Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Journal of Mathematical Economics 39 (2003) 299–315
JOURNAL OF Mathematical ECONOMICS
www.elsevier.com/locate/jmateco

Ellsberg's two-color experiment, portfolio inertia and ambiguity

Sujoy Mukerji [a,b], Jean-Marc Tallon [c,*]

[a] *Department of Economics, University of Oxford, Oxford OX1 3UL, UK*
[b] *University College, Oxford OX1 4BH, UK*
[c] *EUREQua, CNRS, Université Paris I Panthéon-Sorbonne, 75647, Paris Cedex 13, France*

Abstract

Results in this paper relate the observation of an interval of prices at which a decision maker (DM) strictly prefers to hold a zero position on an asset (termed "portfolio inertia") to the DM's perception of the underlying payoff relevant events as *ambiguous*, as the term is defined in [Econometrica 69 (2001) 265]. The connection between portfolio inertia and ambiguity is established without invoking a parametric preference form, such as the Choquet expected utility or the max–min multiple priors model. This allows us to draw an observable distinction between portfolio inertia that may arise purely due to first-order risk aversion type effects, such as those which could arise even if preferences were probabilistically sophisticated, and portfolio inertia that involves ambiguity perceptions.

JEL classification: D81

Keywords: Ellsberg paradox; Portfolio inertia; Testing for ambiguity aversion; Uncertainty aversion; Unforeseen contingencies; Subjective state space

1. Introduction

We shall say that a decision maker (DM) exhibits *portfolio inertia* with respect to a financial asset if he strictly prefers to maintain a zero holding rather than take a non-zero position of the asset when the price of the asset lies in a given (non-trivial) interval. A DM exhibiting portfolio inertia is one whose reservation price to sell an infinitesimal amount of an asset is strictly greater than his reservation price to buy the asset; hence, there is a price interval on which he strictly prefers to hold a zero amount of the asset. We call the interval between the two reservation prices the DM's subjective *portfolio inertia interval*. On the

* Corresponding author. Tel.: +33-1-44-07-82-04; fax: +33-1-44-07-82-31.
E-mail address: jmtallon@univ-paris1.fr (J.-M. Tallon).

0304-4068/03/$ – see front matter
doi:10.1016/S0304-4068(03)00009-0

other hand, for the DM to exhibit *no-portfolio inertia* would mean that there is a unique, subjective, *switch price*, at which the DM is indifferent between holding a "marginal" (positive or negative) unit of the asset or holding none. At any price above the switch price he strictly prefers to go short on at least some amount of the asset while below the switch price he strictly prefers to go long on a marginal unit. Results in this paper relate the presence of portfolio inertia of a DM on an asset to the DM's perception of the underlying payoff relevant events as *ambiguous*, as defined by Epstein and Zhang (2001). What is, perhaps, of note is that the connection between portfolio inertia and ambiguity is established without assuming any parametric preference form, such as the Choquet expected utility (CEU) (Schmeidler, 1989) or the max–min multiple priors model (Gilboa and Schmeidler, 1989).

We explore the link between ambiguity and portfolio inertia by analyzing the modal preference pattern observed in the Ellsberg two-color experiment. First, we show (Proposition 1) that if the Ellsberg bets were offered at a price then a DM with modal preferences would exhibit portfolio inertia for bets on draws from the urn with an unknown mixture of balls. Hence, the very same considerations (playing in the mind of the DM) that give rise to the modal preference pattern are enough to generate portfolio inertia. Next, in Proposition 2, we compare a DM's portfolio inertia across two assets. The two assets offer the same payoff possibilities but have different payoff relevant events. Suppose we were to observe that the DM exhibits portfolio inertia for one of the assets, but with respect to the other asset, either he shows no-portfolio inertia or shows a portfolio inertia interval that falls strictly inside the portfolio inertia interval for the first asset. Then, we prove, the DM's preferences over the assets share a key feature of the modal preferences observed for Ellsberg bets. The key feature is that the DM prefers either side of a bet on one event to the corresponding sides of the same bet on another event. Finally, in Proposition 3, we show that this key feature has several implications regarding the DM's ambiguity perceptions (and attitudes) towards the payoff relevant events; in particular, it implies that some payoff relevant event(s) must be ambiguous.

Why might our findings be of interest? Models with specific functional forms motivated by ideas of ambiguity aversion,[1] such as the CEU, have been used to "demonstrate" that portfolio inertia is a key empirical implication of ambiguity aversion, in turn providing an explanation for many puzzling regularities.[2] However, these demonstrations only link a parametric preference form to portfolio inertia, not ambiguity per se. While it is not even clear if it is the algebra of the functional form which is really responsible for the demonstrated link, the confusion actually goes a lot deeper; it is conceptual. Portfolio inertia can be generated by any preference order that involves first-order risk aversion—such as preferences representing rank dependent expected utility (RDEU) (Quiggin, 1982), betweenness (Dekel, 1986), disappointment aversion (Gul, 1991) or the weighted utility model (Chew,

[1] An alternative terminology for ambiguity aversion is uncertainty aversion, a term used in the pioneering papers of this literature, Gilboa (1987), Gilboa and Schmeidler (1989), Schmeidler (1989). The term ambiguity aversion goes back a long way too, at least to Ellsberg (1961); recently, many researchers have returned to this term, as for instance, Epstein and Zhang (2001), Ghirardato and Marinacci (2002).

[2] Indeed this key, in one form or the other, such as kinkedness of the indifference curve graphing the CEU and max–min multiple prior functionals or the non-additivity of the Choquet integral, is present in almost all applications of these models. The list of papers obtaining empirical implications using this key ingredient include Dow and Werlang (1992), Epstein and Wang (1994, 1995), Ghirardato and Katz (2000), Mukerji (1998), Mukerji and Tallon (2000, 2001).

1989), (Chew et al., 1993), which have nothing to do with ambiguity since they are probabilistically sophisticated. Graphically, in the case of binary gambles, a preference functional that embodies second but not first-order risk aversion has indifference curves that are tangential to the actuarially fair market line at certainty; in the case of first-order risk aversion, there is a kink at the certainty line. Further, it has been widely observed (see, e.g. Segal and Spivak, 1990; Epstein, 1992; Machina, 2000) that this kinked nature of the functional holds the key to the important empirical implications of theories such as RDEU. Indeed Epstein and Zhang (2001) showed that CEU (even with convex capacities) does not necessarily have anything to do with ambiguity attitudes, a result which echoes an earlier finding by Wakker (1990) who showed that the class of subjective RDEU maximizers is identical to the class of probabilistically sophisticated CEU maximizers. Hence, derivation of empirical implications using parametric models begs two questions, at the least. One, "Can portfolio inertia, generally, be caused by ambiguity attitudes pure and simple, i.e. without the aid of effects arising from (probabilistically sophisticated) first-order risk aversion?" And if so, two, "Is there a pattern of portfolio inertia that can arise *only* if ambiguity is involved?" Proposition 1, in conjunction with Proposition 3, answers the first question. Proposition 2, in conjunction with Proposition 3, answers the second question and in addition, generates insights on the conceptual distinctiveness of the portfolio inertia arising from ambