f + E_{ft} + \beta^\omega \omega_{ft} + \varepsilon_{ft} + u_{ft}. \end{aligned} \tag{E.15}$$

Comparing (E.15) with (E.10) in the text, we find several difficulties and discrepancies.

First, deflating revenues by the weighted average of price indices in each destination requires a lot of information. The isomorphism between a multi-segment model and a multi-destination model does not imply similar data requirements. In practice, De Loecker (2011) uses data on Belgium prices only, which ignores the fact that Belgian firms export.

Second, there are 4 terms in (E.15) that are missing in (E.10)— the terms associated with χ_{ft}^ds and τ_t^d, and the terms related to the approximation ($Cons_f$ and E_{ft}). For the first term, De Loecker (2011) assumes a fixed product mix over time within firms to proxy for revenues shares with the number of products N_f. In our setting, this proxy would be equivalent to the number of destinations. But most firms change their set of destinations over time. Assuming all destination-specific revenues shares s_{ft}^ds are observed, we could use that information to proxy for χ_{ft}^ds, or we could proxy it by the maximum number of destinations. The second term that contains trade costs is specific to our setting. The constant is not time-varying, but it is firm specific, hence it could be absorbed by firm fixed effects in the Markov process (which could introduce Nickell bias). But these fixed effects are not present in (E.10), thereby relegating this constant to the error term. As a result, the baseline period export share could generate an omitted variable problem in the specification proposed by De Loecker (2011), and the moment conditions may not hold. Finally, our derivation indicates that the extensive margin for destinations (E_{ft}) should be controlled for.

Note that the first-order approximation holds for a fixed set of destinations over time. If the export strategy Ω_{ft} changes over time, then the equation holds as an approximation, though not necessarily a "first-order" approximation. In this case, nontrivial movements in the quantity shares (χ_{ft}^d) over time could introduce bias. Furthermore, identification of the parameter ρ^d is complicated by all the measurement issues for B_t^d discussed above, as well as the concerns regarding short panels. By exploiting a novel source of identifying variation—the export revenues share—we circumvent both the approximation bias and the measurement issues related to B_t^d.

E.6 Comparison to Almunia et al. (2021)

In recent work, Almunia et al. (2021) also specify a model with multiple destinations, monopolistic competition, and quasi-fixed capital. They also estimate production function parameters and an elasticity of demand. There are four key differences between the production function estimation in Almunia et al. (2021) and ours. First, Almunia et al. (2021) assume that firms are myopic with respect to *ex post* destination specific demand shocks ($E\left[e^{u_{ft}}\right] = 1$). Second, as we explain below, Almunia et al. (2021) implicitly assume constant returns to flexible inputs when estimating the elasticity of demand, which is inconsistent with the thrust of their main findings. Third, it seems that Almunia et al. (2021) do not consider the multi-destination dimension when estimating firm-level productivity (see Appendix F in their paper). Finally, Almunia et al. (2021) assume that the elasticity of demand is identical across all destinations (within an industry), whereas one of our contributions is to allow for variation in elasticities across markets.

In what follows, we analyze their estimator in the context of a version of our model where we impose the same elasticity of demand across destinations, as they do. We demonstrate that their estimator is not consistent with variable (increasing or decreasing) returns to variable inputs.

Almunia et al. (2021) claim (see Appendix F.1 of Almunia et al. (2021)) that the assumption of monopolistic competition implies (in their notation) that

$$R_{it} - C_{it}^{v} = \frac{1}{\sigma} R_{it}, \tag{E.16}$$

where R_{it} is total revenues of firm i at time t, C_{it}^{v} denotes the total variable cost of firm i at time t, and σ is the elasticity of substitution (and demand). Equivalently, we can define in our notation the total variable cost of firm f in time t,

$$Cost_{ft}\left(Q_{ft}\right) = M_{ft} W_{t}^{M}, \tag{E.17}$$

We can then re-write (E.16) in our notation

$$R_{ft} - Cost_{ft}\left(Q_{ft}\right) = (1-\rho) R_{ft}. \tag{E.18}$$

Based on assumption (E.16), and substituting for C_{t}^{v} with expenditures on flexible inputs, Almunia et al. (2021) derive the following moment condition (in their notation)

$$E\left[\ln\left(\frac{\sigma-1}{\sigma}\right) + r_{it}^{obs} - \ln\left(P_{it}^{M} M_{it} + w_{it} L_{it}\right)\right] = 0, \tag{E.19}$$

or in our notation

$$E\left[\ln\rho + r_{ft} - \ln\left(W_{t}^{M} M_{ft}\right)\right] = 0. \tag{E.20}$$

If the assumption in (E.16) were to hold, then indeed the moment condition (E.20) could be exploited to identify the curvature of demand (ρ, in our notation). But as we show below, (E.16) requires both (in our notation) $E\left[e^{u_{ft}}\right] = 1$ and constant marginal costs. This implies that firms are myopic with respect to *ex post* destination specific demand shocks, and that variable returns to scale are unitary.

We can rewrite the optimization problem of a firm from section 2 for a fixed set of destinations (fixed "export strategy") using the variable cost function (E.17):

$$\max_{\boldsymbol{\chi}_{ft}, Q_{ft}} \quad \mathscr{L} \quad = \quad E\left[Q_{ft}^{\rho} \sum_{d\in\Omega_{ft}} \left(\chi_{ft}^{d}\right)^{\rho} D_{t}^{d} \exp\left(\varepsilon_{ft}^{d} + u_{ft}^{d}\right)\right] - Cost_{ft}\left(Q_{ft}\right) + \lambda_{ft}\left(1 - \sum_{d\in\Omega_{ft}} \chi_{ft}^{d}\right) \tag{E.21}$$

which leads to first order condition for Q_{ft}

$$\rho\left(Q_{ft}\right)^{\rho-1}\left[\sum_{d\in\Omega_{ft}}\left(D_t^d\exp\left(\varepsilon_{ft}^d\right)\right)^{\frac{1}{1-\rho}}\right]^{1-\rho}E\left[\exp\left(u\right)\right]=\frac{\partial Cost_{ft}\left(Q_{ft}\right)}{\partial Q_{ft}},\tag{E.22}$$

Multiplying both sides by Q_{ft}, we have

$$\rho\left(Q_{ft}\right)^{\rho}\left[\sum_{d\in\Omega_{ft}}\left(D_t^d\exp\left(\varepsilon_{ft}^d\right)\right)^{\frac{1}{1-\rho}}\right]^{1-\rho}E\left[\exp\left(u\right)\right]=\frac{\partial Cost_{ft}\left(Q_{ft}\right)}{\partial Q_{ft}}Q_{ft},\tag{E.23}$$

and substituting with total revenues

$$\rho E\left[\exp\left(u\right)\right]R_{ft}\psi_{ft}^{-1}=\frac{\partial Cost_{ft}\left(Q_{ft}\right)}{\partial Q_{ft}}Q_{ft},\tag{E.24}$$

Now if we set $E\left[\exp\left(u\right)\right]=1$ and we assume $\frac{\partial Cost_{ft}\left(Q_{ft}\right)}{\partial Q_{ft}}Q_{ft}=Cost_{ft}\left(Q_{ft}\right)$, we get the moment condition (E.20). But with non-constant marginal cost, $\frac{\partial Cost_{ft}\left(Q_{ft}\right)}{\partial Q_{ft}}Q_{ft}\neq Cost_{ft}\left(Q_{ft}\right)$. In particular, with decreasing returns to flexible inputs, which is the necessary condition for cross-market complementarities, $\frac{\partial Cost_{ft}\left(Q_{ft}\right)}{\partial Q_{ft}}Q_{ft}\neq Cost_{ft}\left(Q_{ft}\right)$.

E.7 Building market quantity proxy from price indices

In a single-market estimation model, the demand-side parameter is identified from time series variation in the industry-wide CES demand index. In this section, we discuss how to construct this index.

Essentially, the quantity index can be recovered from expenditure data and industry-wide price deflators. Assuming just a single market (hence dropping the d superscript), and using (1) and (2), we can write

$$B_t^{\rho} = \sum_{f\in\Theta_t}\exp\left(\varepsilon_{ft}+u_{ft}\right)X_{ft}^{\rho}=\sum_{f\in\Theta_t}\frac{R_{ft}}{\Upsilon_t}B_t^{\rho-1}\tag{E.25}$$

This implies

$$B_t = \sum_{f\in\Theta_t}\frac{R_{ft}}{\Upsilon_t}\tag{E.26}$$

Hence, if we observe the true CES price index in levels, we can construct the CES quantity index from aggregate deflated revenues.[4]

But of course, the true CES price index is not observed in levels. First, price indices are almost always reported relative to some base year normalization. This implies

$$B_t=\sum_{f\in\Theta_t}\frac{R_{ft}}{\Upsilon_t}=\Upsilon_0\underbrace{\sum_{f\in\Theta_t}\frac{R_{ft}}{\Lambda_t}}_{\equiv B_t^{proxy}}\tag{E.27}$$

[4]In the only work we are aware of that explains how to construct the CES quantity index, De Loecker (2011) computes the weighted average of deflated revenues (see De Loecker (2011) equation B.1.9 in the appendix), though – as we show in (E.26) – theory indicates the gross sum is called for.

where Λ_t is the empirical analogue to the true CES price index normalized to base-year $t=0$, and Υ_0 is the unobserved base-year normalization.

Second, the CES price index is a theoretical construct that depends on structural parameters. How does this theoretical object correspond to Λ_t? Sato (1976) and Vartia (1976) prove for a symmetric CES with no entry and exit, there exists a set of weights wt_{ft} such that

$$\ln \frac{\Upsilon_t}{\Upsilon_0} = \sum_{f \in \Theta_t} wt_{ft} \ln \left(\frac{p_{ft}}{p_{fi0}} \right) \tag{E.28}$$

I.e., the log change in the true CES price index is a weighted average of the log change in the prices of individual firms. Sato (1976) and Vartia (1976) give the analytical expression for these weights, which ends up being very close to a simple chain weight. Feenstra (1994) extends to the case of entry and exit. Redding & Weinstein (2020) extends to the asymmetric CES (which corresponds to our demand system (1)). If we assume that Λ_t is computed using Weinstein-Redding weights, then (E.27) holds.

F Monte Carlo Simulations

In this appendix, we present details of the Monte Carlo experiments not reported in the main text.

F.1 Multi-Market Monte Carlo Simulations

These Monte Carlo simulations are based on the multi-destination model developed in Section 2. We simulate a panel of firms that produce differentiated products, may export to multiple destination markets, and compete imperfectly under potentially non-constant returns to variable inputs. The simulations are conducted in partial equilibrium on the foreign markets: firms take destination-level demand shifters and aggregate price indices as given when solving their optimization problem. However, we impose market-clearing conditions on the domestic market. In particular, the domestic CES quantity index is determined endogenously from the simulated equilibrium allocation across domestic firms. This structure mirrors the empirical setting considered in the paper, in which researchers observe firms from a single origin country that compete domestically while treating foreign market conditions as exogenous.

F.1.1 Building the cost function

On each market, expenditures on outputs from a particular firm are given by (2), with

$$B_t^d \equiv \left[\sum_f \left(Q_{ft}^d\right)^{\rho^d} \exp(\varepsilon_{ft}^d + u_{ft}^d)\right]^{1/\rho^d} \tag{F.1}$$

$$\Upsilon_t^d \equiv \left[\sum_f \left(P_{ft}^d\right)^{\frac{\rho^d}{\rho^d-1}} \exp\left(\frac{\varepsilon_{ft}^d + u_{ft}^d}{1-\rho^d}\right)\right]^{\frac{\rho^d-1}{\rho^d}}, \tag{F.2}$$

We allow for two different values of ρ. We set $\rho^D \equiv \rho^0 = 0.8$ and $\rho^X \equiv \rho^d = 0.6$ for all $d > 0$.

Total expenditures are equal to the quantity index times the price index

$$Y_t^d = B_t^d \Upsilon_t^d \tag{F.3}$$

which are assumed fixed.

Production is given by (33). With a single variable input, we can simply invert the production function to solve for M_{ft}

$$M_{ft} = \left(\frac{Q_{ft}}{\exp(\omega_{ft}) K_{ft}^{\gamma^K}}\right)^{1/\gamma^M} \tag{F.4}$$

which yields cost function

$$Cost_{ft} \equiv W_t^M M_{ft} = W_t^M \left(\frac{Q_{ft}}{\exp(\omega_{ft}) K_{ft}^{\gamma^K}}\right)^{1/\gamma^M} = c_{ft}\left(Q_{ft}\right)^{\theta+1} \tag{F.5}$$

with $\theta = \frac{1}{\gamma^M} - 1$ and $c_{ft} = W_t^M \exp(-(\theta+1)\omega_{ft}) K_{ft}^{-(\theta+1)\gamma^K}$

F.1.2 Assuming Monopolistic Competition on all markets

Firms choose the vector of $\{Q_{ft}^d\}$ to maximize expected profits, where expectations are taken over future realizations of the ex post demand shocks u_{ft}^d, conditional on firms' beliefs about the aggregate quantity indices. Assuming monopolistic competition, we have

$$\begin{aligned}
\max_{\{Q_{ft}^d\}} \Pi &= \mathbb{E}\Bigg[Y_t^0 \left(Q_{ft}^0\right)^{\rho^D} \left(B_t^0\right)^{-\rho^D} \exp(\varepsilon_{ft}^0 + u_{ft}^0) \\
&+ \sum_{d>0} Y_t^d \left(Q_{ft}^d\right)^{\rho^X} \left(B_t^d\right)^{-\rho^X} \exp(\varepsilon_{ft}^d + u_{ft}^d) \Bigg] - c_{ft}\left(Q_{ft}\right)^{\theta+1} \qquad \text{(F.6)}
\end{aligned}$$

FONC are

$$\rho^D Y_t^0 (Q_{ft}^0)^{\rho^D - 1} \mathbb{E}\left[(B_t^0)^{-\rho^D}\right] \exp(\varepsilon_{ft}^0) \mathbb{E}\left[\exp(u_{ft}^0)\right] = c_{ft}\,(\theta+1)\left(Q_{ft}\right)^{\theta} \qquad \text{(F.7)}$$

$$\rho^X Y_t^d (Q_{ft}^d)^{\rho^X - 1} \mathbb{E}\left[(B_t^d)^{-\rho^X}\right] \exp(\varepsilon_{ft}^d) \mathbb{E}\left[\exp(u_{ft}^d)\right] = c_{ft}\,(\theta+1)\left(Q_{ft}\right)^{\theta} \qquad \text{(F.8)}$$

Taking expectations over the ex post demand shocks is straightforward under the assumed log-normal distribution. But taking expectations over the quantity indices is not as clear.

Taking expectation over the ex post demand shocks

$$\exp(\sigma_u^2/2) \rho^X Y_t^d (Q_{ft}^d)^{\rho^X - 1} \mathbb{E}\left[(B_t^d)^{-\rho^X}\right] \exp(\varepsilon_{ft}^d) = c_{ft}(\theta+1)\left(Q_{ft}\right)^{\theta} \qquad \text{(F.9)}$$

$$\exp(\sigma_u^2/2) \rho^D Y_t^0 (Q_{ft}^0)^{\rho^D - 1} \mathbb{E}\left[(B_t^0)^{-\rho^D}\right] \exp(\varepsilon_{ft}^0) = c_{ft}(\theta+1)\left(Q_{ft}\right)^{\theta} \qquad \text{(F.10)}$$

Where we impose normal distribution for u shocks with variance σ_u^2.

Solving for export strategy To solve for the optimal export strategy, firms take the destination-specific CES quantity indices, B_t^d, as given in all markets when forming expectations over profits. Conditional on these perceived market conditions, each firm evaluates the profitability of all feasible export strategies, where an export strategy corresponds to a particular subset of foreign destinations to serve. Because the number of potential destinations is finite and relatively small in the simulations, firms can explicitly enumerate all feasible destination combinations.

For a given export strategy Ω_{ft}, firms solve the static profit maximization problem described in Section 2, conditional on the realization of productivity and ex ante demand shocks $(\omega_{ft}, \varepsilon_{ft}^d)$. At this stage, firms have not yet observed the ex post demand shocks u_{ft}^d, which are realized only after production and destination-specific quantity allocations are chosen. Consequently, firms compute expected profits by integrating over the distribution of future u_{ft}^d draws.

More formally, for each feasible export strategy Ω_{ft}, firms compute

$$E_u\left[\Pi_{ft}(\Omega_{ft})\right],$$

where expectations are taken over the distribution of ex post demand shocks conditional on the information available at the time production decisions are made. Firms then select the export strategy that yields the highest expected profits net of fixed export costs:

$$\Omega^*_{ft} = \arg\max_{\Omega_{ft}} E_u\left[\Pi_{ft}(\Omega_{ft})\right].$$

At this stage, firms treat the destination-level quantity indices B_t^d as exogenous objects that summarize market conditions. For foreign destinations, this perceived environment coincides with the equilibrium environment because the simulations are conducted in partial equilibrium with respect to foreign demand conditions. However, the same is not true for the domestic market. Firms initially take a conjecture for the domestic quantity index B_t^0 as given when solving for export participation, but this quantity will update endogenously when solving for optimal quantities and prices. Thus, firms' initial conjectures about B_t^0 generally differ from the equilibrium domestic quantity index that ultimately clears the domestic market. In this sense, the simulations impose a fixed-point problem only for the domestic market, while foreign market conditions remain exogenous from the perspective of domestic firms.

Solving for equilibrium, given export strategy Conditional on the export strategy Ω_{ft}, firms solve for the monopolistic competition equilibrium taking destination-level quantity indices B_t^d as given. For foreign destinations, firms correctly treat the foreign quantity indices as fixed because the simulations are conducted in partial equilibrium with respect to foreign demand conditions. For the domestic market, however, we impose the CES adding-up constraint and solve for the domestic quantity index endogenously.

The equilibrium is computed iteratively. Given current values for firm-level aggregate output Q_{ft} and the domestic quantity index B_t^0, firms solve the static profit maximization problem and obtain optimal destination-level sales allocations. Using the definition

$$D_t^d = \frac{Y_t^d}{(B_t^d)^{\rho^d}},$$

the optimal quantity allocations can be written as

$$Q_{ft}^d = \left[\frac{\rho^d D_t^d \exp(\varepsilon_{ft}^d)\mathbb{E}[\exp(u_{ft}^d)]}{(\theta+1)\, c_{ft} Q_{ft}^{\theta}}\right]^{\frac{1}{1-\rho^d}}.$$

Although firms treat the quantity indices parametrically when choosing quantities, evaluating the expectations over the aggregate quantity indices exactly would require integrating over the joint distribution of firm-level demand shocks and aggregate market outcomes. To obtain a tractable approximation, we treat the quantity indices as deterministic objects conditional on the current iteration of the algorithm.

For export destinations, quantities are therefore computed taking the foreign quantity indices B_t^d as fixed. For the domestic market, however, the implied firm-level quantities must satisfy the CES aggregation condition

$$B_t^0 = \left(\sum_f (Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0) \exp(u_{ft}^0) \right)^{1/\rho^D}.$$

Consequently, after all firms choose optimal quantities, we aggregate domestic sales across firms and update the implied domestic quantity index B_t^0. Using the updated quantity index, we recompute the domestic demand shifter

$$D_t^0 = \frac{Y_t^0}{(B_t^0)^{\rho^D}},$$

and firms recompute their optimal quantity allocations conditional on the updated aggregate state variables.

The algorithm therefore iterates between firm-level optimization and aggregation in the domestic market until convergence of both firm-level quantities and the domestic quantity index. In practice, convergence is accelerated using dampened updates of the form

$$X^{(s+1)} = \alpha \tilde{X}^{(s+1)} + (1-\alpha) X^{(s)},$$

where $\tilde{X}^{(s+1)}$ denotes the updated value implied by the current iteration and $\alpha \in (0,1)$ is a relaxation parameter. Iteration continues until

$$|(Q_{ft})' - Q_{ft}| < tol.$$

After firms choose destination-specific quantities, the ex post demand shocks u_{ft}^d are realized. Conditional on these realizations, firms set destination-specific prices according to the inverse demand system

$$P_{ft}^d = D_t^d (Q_{ft}^d)^{\rho^d - 1} \exp(\varepsilon_{ft}^d) \exp(u_{ft}^d),$$

and realized revenues are given by

$$R_{ft}^d = P_{ft}^d Q_{ft}^d = D_t^d (Q_{ft}^d)^{\rho^d} \exp(\varepsilon_{ft}^d) \exp(u_{ft}^d).$$

F.1.3 Assuming Nash in Quantities on the Domestic Market

We next consider an alternative market structure in which firms behave competitively in export markets but play Nash in quantities in the domestic market. Relative to the monopolistic competition case, firms now internalize the effect of their domestic quantity choices on the domestic CES quantity index. As a result, the domestic quantity index becomes an endogenous object in the firm's optimization problem.

Conditional on the export strategy Ω_{ft}, firms solve

$$\max_{Q_{ft}^0} \ \mathbb{E}_u\left[Y_t^0 (Q_{ft}^0)^{\rho^D} (B_t^0)^{-\rho^D} \exp(\varepsilon_{ft}^0 + u_{ft}^0)\right] - c_{ft}\left(Q_{ft}^0 + \sum_{d>0} Q_{ft}^d\right)^{\theta+1}, \tag{F.11}$$

where expectations are taken over future realizations of the ex post demand shocks u_{ft}^d.

The first-order condition for domestic quantities is

$$\begin{aligned}
\frac{\partial \mathbb{E}_u[\Pi_{ft}]}{\partial Q_{ft}^0} &= Y_t^0 \exp(\varepsilon_{ft}^0) \mathbb{E}_u[\exp(u_{ft}^0)] \\
&\times \left(\rho^D (Q_{ft}^0)^{\rho^D - 1} \mathbb{E}_u[(B_t^0)^{-\rho^D}] + (Q_{ft}^0)^{\rho^D} \frac{\partial \mathbb{E}_u[(B_t^0)^{-\rho^D}]}{\partial Q_{ft}^0}\right) \\
&- c_{ft}\,(\theta+1)\,(Q_{ft})^{\theta}.
\end{aligned} \tag{F.12}$$

The key complication relative to the monopolistic competition case is that firms internalize the effect of their domestic quantity choices on the aggregate quantity index B_t^0. Consequently, evaluating the firm's optimization problem requires taking expectations over the inverse CES quantity index, which generally does not admit a closed-form solution.

The domestic quantity index is given by

$$B_t^0 = \left[\sum_j (Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0 + u_{ft}^0)\right]^{1/\rho^D}, \tag{F.13}$$

which implies

$$(B_t^0)^{-\rho^D} = \left[\sum_j (Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0 + u_{ft}^0)\right]^{-1}. \tag{F.14}$$

To obtain a tractable approximation to the equilibrium, we replace the expectation of the inverse CES aggregator with the inverse of the expected CES aggregator:

$$\mathbb{E}_u[(B_t^0)^{-\rho^D}] \approx \frac{1}{\exp(\sigma_u^2/2)} \left[\sum_j (Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0)\right]^{-1}. \tag{F.15}$$

Using this approximation,

$$\frac{\partial \mathbb{E}_u[(B_t^0)^{-\rho^D}]}{\partial Q_{ft}^0} = -\frac{\rho^D (Q_{ft}^0)^{\rho^D - 1} \exp(\varepsilon_{ft}^0)}{\exp(\sigma_u^2/2) \left[\sum_j (Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0)\right]^2}. \tag{F.16}$$

Substituting this expression into the first-order condition yields

$$\rho^D Y_t^0 (Q_{ft}^0)^{\rho^D - 1} \exp(\varepsilon_{ft}^0) \left[\sum_j (Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0)\right]^{-1} \left(1 - \frac{(Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0)}{\sum_j (Q_{ft}^0)^{\rho^D} \exp(\varepsilon_{ft}^0)}\right) = c_{ft}\,(\theta+1)\,(Q_{ft})^{\theta}. \tag{F.17}$$

Define the firm's domestic market share as

$$s_{ft}^{0}=\frac{(Q_{ft}^{0})^{\rho^{D}}\exp(\varepsilon_{ft}^{0})}{\sum_{j}(Q_{ft}^{0})^{\rho^{D}}\exp(\varepsilon_{ft}^{0})}. \tag{F.18}$$

The first-order condition can then be written compactly as

$$\frac{\rho^{D}Y_{t}^{0}s_{ft}^{0}(1-s_{ft}^{0})}{c_{ft}\,(\theta+1)\,(Q_{ft})^{\theta}}=Q_{ft}^{0}. \tag{F.19}$$

Conditional on the export strategy Ω_{ft}, firms solve for the Nash equilibrium iteratively. Given current values for aggregate output Q_{ft}, firms first solve the monopolistic competition first-order conditions for export quantities:

$$Q_{ft}^{d}=\left[\frac{\rho^{X}D_{t}^{d}\exp(\varepsilon_{ft}^{d})\mathbb{E}[\exp(u_{ft}^{d})]}{(\theta+1)\,c_{ft}Q_{ft}^{\theta}}\right]^{\frac{1}{1-\rho^{X}}}, \qquad d>0.$$

Conditional on export quantities, firms then solve equation F.19 for domestic quantities Q_{ft}^{0}. Aggregate output is updated according to

$$(Q_{ft})'=(Q_{ft}^{0})'+\sum_{d\in\Omega_{ft}}Q_{ft}^{d}.$$

After all firms choose quantities, the domestic quantity index is updated using

$$(B_{t}^{0})'=\left[\sum_{j}\left((Q_{ft}^{0})'\right)^{\rho^{D}}\exp(\varepsilon_{ft}^{0}+u_{ft}^{0})\right]^{1/\rho^{D}}.$$

Using the updated quantity index, we recompute the domestic demand shifter

$$D_{t}^{0}=\frac{Y_{t}^{0}}{(B_{t}^{0})^{\rho^{D}}},$$

and firms recompute their optimal quantity allocations conditional on the updated aggregate state variables.

The equilibrium algorithm therefore proceeds as follows:

1. Guess aggregate output Q_{ft}.
2. Solve the monopolistic competition first-order conditions for export quantities $\{Q_{ft}^{d}\}_{d>0}$.
3. Compute domestic market shares s_{ft}^{0}.
4. Solve equation F.19 for updated domestic quantities $(Q_{ft}^{0})'$.
5. Update aggregate output:

$$(Q_{ft})'=(Q_{ft}^{0})'+\sum_{d\in\Omega_{ft}}Q_{ft}^{d}.$$

6. Update the domestic quantity index:

$$(B_t^0)' = \left[\sum_j \left((Q_{ft}^0)'\right)^{\rho^D} \exp(\varepsilon_{ft}^0 + u_{ft}^0)\right]^{1/\rho^D}.$$

7. Iterate until

$$|(Q_{ft})' - Q_{ft}| < tol.$$

After firms choose destination-specific quantities, the ex post demand shocks u_{ft}^d are realized. Conditional on these realizations, firms set destination-specific prices according to the inverse demand system

$$P_{ft}^d = D_t^d (Q_{ft}^d)^{\rho^d - 1} \exp(\varepsilon_{ft}^d) \exp(u_{ft}^d),$$

and realized revenues are given by

$$R_{ft}^d = P_{ft}^d Q_{ft}^d = D_t^d (Q_{ft}^d)^{\rho^d} \exp(\varepsilon_{ft}^d) \exp(u_{ft}^d).$$

F.1.4 Assuming Non-CES Demand on the Domestic Market

We next consider an alternative demand system in which demand on the domestic market deviates from CES. In particular, we replace the CES revenue function with a Kimball-style specification (Kimball, 1995) that allows for state-dependent demand curvature, while maintaining CES demand and monopolistic competition in export markets.

Under this specification, domestic revenues are given by

$$R_{ft}^0 = Y_t^0 \exp(\varepsilon_{ft}^0 + u_{ft}^0) H\left(\frac{Q_{ft}^0}{B_t^0}\right), \tag{F.20}$$

where

$$H(\Xi) = x^{\rho^D} \exp\left(-\psi(\ln \Xi)^2\right), \tag{F.21}$$

which nests the CES case when $\psi = 0$. This specification can be interpreted as a second-order approximation to a general Kimball demand system around the CES benchmark.

Second-Order Approximation. To formalize this interpretation, consider a general Kimball demand system in which revenues can be written as

$$R_{ft} = Y_t \exp(\varepsilon_{ft}) H\left(\frac{Q_{ft}}{B_t}\right), \tag{F.22}$$

where

$$H(\Xi) = \Xi \Psi'(\Xi),$$

and $\Psi(\cdot)$ is the underlying Kimball aggregator. The CES case corresponds to $H(\Xi) = \Xi^{\rho}$.

To study departures from CES, write

$$H(\Xi) = \Xi^{\rho} g(\Xi), \tag{F.23}$$

where $g(\Xi) = 1$ under CES. Taking logs,

$$\ln H(\Xi) = \rho \ln \Xi + \ln g(\Xi). \tag{F.24}$$

Let $z = \ln \Xi$ and define

$$m(z) \equiv \ln g(e^z).$$

Then

$$\ln H(\Xi) = \rho z + m(z). \tag{F.25}$$

Assuming $m(\cdot)$ is twice continuously differentiable, a second-order Taylor expansion around a reference point $\bar{z}$ yields

$$m(z) = m(\bar{z}) + m'(\bar{z})(z-\bar{z}) + \frac{1}{2}m''(\bar{z})(z-\bar{z})^2 + O((z-\bar{z})^3). \tag{F.26}$$

Substituting and collecting terms gives

$$\ln H(\Xi) = c_0 + \tilde{\rho} z + \frac{1}{2}m''(\bar{z})z^2 + O(z^3), \tag{F.27}$$

where

$$c_0 = m(\bar{z}) - m'(\bar{z})\bar{z} + \frac{1}{2}m''(\bar{z})\bar{z}^2, \tag{F.28}$$

$$\tilde{\rho} = \rho + m'(\bar{z}) - m''(\bar{z})\bar{z}. \tag{F.29}$$

The constant c_0 can be absorbed into the aggregate demand shifter Y_t. Defining

$$\psi = -\frac{1}{2}m''(\bar{z}), \tag{F.30}$$

the approximation becomes

$$\ln H(\Xi) = \tilde{\rho} \ln \Xi - \psi(\ln \Xi)^2 + O((\ln \Xi)^3). \tag{F.31}$$

Taking the exponent yields

$$H(\Xi) = \Xi^{\tilde{\rho}} \exp\left[-\psi(\ln \Xi)^2\right] + O((\ln \Xi)^3). \tag{F.32}$$

Differentiating with respect to $\ln \Xi$ implies

$$\frac{d \ln H(\Xi)}{d \ln \Xi} = \tilde{\rho} - 2\psi \ln \Xi + O((\ln \Xi)^2), \tag{F.33}$$

so that, to a second-order approximation, the elasticity of demand varies linearly in $\ln \Xi$.

Hence, the specification

$$H(\Xi) = \Xi^{\rho} \exp\left[-\psi(\ln \Xi)^2\right] \tag{F.34}$$

provides a tractable second-order approximation to a smooth Kimball demand system around the CES benchmark, with ψ governing the extent of departures from constant elasticity.

Optimization. Conditional on the export strategy Ω_{ft}, firms solve

$$\max_{Q^0_{ft}} \ \mathbb{E}_u \left[Y^0_t \exp(\varepsilon^0_{ft} + u^0_{ft}) H\left(\frac{Q^0_{ft}}{B^0_t}\right)\right] - c_{ft}\left(Q^0_{ft} + \sum_{d>0} Q^d_{ft}\right)^{\theta+1}. \tag{F.35}$$

Let $x_{ft} \equiv Q^0_{ft}/B^0_t$. The first-order condition is

$$\frac{\partial \mathbb{E}_u[\Pi_{ft}]}{\partial Q^0_{ft}} \quad = \quad Y^0_t \exp(\varepsilon^0_{ft})\mathbb{E}_u[\exp(u^0_{ft})]\frac{1}{B^0_t}H'(\Xi_{ft}) - c_{ft}(\theta+1)(Q_{ft})^{\theta}. \tag{F.36}$$

Using the Kimball specification,

$$H'(\Xi) = \Xi^{\rho^D-1}\exp\left(-\psi^D(\ln\Xi)^2\right)\left(\rho^D - 2\psi^D\ln\Xi\right), \tag{F.37}$$

so that the first-order condition becomes

$$Y^0_t \exp(\varepsilon^0_{ft})\mathbb{E}_u[\exp(u^0_{ft})]\frac{1}{B^0_t}\left(\frac{Q^0_{ft}}{B^0_t}\right)^{\rho^D-1}\exp\left[-\psi^D\left(\ln\frac{Q^0_{ft}}{B^0_t}\right)^2\right]\left(\rho^D - 2\psi^D\ln\frac{Q^0_{ft}}{B^0_t}\right) = c_{ft}(\theta+1)(Q_{ft})^{\theta} \tag{F.38}$$

Relative to the CES case, the key difference is that the effective demand elasticity

$$\rho^D - 2\psi^D\ln\left(\frac{Q^0_{ft}}{B^0_t}\right)$$

varies across firms depending on their relative size in the domestic market.

The domestic quantity index is given by

$$B^0_t = \left[\sum_j (Q^0_{ft})^{\rho^D}\exp(\varepsilon^0_{ft} + u^0_{ft})\right]^{1/\rho^D}, \tag{F.39}$$

as in the CES case. To obtain a tractable approximation, we replace expectations over the demand shocks with their log-normal moments:

$$\mathbb{E}_u[\exp(u^0_{ft})] = \exp(\sigma_u^2/2). \tag{F.40}$$

Conditional on the export strategy Ω_{ft}, firms solve for equilibrium quantities iteratively. Given current aggregate output Q_{ft}, firms first solve the monopolistic competition first-order conditions for export quantities:

$$Q^d_{ft} = \left[\frac{\rho^X D^d_t \exp(\varepsilon^d_{ft})\mathbb{E}[\exp(u^d_{ft})]}{(\theta+1)c_{ft}Q^{\theta}_{ft}}\right]^{\frac{1}{1-\rho^X}}, \qquad d > 0.$$

Conditional on export quantities, firms then solve the nonlinear first-order condition above for domestic quantities Q^0_{ft}.

Aggregate output is updated according to

$$(Q_{ft})' = (Q^0_{ft})' + \sum_{d\in\Omega_{ft}} Q^d_{ft},$$

and the domestic quantity index is updated using

$$(B^0_t)' = \left[\sum_j \left((Q^0_{ft})'\right)^{\rho^D}\exp(\varepsilon^0_{ft} + u^0_{ft})\right]^{1/\rho^D}.$$

The equilibrium algorithm proceeds as follows:

1. Guess aggregate output Q_{ft}.

2. Solve the monopolistic competition first-order conditions for export quantities $\{Q^d_{ft}\}_{d>0}$.

3. Solve the Kimball first-order condition for domestic quantities $(Q^0_{ft})'$.

4. Update aggregate output:

$$(Q_{ft})' = (Q^0_{ft})' + \sum_{d \in \Omega_{ft}} Q^d_{ft}.$$

5. Update the domestic quantity index:

$$(B^0_t)' = \left[\sum_j \left((Q^0_{ft})' \right)^{\rho^D} \exp(\varepsilon^0_{ft} + u^0_{ft}) \right]^{1/\rho^D}.$$

6. Iterate until

$$|(Q_{ft})' - Q_{ft}| < tol.$$

After quantities are chosen, ex post demand shocks are realized, and prices are given by the inverse demand system implied by the Kimball revenue function:

$$P^0_{ft} = \frac{R^0_{ft}}{Q^0_{ft}}.$$

F.1.5 Details of the Estimation

We use a common practitioners' approach to choosing initial conditions for the nonlinear optimizations based on OLS.

For the factor shares model, we set the initial conditions for the first-step NLLS estimation for M_{ft} based on an OLS estimation of the regression

$$\ln(W^m_{ft} M_{ft}/R_{ft}) = g^m_0 + g^m_m m_{ft} + g^m_k k_{ft} + g^m_{mm} m_{ft} m_{ft} + g^m_{kk} k_{ft} k_{ft} + g^m_{mk} m_{ft} k_{ft} + \vartheta_{ft},$$

where ϑ_{ft} is a regression residual. For our multi-destination procedure, we condition the OLS regression on non-exporters.

For the second step GMM in our multi-destination estimation procedure, we set initial conditions based on an OLS estimation of the regression

$$\tilde{r}^0_{ft} = g_k k_{ft} + g_{kk} k k_{ft} + g_D \ln \left[R^0_{ft} / (R_{ft} \widehat{Z}_{ft}) \right] + \delta_t + \vartheta'_{ft}.$$

For the factor shares model that makes no correction for demand or controls for the aggregate domestic CES quantity index, we adjust the dependent variable and the demand proxy accordingly.

For the control function approach, we set initial conditions for the second-step GMM based on an OLS estimation of the regression

$$\tilde{r}^{CF}_{ft} = g^{CF}_0 + g^{CF}_B \ln B^0_t + g^{CF}_m m_{ft} + g^{CF}_k k_{ft} + g^{CF}_{mm} m_{ft} m_{ft} + g^{CF}_{kk} k_{ft} k_{ft} + g^{CF}_{mk} m_{ft} k_{ft} + \vartheta^{CF}_{ft},$$

where $\widetilde{r}_{ft}^{CF}$ represents log revenues net of the residual from the control function first step, and ϑ_{ft}^{CF} is a regression residual.

F.2 Single-Market Monte Carlo Simulations

In this appendix, we test the performance of the factor shares single-market estimator from Section E.2 and a single-market version of the control function method from Section E.3 when the underlying data generating process features just a single market. We thus simulate data as described in Section 2 assuming there is just a single market. With these simulated data, we estimate output elasticities and the curvature of the demand function. These simulations extend the Monte Carlo experiments from Gandhi et al. (2020) and Ackerberg et al. (2023) to the case of heterogeneous products with missing output price data, and highlight the advantages of the factor share method over the control function approach.

For the data generating process, we impose that firms produce with a Cobb-Douglas production function with one flexible input, materials (M), and one quasi-fixed input, capital (K):

$$Q_{ft} = \exp\left(\omega_{ft}\right) M^{\gamma^M} K^{\gamma^K} \tag{F.41}$$

with $\gamma^M = 0.8$ and $\gamma^K = 0.3$. Capital updates each period according to the law of motion: $K_{ft} = 0.9K_{f,t-1} + \iota_{f,t-1}$, where $\iota_{ft} = \exp(0.8\rho\omega_{ft} + 0.8\varepsilon_{ft})\left(K_{ft}\right)^{0.2}$. We fix $\rho = 0.8$. While we build the data according to these restrictions for simplicity, we obviously need not impose any functional form in the estimation.

Within each replication we draw total expenditures and quantity series, and homogeneous (across firms) material input prices. At the firm level, we draw initial capital stocks $K_{f,1} \sim U(1,201)$, initial productivity shocks $\omega_{f,1} \sim N(0,0.01)$, and initial *ex ante* demand shocks $\varepsilon_{f,1} \sim N(0,0.0009)$. We let ω and ε update according to the same AR(1) process described in the main text, with $h = 0.8$ and where $\widetilde{\omega}_{ft} \sim N(0,0.01)$ and $\widetilde{\varepsilon_{ft}} \sim N(0,0.0009)$. We draw *ex post* demand shocks $u_{ft} \sim N(0,0.0009)$. Firm-period quantities, revenues and inputs M_{ft} are determined given productivity, capital, materials prices and aggregate demand. We simulate 100 samples of a single industry with 500 firms over 50 periods.[5]

We estimate in each sample of simulated data the factor shares approach and the control function approach assuming researchers observe R_{ft}, M_{ft}, K_{ft}, W_t^M, B_t and Υ_t. For the factor shares model, we set initial conditions for the first-step NLLS estimation for M based on an OLS estimation of

[5] In the single-market case, we follow closely the experiments presented in Gandhi et al. (2020). Gandhi et al. (2020) posit a long panel (50 periods) in order to give the control function a reasonable chance to identify the structural parameters. Since the output elasticity for materials is identified purely from time-series variation in the material input price, there is little chance that the control function identifies structural parameters in panels of only 10-15 years (the type of duration one usually observes in balance sheet datasets). In the multi-market simulations below, we can entertain much shorter panels.

the regression

$$\ln\left[\frac{W^m_{ft}M_{ft}}{R_{ft}}\right] = g^m_0 + g^m_m m_{ft} + g^m_k k_{ft} + g^m_{mm} m_{ft} m_{ft} + g^m_{kk} k_{ft} k_{ft} + g^m_{mk} m_{ft} k_{ft} + \vartheta_{ft},$$

where ϑ_{ft} is a regression residual. For the second step GMM, we set initial conditions based on an OLS estimation of the regression

$$\widetilde{r}_{ft} = g_k k_{ft} + g_{kk} kk_{ft} + g_D B_t + \vartheta'_{ft},$$

where ϑ'_{ft} is a regression residual and B_t is the true CES quantity index.

For the control function approach, we set initial conditions for the second-step GMM based on an OLS estimation of the regression

$$\widetilde{r}^{CF}_{ft} = g_0 + g_D B_t + g^m_m m_{ft} + g^m_k k_{ft} + g^m_{mm} m_{ft} m_{ft} + g^m_{kk} k_{ft} k_{ft} + g^m_{mk} m_{ft} k_{ft} + \vartheta''_{ft},$$

where $\widetilde{r}^{CF}_{ft}$ represents log revenues net of the residual from the control function first step, and ϑ''_{ft} is a regression residual. We denote the resulting control function estimates as "CF ls", since the GMM starts from the OLS point estimates. We also start the control function estimation from the true parameter values and refer to the resulting estimates as "CF tr".

In Figure F.1 we present the distribution of estimates of ρ and the average material and capital output elasticities across the 100 samples by estimator, along with the average ("av") and median ("md") of the distributions. In the top row we present the case of high input price variation. True material and capital output elasticities are constant across firms and over time ($\sigma^M = \gamma^M = 0.8$ and $\sigma^K = \gamma^K = 0.3$), and are depicted with vertical black lines.[6]

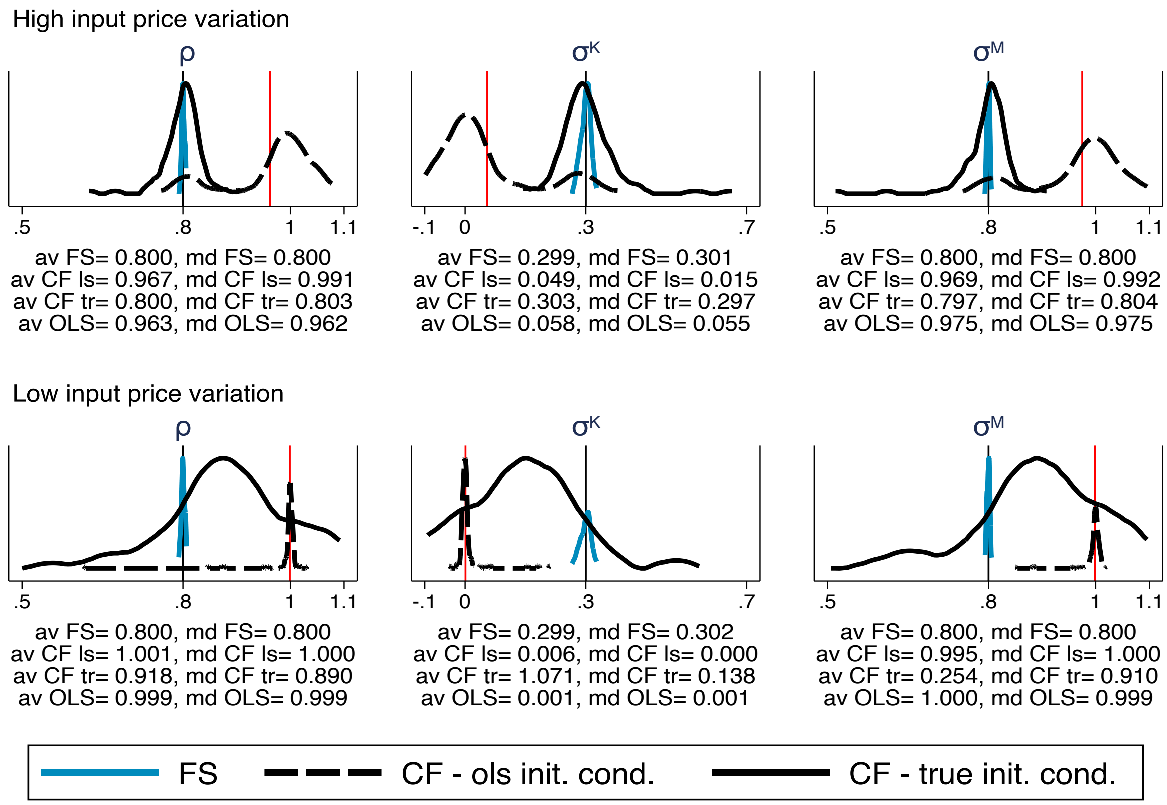
The distribution of the estimates from the factor shares approach is depicted in solid blue. For each empirical object ρ, σ^K , σ^M, the distribution of estimates appears to be centered on the true values. Averages and medians of the distributions are identical with the truth out to at least two decimal places. Similarly, the distribution of the control function estimates *taking true values as the initial conditions* (black solid line) also appears to be centered on the truth, with averages and medians of distributions identical to the truth out to at least two decimal places. The distribution of the solid blue line is clearly narrower than the distribution of the solid black line, indicating that the factor shares approach is more efficient.

In contrast, when the second step of the control function method starts the optimization algorithm from the OLS values (black dashed line), the distributions of estimates are clearly biased. Estimates of ρ and σ^M tend to center around 1, and estimates of σ^K center around 0. These values coincide roughly with the median results from a naïve OLS estimate of the production function (depicted with a vertical red line).

Also in Figure F.1, we present in the bottom row the distribution of estimates for the case of

[6]Estimated material and capital output elasticities vary both due to sampling error and with the level of capital because we allow for higher order terms in capital and interactions between inputs in the estimation process (see equations (24) and (25)).

Figure F.1: Parameter Estimates in the Single-Market Simulations



Notes. The figure reports the distribution of averages of estimates across 100 Monte Carlo samples. Top (bottom) row presents results for high (low) input price variation. True parameter values are depicted as vertical lines. Averages ("av") and medians ("md") of distributions for each estimator are reported below each subfigure. "FS" indicates the factor shares method. "CF - ls" indicates control function method where the GMM optimization starts from the OLS values. "CF - tr" indicates control function method where the GMM optimization starts from the true model parameters. Red line indicates the median estimate based on an OLS regression.

low input price variation. The factor shares method still recovers unbiased estimates of structural parameters. However, the estimates from the control function method are biased even when the GMM optimization starts from the true parameters. As explained by Nelson & Startz (1990), instrumental variables estimators are biased towards OLS in finite samples with weak instruments. We can see this pattern from the medians of the distributions in solid black. The distributions are wide, and outliers severely distort the means, but the medians indicate that the estimates of σ^M are biased up and the estimates of σ^K are biased down, as they are in OLS. This is the same pattern found in Monte Carlo simulations by Gandhi et al. (2020) for the single-market case in which quantities are observed.

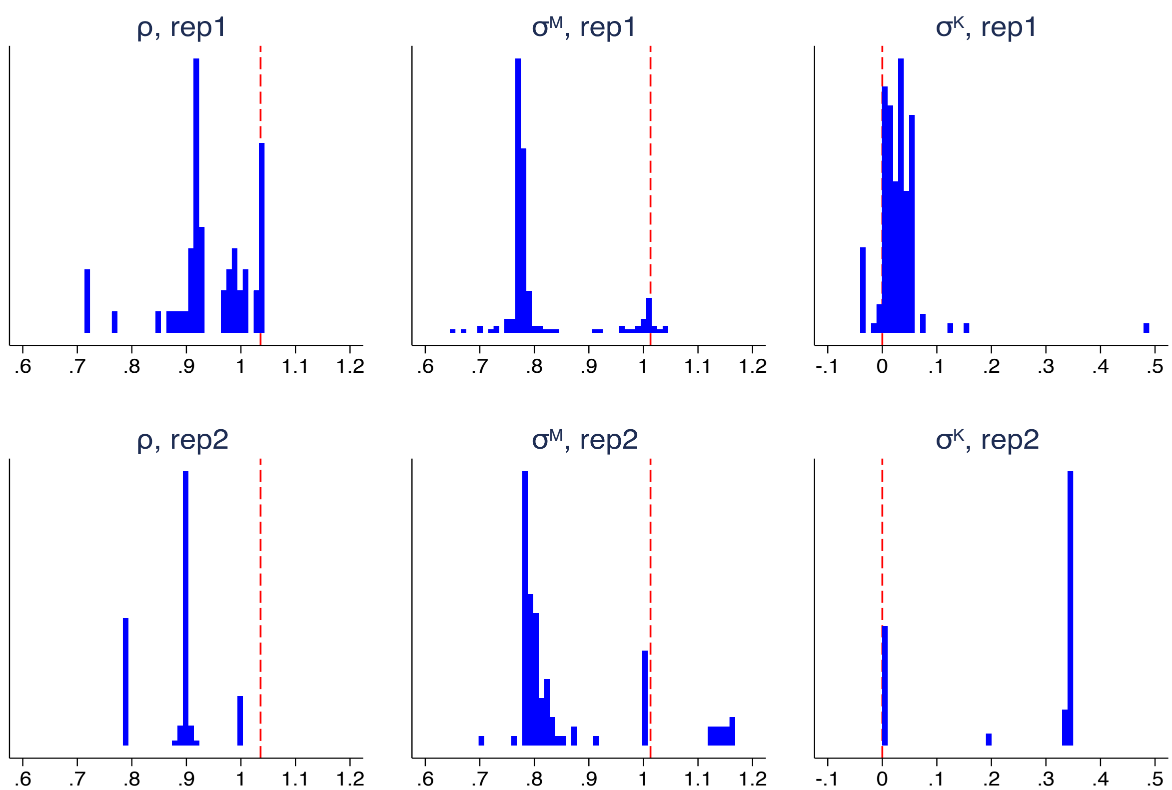
Comparing the solid black line to the dashed black line, it is clear that the control function is sensitive to initial starting conditions. We explore this sensitivity further by plotting the distribution of parameter estimates from the control function method when varying systematically the initial values of the second step of the GMM procedure below. We run the same Monte Carlo simulations as in the main text. We vary initial conditions for $\beta_0^D \in \{0, 0.05, 0.1, 0.15, 0.2, 0.25\}$, $\beta_0^M \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$ and $\beta_0^K \in \{0, 0.1, 0.2, 0.3, 0.4\}$. We compute the GMM solution starting from every combination defined by these three sets. Recall that the true parameter values are $\beta^D = 1 - \rho = 0.2$, $\beta^M = \rho\sigma^M = 0.64$ and $\beta^K = \rho\sigma^K = 0.24$.

In Figure F.2 (F.3), we present results when the data is simulated with a high (low) degree of time series variation in material input prices. We display the distribution of GMM solutions for each parameter for two different Monte Carlo samples, one in each row. Results starting from the OLS estimates are depicted with a vertical red dashed line. Results from all other starting values are depicted in solid bars in blue.

In Figure F.2, we see that the estimates based on the OLS initial values coincide with the results in Figure F.1: starting from the OLS values, the GMM solution tends towards $\rho = 1$, $\sigma^M = 1$, and $\sigma^K = 0$ (red dashed line). In Figure F.2, we also see that when the GMM starts from other initial conditions there is a mass point of convergence around the same values, although we also see other mass points. This bunching pattern is consistent with Ackerberg et al. (2023).

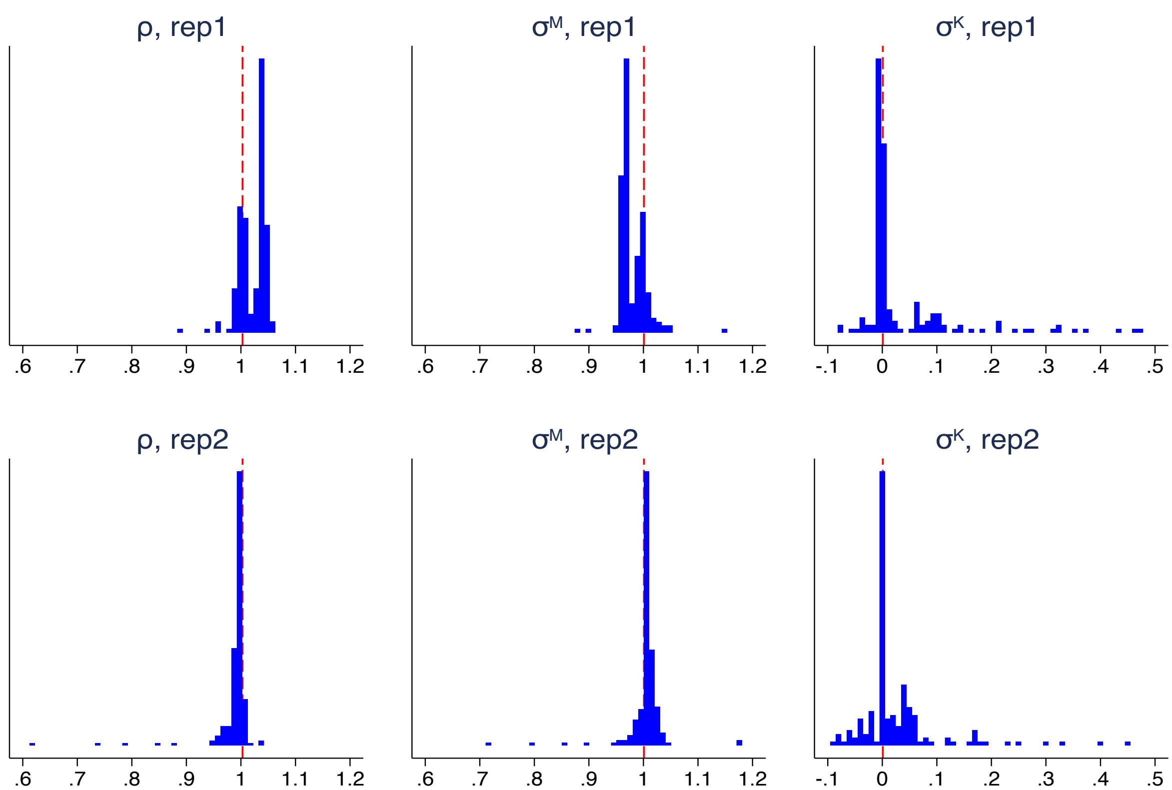
In Figure F.3, we see that the estimates converges more often towards the OLS results. That is, the distribution is bunched more tightly around the OLS estimates than in Figure F.2. This is what we would expect, as with low input price variation, the GMM suffers from weak instruments, which tends to bias the estimates towards the OLS result.

Figure F.2: Different Starting Values for Control Function Estimation, High Material Price Variation



Notes. The figure reports the distribution of estimates of ρ and averages of estimates of factor output elasticities resulting from the control function method taking different parameter vectors as starting values for the second-step GMM procedure. Each row reports results for a single Monte Carlo sample. Data is generated based on the single-market scenario described in Section F.2, with high input price variation. The true parameter values are $\rho = 0.8$, $\sigma^M = 0.8$ and $\sigma^K = 0.3$.

Figure F.3: Different Starting Values for Control Function Estimation, Low Material Price Variation

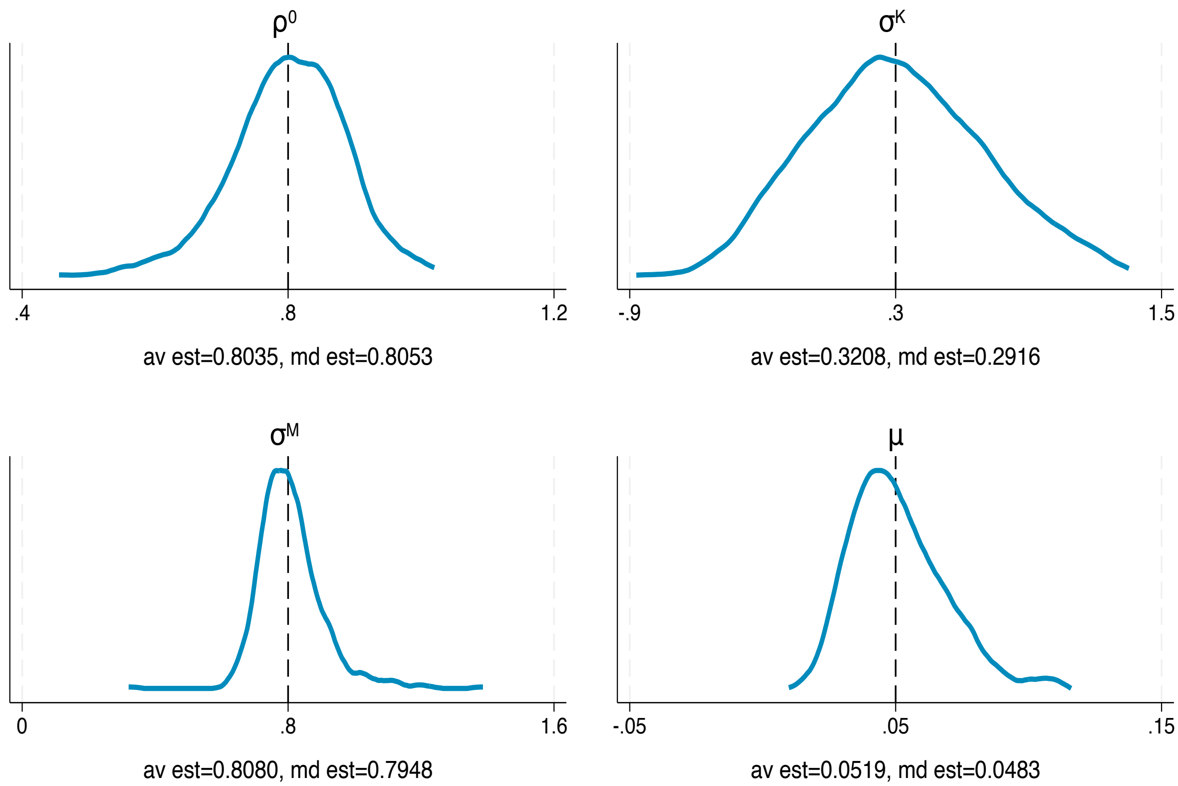


Notes. The figure reports the distribution of estimates of ρ and averages of estimates of factor output elasticities resulting from the control function method taking different parameter vectors as starting values for the second-step GMM procedure. Each row reports results for a single Monte Carlo sample. Data is generated based on the single-market scenario described in Section F.2, with low input price variation. The true parameter values are $\rho = 0.8$, $\sigma^M = 0.8$ and $\sigma^K = 0.3$.

G Additional Monte Carlo Simulation Results

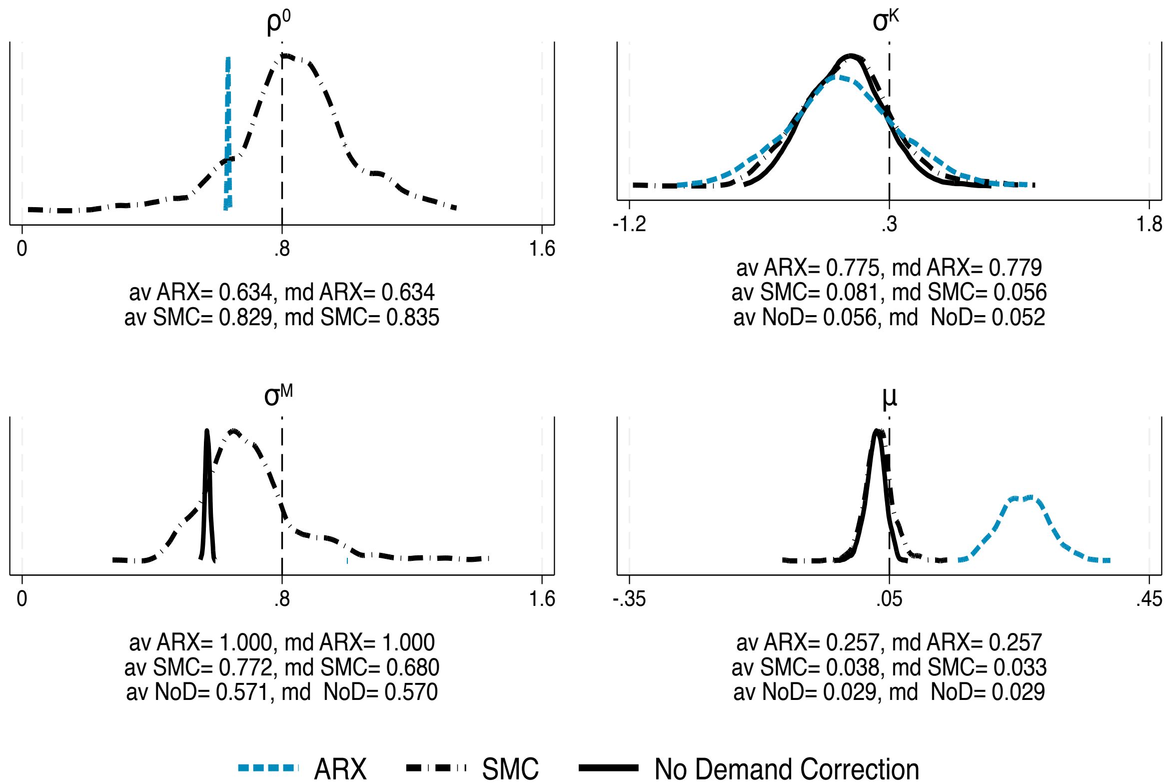
G.1 Baseline Simulations

Figure G.1: Distribution of Parameter Estimates, Multi-Destination Factor Shares Approach



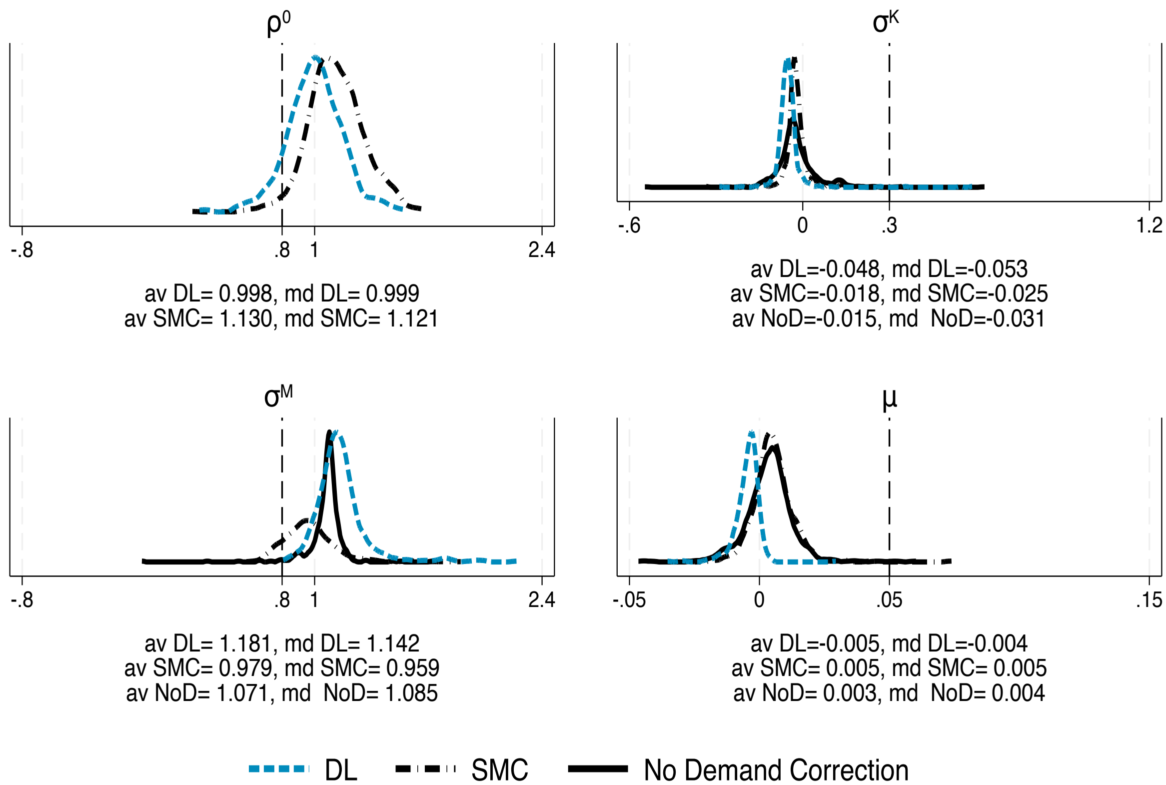
Notes. The figure reports distributions of point estimates across 500 simulations estimated by our multi-destination factor shares approach. Means ("av est") and medians ("md est") of each distribution is reported below each subfigure.

Figure G.2: Distribution of Parameter Estimates, Alternative Factor Shares Approach



Notes. The figure reports distributions of point estimates across 500 simulations estimated by alternative factor shares approaches. Means ("av est") and medians ("md est") of each distribution is reported below each subfigure. "ARX" refers to the estimator from Aw et al. (2011) that imposes constant returns to variable inputs. "SMC" refers to the single market correction model (appendix of Gandhi et al. (2020)) "NoD" refers to the estimator in which revenues are deflates by an industry wide deflator and the resulting series is treated as if were a quantities series (main text of Gandhi et al. (2020)).

Figure G.3: Distribution of Parameter Estimates, Control Function Methods



Notes. The figure reports distributions of point estimates across 500 simulations. Means ("av est") and medians ("md est") of each distribution is reported below each subfigure. "DL" refers to the multi-destination control function estimator of De Loecker (2011). "SMC" refers to the single market correction model, estimated by control function. "NoD" refers to the estimator in which revenues are deflates by an industry wide deflator and the resulting series is treated as if were a quantities series (control function approach described in the main text of Gandhi et al. (2020)).

Table G.1: Monte Carlo Experiments, DGP Oligopoly, 200 Firms

	σ^M	σ^K	ρ^0	μ
True Value	0.8000	0.3000	0.8000	0.0500
Factor Shares				
Multi-Market Demand Correction (BOR)	0.7946	0.2981	0.8010	0.0492
	(0.0988)	(0.4273)	(0.0888)	(0.0211)
Multi-Market, Constant Variable Returns to Scale (ARX)	-	0.7832	0.6260	0.2577
		(0.1034)	(0.0023)	(0.0363)
Single Market Demand Correction (SMC)	0.6810	0.0756	0.8337	0.0325
	(5.7047)	(2.9753)	(0.2159)	(0.4084)
No Demand Correction (NoD)	0.5689	0.0702	-	0.0292
	(0.0072)	(0.2209)		(0.0151)
Control Function				
Multi-Market Demand Correction (DL)	1.1367	-0.0516	0.9996	-0.0036
	(0.50)	(0.07)	(0.192)	(0.005)
Single Market Demand Correction (CF-SMC)	0.9504	-0.0252	1.1323	0.0047
	(0.22)	(0.0663)	(0.1794)	(0.0076)
No Demand Correction (CF-NoD)	1.0873	-0.0300	-	0.0039
	(0.0884)	(0.1295)		(0.0106)

Notes: Table reports median and standard deviation of parameter estimates across 500 simulated data samples, by estimator. Data generating process includes 4 destination markets, 200 firms, and 10 time periods. True parameter values are reported in the first row. Additionally, all foreign $\rho^d = 0.6$ for $d \neq 0$. Firms play Nash in quantities on the Domestic market, but monopolistic competition on the foreign markets.

Table G.2: Monte Carlo Experiments, DGP Oligopoly, 40 Firms

	σ^M	σ^K	ρ^0	μ
True Value	0.8000	0.3000	0.8000	0.0500
Factor Shares				
Multi-Market Demand Correction (BOR)	0.8047 (2.1549)	0.3455 (0.7614)	0.7705 (0.2282)	0.0552 (0.1853)
Multi-Market, Constant Variable Returns to Scale (ARX)	-	0.8331 (0.2213)	0.5963 (0.0055)	0.2567 (0.0556)
Single Market Demand Correction (SMC)	0.7492 (0.5387)	0.2746 (0.4966)	0.7775 (0.2273)	0.0369 (0.0378)
No Demand Correction (NoD)	0.5835 (0.0081)	0.2267 (0.6127)	-	0.0317 (0.0216)
Control Function				
Multi-Market Demand Correction (DL)	1.0934 (0.32)	-0.0535 (0.11)	1.0612 (0.250)	-0.0006 (0.004)
Single Market Demand Correction (CF-SMC)	0.9744 (0.35)	0.0034 (0.4728)	1.0684 (0.1900)	0.0021 (0.0590)
No Demand Correction (CF-NoD)	1.0874 (0.2325)	-0.0093 (0.3630)	-	-0.0014 (0.0331)

Notes: Table reports median and standard deviation of parameter estimates across 500 simulated data samples, by estimator. Data generating process includes 4 destination markets, 40 firms, and 10 time periods. True parameter values are reported in the first row. Additionally, all foreign $\rho^d = 0.6$ for $d \neq 0$. Firms play Nash in quantities on the Domestic market, but monopolistic competition on the foreign markets.

Table G.3: Monte Carlo Experiments, Kimball Demand, $\psi = .005$

	σ^M	σ^K	ρ^0	μ
True Value	0.8000	0.3000	0.8000	0.0500
Factor Shares				
Multi-Market Demand Correction (BOR)	0.7969 (0.0825)	0.2939 (0.3939)	0.8677 (0.0812)	0.0496 (0.0239)
Multi-Market, Constant Variable Returns to Scale (ARX)	-	0.9365 (0.2458)	0.6784 (0.0027)	0.3362 (0.0876)
Single Market Demand Correction (SMC)	0.6742 (0.8831)	0.0798 (0.7946)	0.8750 (0.2453)	0.0400 (0.0684)
No Demand Correction (NoD)	0.5900 (0.0103)	0.0806 (0.5379)	-	0.0350 (0.0176)
Control Function				
Multi-Market Demand Correction (DL)	1.1787 (1.21)	-0.0646 (0.11)	0.9694 (0.247)	-0.0054 (0.011)
Single Market Demand Correction (CF-SMC)	0.9763 (0.23)	-0.0399 (0.0616)	1.1085 (0.1778)	0.0035 (0.0093)
No Demand Correction (CF-NoD)	1.1108 (0.1713)	-0.0519 (0.3978)	-	0.0026 (0.0153)

Notes: Table reports median and standard deviation of parameter estimates across 500 simulated data samples, by estimator. Data generating process includes 4 destination markets, 200 firms, and 10 time periods. True parameter values are reported in the first row. Additionally, all foreign $\rho^d = 0.6$ for $d \neq 0$. Demand follows the Kimball-inspired extension from Appendix F.1.4 on the Domestic market with $\psi = .005$, but CES on the foreign markets.

H Data

Firm-level balance sheet information is reported in the FICUS (*Fichier complet unifié de SUSE*) and FARE (*Fichier Approché des Résultats ESANE*) datasets, which cover the periods 1994–2007 and 2008–2016, respectively. These data originate in tax declarations of all firms in France, and are collected by the French National Institute of Statistics and Economic Studies, INSEE. We use total revenue, expenditure on materials, employment and the book value of capital.

We construct capital stocks following the methodology proposed by Bonleu et al. (2013) and Cette et al. (2015). We start with the book value of capital. Since the stocks are recorded at historical cost, i.e., the value at the time of entry into the firm i's balance sheet, an adjustment has to be made to move from stocks valued at historic cost ($K^{BV}_{i,s,t}$) to stocks valued at current prices ($K_{i,s,t}$). We deflate K^{BV} by an industry-specific price index (sourced from INSEE) that assumes that the price of capital is equal to the sectoral price of investment T years before the date when the first book value was available, where T is the corrected average age of capital, hence $p^K_{s,t+1} = p^I_{s,t-T}$. The average age of capital is computed using the share of depreciated capital, $DK^{BV}_{i,s,t}$ in the capital stock at historical cost:

$$T = \frac{DK^{BV}_{i,s,t}}{K^{BV}_{i,s,t}} \times \widetilde{A}$$

where

$$\widetilde{A} = median_{i \in S} \left(\frac{K^{BV}_{i,s,t}}{\Delta DK^{BV}_{i,s,t}} \right)$$

where S the set of firms in a sector. We use the median value $\widetilde{A}$ to reduce the volatility in the data, as investments within firms are discrete events.

Data on firms' exports are from the French Customs. For each observation, we know the value of exports of the firm. We use the firm-level SIREN identifier to match the trade data to FICUS/FARE. This match is not perfect. The imperfect match is because there are SIRENs in the trade data for which there is no corresponding SIREN in FICUS/FARE. This may lead to measurement error: for some firms, we will observe zero exports even when true exports are positive. This is not a big concern because most of the missing values are in the oil refining industry, which we drop from our sample.

Table H.1: Sample Descriptive Statistics

No.	Industry		Revenue (mn euros)	Labor (employment)	Materials (mn euros)	Capital (mn euros)	Obs.	No. firms	No. exporters	Percent exporters
1	Autos and transport equipment	Mean	40.11	165	26.73	23.05	49787	5442	2765	50.8
		Median	1.06	10	0.42	0.23				
2	Chemical products	Mean	52.45	98	29.64	28.07	51026	4863	3457	71.1
		Median	2.36	14	0.94	0.56				
3	Computer, electronics	Mean	11.36	62	5.18	4.96	51259	5610	3116	55.5
		Median	0.84	8	0.27	0.13				
4	Electrical equipment	Mean	13.16	71	7.02	5.17	41656	4524	2293	50.7
		Median	0.97	9	0.35	0.13				
5	Food, beverage, tobacco	Mean	3.13	13	1.84	1.21	877611	112159	7481	6.7
		Median	0.24	4	0.08	0.1				
6	Machinery and equipment	Mean	3.83	23	1.75	1.03	317346	34333	11168	32.5
		Median	0.56	5	0.18	0.09				
7	Basic metal and fabricated metal	Mean	4.33	27	1.9	2.19	353198	33830	12936	38.2
		Median	0.8	9	0.16	0.23				
8	Other manufacturing	Mean	1.56	12	0.61	0.55	245180	30513	6516	21.4
		Median	0.22	3	0.05	0.06				
9	Rubber and plastic	Mean	6.95	39	3.07	4.16	161240	15941	6951	43.6
		Median	0.89	8	0.3	0.27				
10	Textiles, wearing apparel	Mean	3.32	24	1.41	0.99	147372	21285	9055	42.5
		Median	0.49	6	0.14	0.08				
11	Wood, paper products	Mean	2.75	17	1.21	1.62	293482	32161	9739	30.3
		Median	0.47	5	0.11	0.14				
	Total	Mean	5.45	25	2.88	2.56	2589157	300661	75477	25.1
		Median	0.38	5	0.11	0.11				

Notes. The table reports descriptive statistics for the estimation sample, where capital is the book value reported by the firm and materials are expenditures. Exporters are defined as firms that exported at least once during the sample. Source: FICUS/FARE datasets and French Customs.

Table H.2: Percent exports in revenue for exporters

No.	Industry	Mean	p5	p10	p50	p90	p95	Pct. exporters
1	Autos and transport equipment	14.7	0.2	0.4	5.9	43.1	55.9	50.8
2	Chemical products	22.5	0.2	0.6	11.4	64.6	77.5	71.1
3	Computer, electronics	19.6	0.2	0.5	8.5	58.3	74.1	55.5
4	Electrical equipment	15.5	0.2	0.5	6.1	46.5	60.8	50.7
5	Food, beverage, tobacco	10.2	0.1	0.1	2.8	31.4	48.6	6.7
6	Machinery and equipment	12.4	0.1	0.3	4.1	38.5	57.2	32.5
7	Basic metal and fabricated metal	10.5	0.1	0.3	3.5	31.7	47.8	38.2
8	Other manufacturing	12.8	0.2	0.5	4.9	38.2	53.2	21.4
9	Rubber and plastic	11.5	0.1	0.2	3.7	36	52.2	43.6
10	Textiles, wearing apparel	17.9	0.4	0.8	9.5	48.6	62.3	42.5
11	Wood, paper products	7.8	0.1	0.1	1.6	23.7	41.9	30.3
	Total	12.8	0.1	0.3	4.3	39.8	56.7	25.1

Notes. The table reports the distribution of the percent of exports in revenue for exporters in the estimation sample. The last column reports the percent of exporters. Percent exports in revenue for exporters is computed for firms and years in which exports are positive. Source: FICUS/FARE datasets and French Customs.

I Partial Equilibrium Quantification

In this appendix, we derive the equations from the main text for the partial equilibrium quantification exercise. The goal is to quantify the effect of a change in τ_t^d and τ_t (homogeneous tariff change across d) on destination-specific and total revenues of French firms.

We first derive the expressions that are consistent with the multi-destination model from Section 2. These expressions nest the model that assumes constant returns to variable inputs. For the case of a destination-specific tariff change, the single-market model is misspecified, but we discuss how to approach quantification with this model.

We begin with the most general formulation, and then simplify when required. We will use the FONC from the text to derive expressions for firm-destination export value and total firm-level revenues in terms of trade costs (τ_t^d), observables (χ_{ft}^d), and parameters.

Starting from the multi-destination model from Section 2, re-writing equation (9), for any destination $d \in 0 \bigcup \Omega_{ft}$:

$$(Q_{ft})^{1+\frac{1}{\eta^d}} (\chi_{ft}^d)^{\frac{1}{\eta^d}} \tilde{D}_{ft}^d = \lambda_{ft}. \tag{I.1}$$

with $\tilde{D}_{ft}^d \equiv \left(1 + 1/\eta^d\right) D_t^d \exp(\varepsilon_{ft}^d) \mathbb{E}[\exp(u)]$ and where λ_{ft} is the lagrangian associated to the constraint. Recall that $\eta^d \equiv \frac{1}{\rho^d - 1} < -1$. We need to solve for λ_{ft} in terms of σ_{ft}^M and substitute back into this expression.

Substituting with the definition of χ_{ft}^d and rearranging we find

$$Q_{ft}^d = \left[Q_{ft} \tilde{D}_{ft}^d\right]^{-\eta^d} \lambda_{ft}^{\eta^d}. \tag{I.2}$$

Substituting (I.1) into the materials FOC (7):

$$W_t^M = \lambda_{ft} \frac{\partial Q_{ft}}{\partial M_{ft}}. \tag{I.3}$$

Using

$$\sigma_{ft}^M \equiv \frac{\partial \ln Q_{ft}}{\partial \ln M_{ft}} = \frac{\partial Q_{ft}}{\partial M_{ft}} \cdot \frac{M_{ft}}{Q_{ft}}, \tag{I.4}$$

we obtain:

$$\lambda_{ft} = \frac{W_t^M M_{ft}}{\sigma_{ft}^M}. \tag{I.5}$$

We now eliminate M_{ft} using the elasticity definition. Integrating (I.4) yields:

$$\ln Q_{ft} = \sigma_{ft}^M \ln M_{ft} + c_{ft}, \tag{I.6}$$

where c_{ft} is a constant of integration. Now exponentiating and inverting

$$M_{ft} = \kappa_{ft}^{-1/\sigma_{ft}^M} Q_{ft}^{1/\sigma_{ft}^M}. \tag{I.7}$$

with $\kappa_{ft} = \exp(c_{ft})$

Substituting into λ_{ft}:

$$\lambda_{ft} = \frac{W_t^M}{\sigma_{ft}^M} \kappa_{ft}^{-1/\sigma_{ft}^M} Q_{ft}^{\frac{1}{\sigma_{ft}^M}}. \tag{I.8}$$

Define:

$$\tilde{\kappa}_{ft} \equiv \frac{W_t^M}{\sigma_{ft}^M} \kappa_{ft}^{-1/\sigma_{ft}^M}, \tag{I.9}$$

which does not depend on τ_t^d. Then:

$$\lambda_{ft} = \tilde{\kappa}_{ft} Q_{ft}^{\frac{1}{\sigma_{ft}^M}} = \tilde{\kappa}_{ft} Q_{ft}^{1+\theta_{ft}}. \tag{I.10}$$

with $1/\sigma_{ft}^M \equiv 1 + \theta_{ft}$. The term θ_{ft} reflects the elasticity of marginal cost with respect to output when labor and capital do not adjust. If $\theta_{ft} = 0$, then we have constant marginal costs with respect to M.

Now substitute back into (I.2)

$$\begin{aligned} Q_{ft}^d &= \left[\tilde{D}_{ft}^d\right]^{-\eta^d} Q_{ft}^{-\eta^d} \left(\tilde{\kappa}_{ft} Q_{ft}^{1+\theta_{ft}}\right)^{\eta^d} \\ &= \left(\tilde{\kappa}_{ft}\right)^{\eta^d} \left(\tilde{D}_{ft}^d\right)^{-\eta^d} \left(Q_{ft}\right)^{\eta^d \theta_{ft}}. \end{aligned} \tag{I.11}$$

Summing over $d \in 0 \bigcup \Omega_{ft}$ and dividing by Q_{ft}:

$$1 = \sum_d \left(\tilde{\kappa}_{ft}\right)^{\eta^d} \left(\tilde{D}_{ft}^d\right)^{-\eta^d} Q_{ft}^{\eta^d \theta_{ft} - 1}. \tag{I.12}$$

Taking logs

$$0 = \ln \left[\sum_d e^{\left(\eta^d\right) \ln \tilde{\kappa}_{ft}} e^{\left(-\eta^d\right) \ln \tilde{D}_{ft}^d} e^{\left(\eta^d \theta_{ft} - 1\right) \ln Q_{ft}}\right] \tag{I.13}$$

Substituting (I.11) into (2),

$$R_{ft}^d = \left(\tilde{\kappa}_{ft}\right)^{\eta^d} \left(\tilde{D}_{ft}^d\right)^{-\eta^d} Q_{ft}^{\left(1+\eta^d\right)\theta_{ft}} \frac{\exp(u_{ft}^d)}{\left(1 + \frac{1}{\eta^d}\right) \mathbb{E}[\exp(u)]}. \tag{I.14}$$

and aggregating over destinations:

$$R_{ft} = \sum_k R_{ft}^k = \sum_k \left[\left(\tilde{\kappa}_{ft}\right)^{\eta^k} \left(\tilde{D}_{ft}^k\right)^{-\eta^k} Q_{ft}^{\left(1+\eta^k\right)\theta_{ft}} \frac{\exp(u_{ft}^k)}{\left(1 + \frac{1}{\eta^k}\right) \mathbb{E}[\exp(u)]}\right]. \tag{I.15}$$

I.1 Elasticities with respect to destination-specific tariff cut

Totally differentiating (I.13) we obtain:

$$0 = \sum_d \chi_{ft}^d \left[\eta^d d \ln \tilde{\kappa}_{ft} - \eta^d d \ln \tilde{D}_{ft}^d + \left(\eta^d \theta_{ft} - 1\right) d \ln Q_{ft}\right]. \tag{I.16}$$

Since $\tilde{D}^d_{ft} \propto \left(\tau^d_t\right)^{-\left(1+\frac{1}{\eta^d}\right)}$ we have:

$$\frac{d\ln Q_{ft}}{d\ln\tau^d_t} = \frac{\chi^d_{ft}\left(1+\eta^d\right)}{\sum_k \chi^k_{ft}\left(-\eta^k\theta_{ft}+1\right)} < 0. \tag{I.17}$$

Taking derivatives of destination-specific flows we obtain

$$\frac{d\ln R^d_{ft}}{d\ln\tau^d_t} = \left(1+\eta^d\right) + \left(1+\eta^d\right)\theta_{ft}\frac{d\ln Q_{ft}}{d\ln\tau^d_t} < 0 \tag{I.18}$$

and for $j \neq d$

$$\frac{d\ln R^j_{ft}}{d\ln\tau^d_t} = \left(1+\eta^j\right)\theta_{ft}\frac{d\ln Q_{ft}}{d\ln\tau^d_t} > 0 \tag{I.19}$$

An increase in τ^d_t effects sales of firm f in market d via two channels. The first term in (I.18) is negative and captures the direct effect via optimal pricing, holding marginal costs constant. The second term in (I.18) is positive and captures the effect of lower sales on marginal costs. While the second term dampens the main negative effect, it does not overturn it. Hence, an increase in τ^d_t must lower sales in d. An increase in τ^d_t affects sales in market f only via the reduction in marginal costs, and hence is always positive.

Combining the two effects, we can also compute the effect on total firm-level revenues, which can be negative or positive. Taking logs of I.15 and taking derivatives w.r.t. $\ln\tau^d_t$, and defining $\Xi^k_{ft} \equiv \left(\tilde{\kappa}_{ft}\right)^{\eta^k}\left(\tilde{D}^k_{ft}\right)^{-\eta^k}\frac{\exp(u^k_{ft})\left(1+\frac{1}{\eta^k}\right)}{E[\exp(u)]}$, we have

$$\begin{aligned}\frac{\ln R_{ft}}{\ln\tau^d_t} &= \frac{(1+\eta^d)\left(\tau^d_t\right)^{1+\eta^d} Q_{ft}^{\theta_{ft}(1+\eta^d)}\Xi^d_{ft} + \sum_k\left(\tau^k_t\right)^{1+\eta^k}\Xi^k_{ft}\theta_{ft}(1+\eta^k)Q_{ft}^{\theta_{ft}(1+\eta^k)}\frac{d\ln Q_{ft}}{d\ln\tau^d_t}}{R_{ft}} \\ &= \chi^d_{ft}(1+\eta^d) + \theta_{ft}\frac{d\ln Q_{ft}}{d\ln\tau^d_t}\left[\sum_k \chi^k_{ft}(1+\eta^k)\right]\end{aligned} \tag{I.20}$$

Setting $\eta = \eta^k$ for all k, and substituting with (I.17)

$$\frac{\ln R_{ft}}{\ln\tau^d_t} = (1+\eta)\left(\frac{1+\theta_{ft}}{1-\eta\theta_{ft}}\right)\chi^d_{ft} \tag{I.21}$$

which is what we actually compute in the data.

With estimates of η^d, σ^M_{ft}, and data on χ^d_{ft}, we can compute the change in destination-specific and total revenues of French firms for a change in τ^d_t. We use R^d_{ft}/R_{ft} as an approximation for χ^d_{ft} and we use our estimate of η from the domestic market for all destinations.

Since the multi-market model nests the constant-VRTS model from Aw et al. (2011), we can compute the analogous prediction based on this estimator by simply setting $\theta_{ft} = 0$ in (I.18) and (I.19), and using the estimates of η from J.5.

The single-market model from Klette & Griliches (1996) is misspecified when firms in fact serve multiple destination markets. Nevertheless, to facilitate comparison across estimators, we substitute the estimated values of θ_{ft} and η^d from J.3 into (I.18) and (I.19) and compute the implied revenue elasticities. These quantifications should therefore be interpreted with caution, since the underlying empirical exercise is not fully consistent with the economic environment assumed by the model.

I.2 Elasticities with respect to homogeneous tariff cut

Starting from (I.13), and taking derivative with respect to a homogeneous change in tariffs $d\ln\tau_t = d\ln\tau_t^d$ for all d

$$\frac{d\ln Q_{ft}}{d\ln\tau_t} = \frac{\sum_k \chi_{ft}^k\left(1+\eta^k\right)}{\sum_k \chi_{ft}^k\left(-\eta^k\theta_{ft}+1\right)} < 0. \tag{I.22}$$

Taking derivatives of destination-specific flows with respect to a homogeneous change in tariffs $d\ln\tau_t = d\ln\tau_t^d$ for all d yields

$$\frac{d\ln R_{ft}^d}{d\ln\tau_t} = \left(1+\eta^d\right) + \left(1+\eta^d\right)\theta_{ft}\frac{d\ln Q_{ft}}{d\ln\tau_t} < 0 \tag{I.23}$$

for any destination d, but with $\frac{d\ln Q_{ft}}{d\ln\tau_t}$ given by (I.22).

This expression turns out to be equal to the elasticity of total firm-level revenues with respect to $d\ln\tau_t$ when η^k is homogeneous across destinations.

Taking logs of I.15 and taking derivatives w.r.t. $\ln\tau_t$

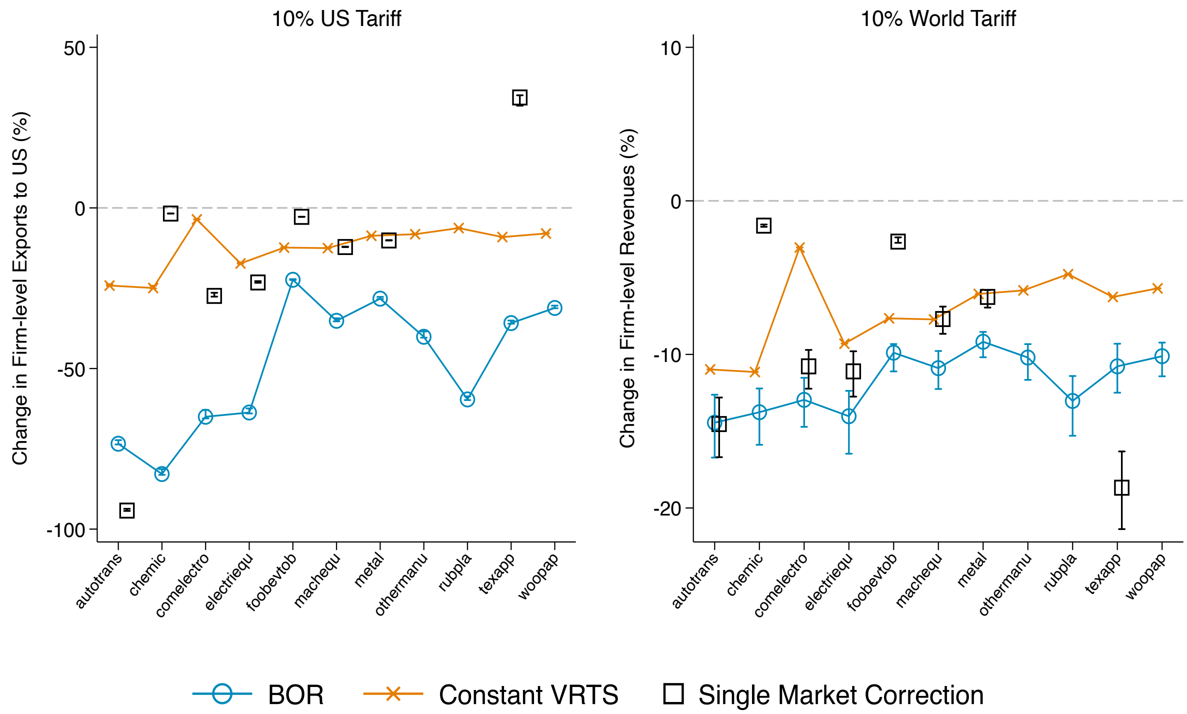
$$\begin{aligned}\frac{\ln R_{ft}}{\ln\tau_t} &= \frac{\sum_k(1+\eta^k)\left(\tau_t\right)^{1+\eta^k} Q_{ft}^{\theta_{ft}(1+\eta^k)}\Xi_{ft}^k + \sum_k\left(\tau_t\right)^{1+\eta^k}\Xi_{ft}^k\theta_{ft}(1+\eta^k)Q_{ft}^{\theta_{ft}(1+\eta^k)}\frac{d\ln Q_{ft}}{d\ln\tau_t}}{R_{ft}} \\ &= \sum_k \chi_{ft}^k(1+\eta^k) + \theta_{ft}\frac{d\ln Q_{ft}}{d\ln\tau_t}\left[\sum_k \chi_{ft}^k(1+\eta^k)\right]\end{aligned} \tag{I.24}$$

Setting $\eta = \eta^k$ for all k, and substituting with (I.22)

$$\frac{\ln R_{ft}^d}{\ln\tau_t} = \frac{\ln R_{ft}}{\ln\tau_t} = (1+\eta)\left(\frac{1+\theta_{ft}}{1-\eta\theta_{ft}}\right) \tag{I.25}$$

I.3 Quantification results

Figure I.1: Responses to Tariff Increase Experiments across Estimators



Notes. The figure reports responses of French firm exports to the US from a 10% increase in US import tariffs (left) and total firm-level revenues to a 10% increase in world-wide import tariffs (right). Estimates based on data from 2015. Markers indicate median estimates across firms within the industry. Vertical bars represent the inner quartile of estimates across firms within the industry. Results for Other manufacturing, Rubber and Plastics, and Wood and Paper for Single Market Correction are not defined, since the estimate $\eta > -1$.

J Additional Results from French Manufacturing

In this section, we first report the detailed results that are presented in the paper graphically and then report two additional sets of results. The first allows for dynamic, partial adjustment for labor and serves as a robustness check for the main results. The second set of results applies the control function method, which is just for comparison, as we expect finite sample bias and weak moments problems.

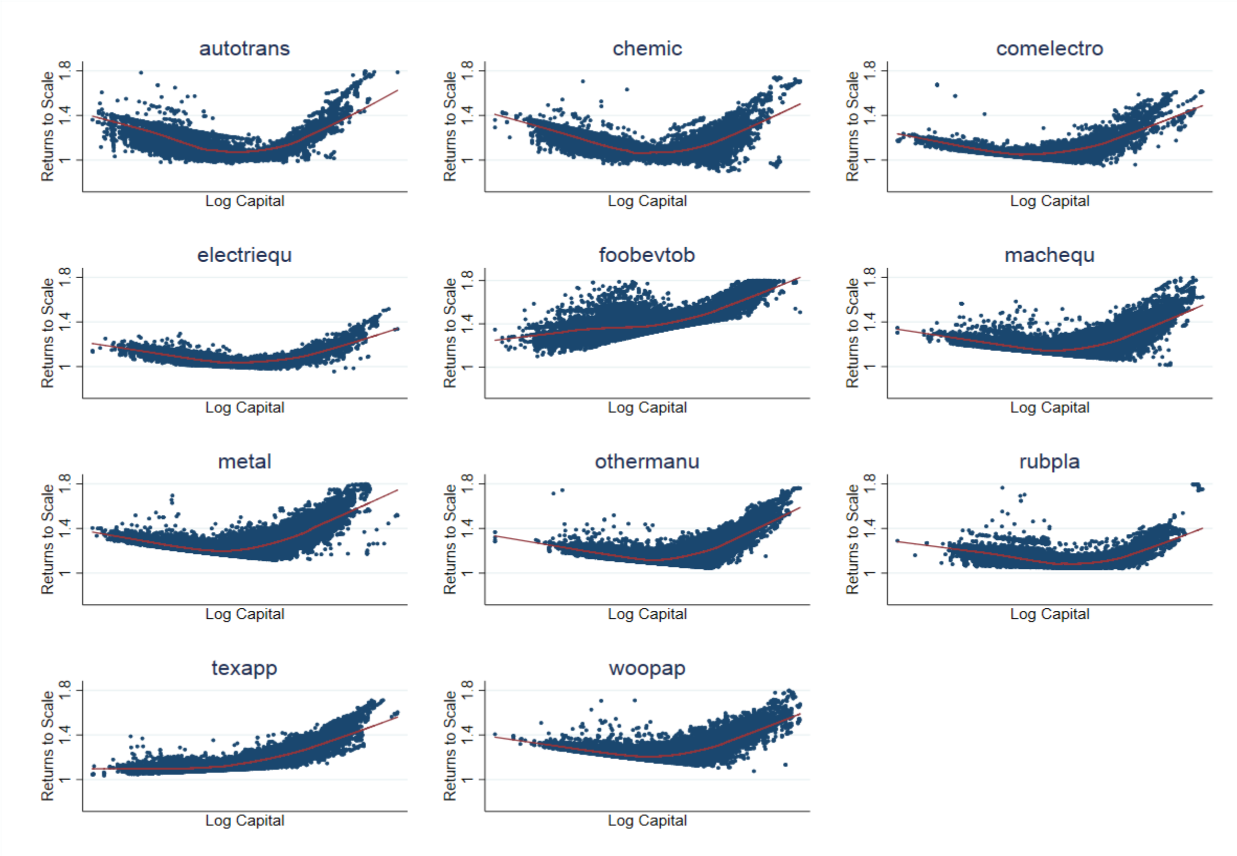
J.1 Results with pre-determined labor

Table J.1: Estimates using Multi-Market Estimator, Predetermined Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ^0	$\eta^0 = \frac{1}{(\rho^0-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.403	0.524	0.170	1.096	0.930	-14.340	0.014	0.818	0.075
	(0.008)	(0.017)	(0.011)	(0.022)	(0.018)	(4.419)	(0.003)	(0.013)	(0.014)
Chemical products	0.387	0.536	0.177	1.100	0.947	-18.815	0.013	0.875	0.105
	(0.009)	(0.017)	(0.011)	(0.017)	(0.015)	(8.004)	(0.002)	(0.016)	(0.016)
Computer, electronics	0.334	0.576	0.137	1.046	0.914	-11.675	0.019	0.845	0.119
	(0.009)	(0.017)	(0.008)	(0.025)	(0.022)	(5.379)	(0.003)	(0.008)	(0.017)
Electrical equipment	0.387	0.519	0.141	1.047	0.911	-11.229	0.015	0.839	0.095
	(0.012)	(0.018)	(0.009)	(0.029)	(0.026)	(6.433)	(0.003)	(0.011)	(0.016)
Food, beverage, tobacco	0.489	0.537	0.328	1.354	0.718	-3.541	-0.005	0.783	-0.022
	(0.009)	(0.010)	(0.007)	(0.025)	(0.014)	(0.169)	(0.002)	(0.006)	(0.010)
Machinery and equipment	0.343	0.686	0.146	1.175	0.813	-5.361	0.028	0.796	0.137
	(0.006)	(0.013)	(0.005)	(0.021)	(0.014)	(0.391)	(0.002)	(0.006)	(0.007)
Basic metal and fabricated metal	0.279	0.765	0.231	1.275	0.770	-4.342	0.008	0.845	0.053
	(0.006)	(0.018)	(0.007)	(0.029)	(0.018)	(0.324)	(0.001)	(0.004)	(0.007)
Other manufacturing	0.270	0.617	0.219	1.106	0.838	-6.182	0.025	0.852	0.167
	(0.005)	(0.012)	(0.006)	(0.021)	(0.016)	(0.634)	(0.001)	(0.006)	(0.009)
Rubber and plastic	0.377	0.528	0.201	1.106	0.901	-10.129	-0.001	0.870	-0.005
	(0.008)	(0.012)	(0.009)	(0.024)	(0.020)	(2.751)	(0.001)	(0.008)	(0.008)
Textiles, wearing apparel	0.335	0.531	0.226	1.092	0.817	-5.476	0.040	0.895	0.380
	(0.020)	(0.031)	(0.015)	(0.065)	(0.047)	(1.680)	(0.003)	(0.004)	(0.019)
Wood, paper products	0.314	0.731	0.199	1.245	0.789	-4.744	0.018	0.837	0.112
	(0.005)	(0.012)	(0.005)	(0.020)	(0.013)	(0.283)	(0.001)	(0.006)	(0.007)

Notes. The table reports estimates based on the multi-market estimator, treating labor as predetermined (like capital): average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity on the domestic market $\eta^0 = 1/(\rho^0 - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Figure J.1: Returns to Scale by Industry



Notes. This figure presents estimated returns to scale via the multi-market factor shares estimator by firm-year against log of capital. These are built using the same specification in Table J.1, where labor is assumed to be pre-determined.

Table J.2: Estimates using no Demand Correction, Predetermined Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.363	0.544	0.147	1.054	-	-	0.043	0.932	0.629
	(0.004)	(0.010)	(0.007)	(0.009)			(0.004)	(0.010)	(0.057)
Chemical products	0.367	0.512	0.165	1.045	-	-	0.024	0.968	0.749
	(0.004)	(0.013)	(0.010)	(0.006)			(0.004)	(0.006)	(0.077)
Computer, electronics	0.300	0.534	0.148	0.981	-	-	0.038	0.939	0.627
	(0.005)	(0.010)	(0.006)	(0.007)			(0.003)	(0.006)	(0.048)
Electrical equipment	0.340	0.485	0.145	0.970	-	-	0.034	0.953	0.713
	(0.004)	(0.010)	(0.008)	(0.007)			(0.004)	(0.005)	(0.062)
Food, beverage, tobacco	0.346	0.401	0.160	0.907	-	-	0.092	0.922	1.187
	(0.001)	(0.004)	(0.002)	(0.005)			(0.003)	(0.002)	(0.039)
Machinery and equipment	0.283	0.558	0.109	0.950	-	-	0.054	0.880	0.451
	(0.003)	(0.006)	(0.003)	(0.007)			(0.002)	(0.007)	(0.019)
Basic metal and fabricated metal	0.218	0.596	0.155	0.969	-	-	0.035	0.930	0.501
	(0.001)	(0.004)	(0.003)	(0.002)			(0.001)	(0.002)	(0.012)
Other manufacturing	0.224	0.531	0.168	0.922	-	-	0.039	0.943	0.691
	(0.003)	(0.006)	(0.004)	(0.007)			(0.002)	(0.002)	(0.039)
Rubber and plastic	0.334	0.493	0.181	1.007	-	-	0.016	0.971	0.551
	(0.003)	(0.006)	(0.005)	(0.004)			(0.002)	(0.002)	(0.035)
Textiles, wearing apparel	0.269	0.456	0.170	0.894	-	-	0.041	0.951	0.829
	(0.002)	(0.005)	(0.003)	(0.005)			(0.002)	(0.002)	(0.025)
Wood, paper products	0.251	0.579	0.147	0.977	-	-	0.035	0.950	0.700
	(0.001)	(0.004)	(0.003)	(0.003)			(0.002)	(0.002)	(0.021)

Notes. The table reports estimates without correcting for demand at all, using the factor share approach, treating labor as predetermined (like capital): average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.3: Estimates using Single-Market Estimator (SMC), Predetermined Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.376	0.557	0.161	1.094	0.966	-29.731	0.024	0.795	0.117
	(0.005)	(0.011)	(0.007)	(0.009)	(0.004)	(3.731)	(0.003)	(0.019)	(0.012)
Chemical products	2.447	3.411	1.127	6.986	0.150	-1.176	0.097	0.872	0.752
	(0.426)	(0.605)	(0.208)	(1.220)	(0.025)	(0.035)	(0.020)	(0.018)	(0.173)
Computer, electronics	0.392	0.711	0.184	1.287	0.764	-4.239	0.031	0.837	0.193
	(9.733)	(21.734)	(5.667)	(37.134)	(0.099)	(1.287)	(1.277)	(0.010)	(9.842)
Electrical equipment	0.468	0.689	0.186	1.343	0.726	-3.656	0.030	0.834	0.184
	(0.021)	(0.030)	(0.012)	(0.054)	(0.029)	(0.393)	(0.003)	(0.012)	(0.017)
Food, beverage, tobacco	1.579	1.812	0.773	4.164	0.219	-1.281	0.158	0.849	1.046
	(0.059)	(0.071)	(0.033)	(0.160)	(0.008)	(0.014)	(0.008)	(0.005)	(0.051)
Machinery and equipment	0.500	0.990	0.194	1.684	0.566	-2.302	0.061	0.793	0.297
	(0.077)	(0.157)	(0.041)	(0.274)	(0.048)	(0.164)	(0.007)	(0.014)	(0.081)
Basic metal and fabricated metal	0.421	1.090	0.349	1.861	0.517	-2.071	0.026	0.850	0.177
	(0.012)	(0.028)	(0.013)	(0.051)	(0.015)	(0.063)	(0.001)	(0.006)	(0.011)
Other manufacturing	-0.370	-0.866	-0.324	-1.560	-0.605	-0.623	-0.036	0.876	-0.291
	(0.075)	(0.178)	(0.062)	(0.316)	(0.182)	(0.050)	(0.008)	(0.007)	(0.053)
Rubber and plastic	-0.849	-1.243	-0.480	-2.572	-0.393	-0.718	-0.008	0.872	-0.064
	(0.092)	(0.142)	(0.045)	(0.277)	(0.051)	(0.024)	(0.002)	(0.009)	(0.016)
Textiles, wearing apparel	0.180	0.308	0.113	0.601	1.495	2.020	0.027	0.894	0.255
	(0.003)	(0.006)	(0.003)	(0.011)	(0.026)	(0.105)	(0.001)	(0.004)	(0.009)
Wood, paper products	-39.831	-90.764	-24.737	-155.332	-0.006	-0.994	-2.691	0.850	-17.947
	(1682.106)	(3850.091)	(1048.955)	(6581.140)	(0.056)	(0.056)	(111.573)	(0.007)	(754.691)

Notes. The table reports estimates based on the single-market estimator *à la* Klette & Griliches (1996), using the factor share approach, treating labor as predetermined (like capital): average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.4: Estimates using Single-Market Estimator (SMC) on Sample of Non-Exporters, Predetermined Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.376	0.568	0.171	1.115	0.966	-29.155	-	0.801	-
	(0.005)	(0.011)	(0.007)	(0.010)	(0.004)	(3.631)		(0.019)	
Chemical products	2.704	3.841	1.282	7.826	0.136	-1.157	-	0.872	-
	(0.542)	(0.783)	(0.271)	(1.579)	(0.025)	(0.034)		(0.018)	
Computer, electronics	0.465	0.858	0.238	1.560	0.645	-2.816	-	0.843	-
	(0.180)	(0.388)	(0.112)	(0.679)	(0.108)	(0.576)		(0.009)	
Electrical equipment	0.479	0.715	0.209	1.404	0.710	-3.448	-	0.838	-
	(0.022)	(0.032)	(0.013)	(0.058)	(0.029)	(0.358)		(0.012)	
Food, beverage, tobacco	1.522	1.760	0.782	4.063	0.228	-1.295	-	0.856	-
	(0.056)	(0.068)	(0.032)	(0.154)	(0.008)	(0.014)		(0.005)	
Machinery and equipment	0.513	1.035	0.219	1.767	0.551	-2.227	-	0.804	-
	(0.103)	(0.213)	(0.059)	(0.375)	(0.052)	(0.163)		(0.014)	
Basic metal and fabricated metal	0.428	1.118	0.368	1.914	0.509	-2.037	-	0.853	-
	(0.012)	(0.029)	(0.014)	(0.054)	(0.015)	(0.062)		(0.006)	
Other manufacturing	-0.321	-0.773	-0.291	-1.385	-0.697	-0.589	-	0.881	-
	(0.066)	(0.162)	(0.058)	(0.286)	(0.309)	(0.148)		(0.006)	
Rubber and plastic	-0.847	-1.248	-0.482	-2.577	-0.394	-0.717	-	0.871	-
	(0.091)	(0.142)	(0.045)	(0.276)	(0.050)	(0.024)		(0.009)	
Textiles, wearing apparel	0.176	0.315	0.127	0.618	1.526	1.900	-	0.913	-
	(0.003)	(0.007)	(0.004)	(0.012)	(0.029)	(0.104)		(0.003)	
Wood, paper products	-5.893	-13.578	-3.828	-23.300	-0.043	-0.959	-	0.853	-
	(974.575)	(2249.828)	(626.628)	(3850.850)	(0.056)	(0.053)		(0.007)	

Notes. The table reports estimates based on the single-market estimator *à la* Klette & Griliches (1996), using the factor share approach, treating labor as predetermined (like capital), where we restrict the sample to non-exporting firms: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.5: Estimates using the approach of Aw et al. (2011), Predetermined Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ^0	$\eta^0 = \frac{1}{(\rho^0-1)}$	ρ^x	$\eta^x = \frac{1}{(\rho^x-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	1.000	1.016	0.245	2.261	0.735	-3.779	0.610	-2.562	0.036	0.814	0.190
		(0.944)	(0.228)	(1.170)	(0.001)	(0.296)	(0.002)	(0.071)	(0.040)	(0.010)	(0.213)
Chemical products	1.000	0.930	0.341	2.271	0.743	-3.887	0.402	-1.671	-0.018	0.866	-0.134
		(0.111)	(0.041)	(0.143)	(0.001)	(0.187)	(0.003)	(0.023)	(0.009)	(0.007)	(0.068)
Computer, electronics	1.000	2.671	0.591	4.262	0.267	-1.365	0.488	-1.954	-0.034	0.825	-0.192
		(0.643)	(0.151)	(0.785)	(0.003)	(0.009)	(0.002)	(0.028)	(0.023)	(0.009)	(0.136)
Electrical equipment	1.000	1.040	0.298	2.338	0.656	-2.907	0.494	-1.975	0.022	0.842	0.138
		(0.146)	(0.050)	(0.188)	(0.002)	(0.126)	(0.002)	(0.033)	(0.009)	(0.011)	(0.056)
Food, beverage, tobacco	1.000	0.923	0.534	2.458	0.569	-2.322	0.608	-2.554	0.328	0.793	1.585
		(0.055)	(0.032)	(0.087)	(0.000)	(0.009)	(0.001)	(0.029)	(0.021)	(0.003)	(0.096)
Machinery and equipment	1.000	1.222	0.248	2.469	0.573	-2.342	0.425	-1.739	0.055	0.789	0.262
		(0.108)	(0.025)	(0.132)	(0.001)	(0.025)	(0.001)	(0.007)	(0.007)	(0.004)	(0.035)
Basic metal and fabricated metal	1.000	1.493	0.407	2.900	0.476	-1.909	0.479	-1.921	0.030	0.817	0.163
		(0.334)	(0.092)	(0.426)	(0.001)	(0.011)	(0.001)	(0.007)	(0.009)	(0.004)	(0.048)
Other manufacturing	1.000	1.290	0.523	2.813	0.462	-1.858	0.465	-1.869	0.098	0.814	0.526
		(0.062)	(0.028)	(0.088)	(0.000)	(0.006)	(0.001)	(0.007)	(0.009)	(0.005)	(0.047)
Rubber and plastic	1.000	1.486	0.925	3.411	0.393	-1.648	0.457	-1.842	-0.036	0.857	-0.255
		(0.181)	(0.110)	(0.288)	(0.001)	(0.004)	(0.001)	(0.009)	(0.007)	(0.004)	(0.050)
Textiles, wearing apparel	1.000	1.107	0.518	2.626	0.488	-1.954	0.418	-1.717	0.149	0.866	1.114
		(0.077)	(0.038)	(0.111)	(0.001)	(0.008)	(0.000)	(0.004)	(0.014)	(0.004)	(0.094)
Wood, paper products	1.000	1.508	0.526	3.034	0.454	-1.830	0.569	-2.318	0.084	0.807	0.436
		(0.158)	(0.057)	(0.213)	(0.000)	(0.005)	(0.001)	(0.021)	(0.010)	(0.005)	(0.053)

Notes. The table reports estimates based on the estimator of Aw et al. (2011), using the factor share approach, treating labor as predetermined (like capital): average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the domestic demand elasticity $\eta^0 = 1/(\rho^0 - 1)$, the demand elasticity on foreign markets $\eta^x = 1/(\rho^x - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses, except for η^0, ρ^0, η^x, ρ^x, where standard errors are clustered by firm but need not be bootstrapped.

J.2 Results allowing partial adjustment for labor

We present results that allow labor to partially adjust to contemporaneous productivity and demand shocks. This permits entertaining the possibility that firms have the ability to flexibly adjust part of employment, while another part is pre-determined within the period. This is particularly interesting in the context of the French dual labor market, which features both short-term fixed employment contracts and long term indefinite duration contracts. Even though the French dual labor market is known for its rigidity, it is certainly possible that French firms adjust the current labor stock to contemporaneous supply and demand shocks, even if not completely (Saint-Paul, 1996; Reshef et al., 2022). To allow for this possibility, we need only adjust the factor shares second step moment condition (23) to replace all contemporaneous labor measures with lagged measures.

We report detailed results for the four models estimated above (multi-market, no demand correction, single market, and single market with no exporters) in Tables J.6–J.9. The results for the multi-market estimator are quite similar to our main specification, in which we treat labor as quasi-fixed. We estimate slightly higher returns to scale and lower elasticities of demand for the multi-market estimator, and a slightly larger range of values for LBE (-0.017 to 0.042). Estimates based on the single-market correction estimator still vary wildly by industry, and the estimates of LBE still appear biased up in the two misspecified estimators.

Table J.6: Estimates using Multi-Market Estimator, Partially Adjusting Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ^0	$\eta^0 = \frac{1}{(\rho^0-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.402	0.615	0.116	1.133	0.934	-15.074	0.004	0.788	0.020
	(0.008)	(0.026)	(0.012)	(0.025)	(0.018)	(4.285)	(0.004)	(0.011)	(0.016)
Chemical products	0.395	0.720	0.096	1.211	0.928	-13.961	-0.000	0.827	-0.003
	(0.009)	(0.031)	(0.013)	(0.024)	(0.014)	(3.165)	(0.003)	(0.011)	(0.018)
Computer, electronics	0.338	0.697	0.078	1.113	0.903	-10.261	0.008	0.815	0.046
	(0.009)	(0.031)	(0.010)	(0.032)	(0.022)	(3.146)	(0.004)	(0.007)	(0.019)
Electrical equipment	0.390	0.640	0.080	1.110	0.903	-10.309	0.009	0.803	0.048
	(0.011)	(0.030)	(0.010)	(0.036)	(0.026)	(3.210)	(0.004)	(0.010)	(0.018)
Food, beverage, tobacco	0.488	0.819	0.193	1.501	0.718	-3.545	0.003	0.715	0.011
	(0.010)	(0.016)	(0.005)	(0.029)	(0.015)	(0.181)	(0.002)	(0.004)	(0.008)
Machinery and equipment	0.351	0.842	0.090	1.283	0.794	-4.862	0.017	0.766	0.072
	(0.006)	(0.021)	(0.004)	(0.026)	(0.015)	(0.325)	(0.002)	(0.005)	(0.007)
Basic metal and fabricated metal	0.286	0.955	0.130	1.371	0.751	-4.014	0.003	0.805	0.016
	(0.005)	(0.022)	(0.005)	(0.027)	(0.014)	(0.217)	(0.002)	(0.003)	(0.008)
Other manufacturing	0.291	0.943	0.135	1.369	0.777	-4.476	0.008	0.773	0.037
	(0.005)	(0.027)	(0.005)	(0.031)	(0.015)	(0.301)	(0.003)	(0.004)	(0.011)
Rubber and plastic	0.393	0.743	0.117	1.252	0.865	-7.391	-0.017	0.816	-0.092
	(0.008)	(0.020)	(0.006)	(0.027)	(0.017)	(1.014)	(0.002)	(0.006)	(0.011)
Textiles, wearing apparel	0.372	0.757	0.176	1.305	0.736	-3.783	0.042	0.850	0.284
	(0.018)	(0.059)	(0.010)	(0.074)	(0.034)	(0.513)	(0.003)	(0.005)	(0.019)
Wood, paper products	0.326	0.975	0.104	1.405	0.761	-4.192	0.011	0.780	0.048
	(0.005)	(0.018)	(0.004)	(0.022)	(0.011)	(0.191)	(0.002)	(0.004)	(0.007)

Notes. The table reports estimates based on the multi-market estimator, allowing labor to partially adjust to current period shocks: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity on the domestic market $\eta^0 = 1/(\rho^0 - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.7: Estimates using Single-Market Estimator (SMC), Partially Adjusting Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.377	0.633	0.126	1.137	0.963	-26.725	0.012	0.772	0.053
	(0.005)	(0.018)	(0.010)	(0.012)	(0.005)	(3.967)	(0.003)	(0.016)	(0.011)
Chemical products	13.778	26.114	3.268	43.159	0.027	-1.027	-0.164	0.834	-0.987
	(39.681)	(77.447)	(7.914)	(124.925)	(0.100)	(0.080)	(0.766)	(0.015)	(4.586)
Computer, electronics	0.419	0.862	0.151	1.432	0.715	-3.515	0.020	0.811	0.108
	(0.236)	(0.609)	(0.086)	(0.929)	(0.100)	(0.738)	(0.018)	(0.009)	(0.118)
Electrical equipment	0.480	0.824	0.134	1.438	0.709	-3.435	0.022	0.800	0.109
	(0.024)	(0.052)	(0.016)	(0.070)	(0.032)	(0.396)	(0.005)	(0.013)	(0.021)
Food, beverage, tobacco	0.655	1.251	0.126	2.033	0.529	-2.122	0.077	0.775	0.341
	(0.025)	(0.035)	(0.014)	(0.069)	(0.022)	(0.118)	(0.004)	(0.003)	(0.017)
Machinery and equipment	0.555	1.223	0.171	1.949	0.509	-2.037	0.056	0.776	0.251
	(0.116)	(0.289)	(0.043)	(0.448)	(0.051)	(0.143)	(0.009)	(0.011)	(0.066)
Basic metal and fabricated metal	0.330	0.840	0.255	1.424	0.660	-2.945	0.023	0.807	0.121
	(0.008)	(0.033)	(0.012)	(0.041)	(0.017)	(0.145)	(0.002)	(0.004)	(0.010)
Other manufacturing	2.068	6.287	1.334	9.690	0.108	-1.121	0.225	0.804	1.148
	(6.995)	(21.319)	(4.356)	(32.668)	(0.066)	(0.089)	(0.660)	(0.005)	(3.486)
Rubber and plastic	-0.847	-1.622	-0.313	-2.782	-0.394	-0.717	0.020	0.828	0.115
	(0.092)	(0.164)	(0.036)	(0.288)	(0.059)	(0.026)	(0.003)	(0.008)	(0.015)
Textiles, wearing apparel	0.169	0.289	0.114	0.571	1.595	1.681	0.028	0.854	0.192
	(0.003)	(0.012)	(0.007)	(0.012)	(0.034)	(0.096)	(0.002)	(0.006)	(0.013)
Wood, paper products	-25.843	-72.424	-10.339	-108.606	-0.010	-0.990	-1.274	0.810	-6.719
	(210.479)	(595.062)	(80.585)	(886.105)	(0.046)	(0.045)	(9.881)	(0.005)	(51.502)

Notes. The table reports estimates based on the single-market estimator *à la* Klette & Griliches (1996), using the factor share approach, allowing labor to partially adjust to current period shocks: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.8: Estimates using Single-Market Estimator (SMC) on Sample of Non-Exporters, Partially Adjusting Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.378	0.647	0.126	1.151	0.962	-26.371	-	0.775	-
	(0.005)	(0.018)	(0.010)	(0.013)	(0.006)	(3.963)		(0.015)	
Chemical products	12.475	23.534	2.976	38.985	0.029	-1.030	-	0.834	-
	(132.540)	(248.825)	(37.153)	(418.182)	(0.066)	(0.061)		(0.014)	
Computer, electronics	0.458	0.970	0.165	1.593	0.655	-2.900	-	0.813	-
	(0.151)	(0.391)	(0.056)	(0.595)	(0.106)	(0.517)		(0.009)	
Electrical equipment	0.488	0.863	0.139	1.489	0.697	-3.304	-	0.801	-
	(0.024)	(0.054)	(0.017)	(0.073)	(0.032)	(0.370)		(0.012)	
Food, beverage, tobacco	0.627	1.224	0.124	1.974	0.553	-2.236	-	0.777	-
	(0.025)	(0.034)	(0.014)	(0.068)	(0.023)	(0.146)		(0.003)	
Machinery and equipment	0.580	1.319	0.186	2.084	0.487	-1.951	-	0.783	-
	(0.143)	(0.368)	(0.054)	(0.564)	(0.052)	(0.135)		(0.010)	
Basic metal and fabricated metal	0.333	0.869	0.260	1.463	0.653	-2.880	-	0.808	-
	(0.009)	(0.033)	(0.012)	(0.043)	(0.017)	(0.141)		(0.004)	
Other manufacturing	2.717	8.599	1.716	13.033	0.082	-1.090	-	0.805	-
	(46.854)	(149.803)	(27.376)	(224.029)	(0.087)	(0.096)		(0.005)	
Rubber and plastic	-0.858	-1.618	-0.319	-2.795	-0.389	-0.720	-	0.829	-
	(0.093)	(0.167)	(0.036)	(0.293)	(0.058)	(0.026)		(0.008)	
Textiles, wearing apparel	0.160	0.306	0.113	0.578	1.685	1.461	-	0.852	-
	(0.926)	(0.840)	(0.960)	(2.726)	(0.313)	(0.495)		(0.014)	
Wood, paper products	-8.117	-23.079	-3.279	-34.475	-0.031	-0.970	-	0.811	-
	(34.913)	(98.525)	(14.678)	(148.096)	(0.046)	(0.043)		(0.004)	

Notes. The table reports estimates based on the single-market estimator *à la* Klette & Griliches (1996), using the factor share approach, allowing labor to partially adjust to current period shocks, where we restrict the sample to non-exporting firms: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.9: Estimates using no Demand Correction, Partially Adjusting Labor Input

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.363	0.608	0.116	1.088	-	-	0.045	0.941	0.774
	(0.004)	(0.015)	(0.009)	(0.010)			(0.004)	(0.008)	(0.078)
Chemical products	0.367	0.807	0.032	1.205	-	-	0.013	0.864	0.097
	(0.004)	(0.257)	(0.124)	(0.134)			(0.078)	(0.031)	(0.865)
Computer, electronics	0.300	0.604	0.119	1.023	-	-	0.044	0.904	0.459
	(0.005)	(0.024)	(0.011)	(0.015)			(0.003)	(0.018)	(0.068)
Electrical equipment	0.340	0.573	0.105	1.019	-	-	0.052	0.923	0.678
	(0.004)	(0.034)	(0.019)	(0.019)			(0.006)	(0.031)	(0.143)
Food, beverage, tobacco	0.346	0.634	0.070	1.051	-	-	0.142	0.860	1.010
	(0.001)	(0.011)	(0.006)	(0.007)			(0.005)	(0.004)	(0.046)
Machinery and equipment	0.283	0.624	0.085	0.992	-	-	0.056	0.857	0.389
	(0.003)	(0.005)	(0.003)	(0.004)			(0.002)	(0.007)	(0.020)
Basic metal and fabricated metal	0.218	0.628	0.129	0.974	-	-	0.042	0.895	0.396
	(0.001)	(0.010)	(0.005)	(0.006)			(0.002)	(0.007)	(0.022)
Other manufacturing	0.224	0.679	0.131	1.034	-	-	0.054	0.922	0.688
	(0.001)	(0.012)	(0.005)	(0.009)			(0.002)	(0.003)	(0.034)
Rubber and plastic	0.334	0.682	0.101	1.116	-	-	0.026	0.944	0.467
	(0.003)	(0.023)	(0.010)	(0.013)			(0.002)	(0.005)	(0.045)
Textiles, wearing apparel	0.269	0.439	0.191	0.899	-	-	0.045	0.948	0.874
	(0.002)	(0.021)	(0.008)	(0.014)			(0.003)	(0.003)	(0.038)
Wood, paper products	0.251	0.706	0.094	1.051	-	-	0.046	0.909	0.513
	(0.001)	(0.011)	(0.005)	(0.007)			(0.001)	(0.005)	(0.027)

Notes. The table reports estimates without correcting for demand at all, using the factor share approach, allowing labor to partially adjust to current period shocks: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

J.3 Results using the control function method in French manufacturing data

We report here estimates of production functions, demand parameters and controlled Markov processes across different models using the control function method. We consider the model without correction for demand (Table J.10), the single market model with all firms (Table J.11) and the single market model in a sample without exporters (Table J.12). In doing so we cannot apply a quasi-non-parametric approach as we did above when using the factor shares approach. Instead, we must make a slightly stronger assumption on the structure of the production function. Since the data clearly reject a Cobb-Douglas production function, we apply a translog, which is a second order approximation.

We use OLS estimates for setting the initial values for the GMM search, which is a common practice. This procedure is prone to the Ackerberg et al. (2023) critique, whereby the GMM search tends not to move away from the OLS point estimates. However, this does not restrict the results to be similar across different models. Indeed, the results are distinct across models.

From Tables J.11–J.12, we find that the models that apply some correction for demand (single market and single-market while excluding exporters) yield some really high or really low returns to scale and output elasticities, or some very high or very low demand curvatures, or both. The single market correction model yields erratic estimates of both returns to scale and demand elasticities, regardless of whether we exclude exporters. The model with no correction for demand yields mostly plausible estimates of returns to scale (except for Wood and paper products), but with quite low estimates of returns to capital and high returns to materials—a telltale sign of transmission bias. We conclude that the control function approach delivers, in practice, a poor estimator of the production function and demand parameters.

Table J.10: Estimates using No Demand Correction, Control Function Estimator

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.613	0.295	0.093	1.000	-	-	0.022	0.924	0.293
	(0.052)	(0.077)	(0.022)	(0.010)			(0.022)	(0.037)	(0.190)
Chemical products	0.594	0.291	0.117	1.002	-	-	0.022	0.892	0.203
	(0.046)	(0.106)	(0.030)	(0.037)			(0.017)	(0.047)	(0.389)
Computer, electronics	0.504	0.394	0.092	0.990	-	-	0.012	0.954	0.268
	(0.221)	(0.281)	(0.053)	(0.021)			(0.060)	(0.050)	(0.479)
Electrical equipment	0.450	0.515	0.010	0.975	-	-	0.166	0.867	1.253
	(0.181)	(0.248)	(0.040)	(0.053)			(0.070)	(0.044)	(0.581)
Food, beverage, tobacco	0.612	0.263	0.095	0.970	-	-	0.027	0.947	0.508
	(0.004)	(0.010)	(0.005)	(0.006)			(0.002)	(0.003)	(0.058)
Machinery and equipment	1.141	-0.332	0.125	0.934	-	-	-0.056	0.818	-0.308
	(0.770)	(0.866)	(0.052)	(0.049)			(0.099)	(0.037)	(0.566)
Basic metal and fabricated metal	0.384	0.467	0.125	0.976	-	-	0.011	0.958	0.263
	(0.014)	(0.022)	(0.005)	(0.006)			(0.002)	(0.002)	(0.033)
Other manufacturing	0.366	0.495	0.117	0.979	-	-	0.015	0.944	0.259
	(0.046)	(0.092)	(0.020)	(0.029)			(0.012)	(0.033)	(0.093)
Rubber and plastic	0.402	0.728	0.060	1.190	-	-	0.020	0.887	0.177
	(0.177)	(0.461)	(0.041)	(0.258)			(0.022)	(0.025)	(0.181)
Textiles, wearing apparel	0.537	0.297	0.116	0.950	-	-	0.005	0.967	0.153
	(0.032)	(0.063)	(0.011)	(0.034)			(0.012)	(0.020)	(0.194)
Wood, paper products	0.422	0.468	0.081	0.970	-	-	0.012	0.952	0.259
	(0.553)	(0.947)	(0.046)	(0.351)			(0.040)	(0.054)	(0.243)

Notes. The table reports estimates without correcting for demand at all, using the control function estimator: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.11: Estimates using Single-Market Model (SMC), Control Function Estimator

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.187	0.925	0.040	1.152	0.941	-17.006	0.008	0.832	0.048
	(0.329)	(0.471)	(0.041)	(0.108)	(0.039)	(501.349)	(0.010)	(0.019)	(0.058)
Chemical products	1.406	0.218	0.346	1.969	0.478	-1.915	0.032	0.848	0.213
	(1.147)	(1.890)	(0.227)	(2.802)	(0.215)	(1.824)	(0.097)	(0.044)	(0.641)
Computer, electronics	0.489	0.456	0.084	1.029	0.972	-36.325	0.001	0.756	0.003
	(0.014)	(0.082)	(0.014)	(0.073)	(0.080)	(93.192)	(0.002)	(0.035)	(0.011)
Electrical equipment	0.500	0.426	0.050	0.977	1.012	81.222	-0.013	0.835	-0.077
	(0.065)	(0.232)	(0.016)	(0.168)	(0.112)	(184.529)	(0.006)	(0.023)	(0.038)
Food, beverage, tobacco	1.772	1.229	0.014	3.015	0.375	-1.599	-0.009	0.799	-0.043
	(1.523)	(0.347)	(0.237)	(2.006)	(0.214)	(1.653)	(0.012)	(0.032)	(0.054)
Machinery and equipment	1.241	1.133	0.188	2.562	0.387	-1.631	-0.013	0.799	-0.062
	(0.119)	(1.140)	(0.032)	(1.156)	(0.222)	(36.679)	(0.040)	(0.015)	(0.171)
Basic metal and fabricated metal	0.671	0.045	0.212	0.928	0.924	-13.148	-0.004	0.930	-0.055
	(0.284)	(7.123)	(0.166)	(6.711)	(0.389)	(4.791)	(0.026)	(0.007)	(0.404)
Other manufacturing	1.781	1.729	0.575	4.085	0.234	-1.305	-0.016	0.795	-0.076
	(0.907)	(2.156)	(0.247)	(3.137)	(1.394)	(1.752)	(0.031)	(0.009)	(0.148)
Rubber and plastic	-1.133	-1.486	-0.198	-2.817	-0.365	-0.733	-0.003	0.803	-0.015
	(237.613)	(14.813)	(41.329)	(293.615)	(0.266)	(0.171)	(10.877)	(0.039)	(77.125)
Textiles, wearing apparel	0.121	0.501	-0.003	0.619	2.085	0.921	-0.024	0.819	-0.130
	(0.204)	(0.244)	(0.074)	(0.044)	(0.406)	(8.529)	(0.019)	(0.011)	(0.105)
Wood, paper products	0.228	-0.941	-0.002	-0.715	-2.144	-0.318	0.004	0.820	0.019
	(0.365)	(0.768)	(0.094)	(1.198)	(1.324)	(10.984)	(0.007)	(0.019)	(0.051)

Notes. The table reports estimates based on the single-market model *à la* Klette & Griliches (1996), using the control function estimator: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

Table J.12: Estimates using Single-Market Model (SMC), on Sample of Non-Exporters, Control Function Estimator

Industry	σ^M	σ^L	σ^K	RTS	ρ	$\eta = \frac{1}{(\rho-1)}$	μ	h	$\mu/(1-h)$
Autos and transport equipment	0.701	0.177	0.103	0.981	1.010	97.819	-	0.827	-
	(0.240)	(0.352)	(0.057)	(0.086)	(0.035)	(497.709)		(0.025)	
Chemical products	0.955	-0.631	0.332	0.656	1.097	10.265	-	0.849	-
	(2.550)	(4.795)	(0.675)	(6.673)	(0.136)	(0.282)		(0.051)	
Computer, electronics	0.490	0.457	0.084	1.031	0.972	-35.580	-	0.755	-
	(0.013)	(0.066)	(0.012)	(0.063)	(0.064)	(67.912)		(0.034)	
Electrical equipment	0.537	0.386	0.081	1.005	0.992	-119.814	-	0.860	-
	(0.029)	(0.103)	(0.011)	(0.089)	(0.070)	(1397.845)		(0.024)	
Food, beverage, tobacco	1.774	1.230	0.015	3.019	0.373	-1.595	-	0.799	-
	(1.287)	(0.351)	(0.231)	(1.716)	(0.300)	(0.665)		(0.034)	
Machinery and equipment	1.184	0.977	0.176	2.338	0.421	-1.728	-	0.804	-
	(0.191)	(40.298)	(0.758)	(39.411)	(0.418)	(8.657)		(0.015)	
Basic metal and fabricated metal	0.670	0.034	0.208	0.912	0.932	-14.814	-	0.930	-
	(0.158)	(1.144)	(0.081)	(1.068)	(0.463)	(5.233)		(0.007)	
Other manufacturing	0.237	-1.029	0.095	-0.697	-2.293	-0.304	-	0.800	-
	(1.383)	(3.266)	(0.286)	(4.852)	(1.731)	(0.705)		(0.009)	
Rubber and plastic	-1.238	-1.539	-0.216	-2.993	-0.342	-0.745	-	0.803	-
	(47.666)	(6.506)	(4.808)	(58.789)	(0.250)	(0.199)		(0.039)	
Textiles, wearing apparel	0.237	0.365	0.039	0.641	1.672	1.489	-	0.818	-
	(0.185)	(0.189)	(0.058)	(0.062)	(0.347)	(1.987)		(0.010)	
Wood, paper products	0.289	-1.106	0.005	-0.813	-1.912	-0.343	-	0.819	-
	(0.357)	(0.741)	(0.094)	(1.178)	(1.349)	(20.972)		(0.021)	

Notes. The table reports estimates based on the single-market model *à la* Klette & Griliches (1996), using the control function estimator, where we restrict the sample to non-exporting firms: average output elasticities σ^j for materials input ($j = M$), labor ($j = L$) and capital ($j = M$), overall returns to scale (RTS), the demand elasticity $\eta = 1/(\rho - 1)$, the coefficient to learning by exporting μ, the persistence parameter in the controlled Markov h, and the long run effect of exporting $\mu/(1-h)$. Bootstrap standard errors clustered by firm in parentheses.

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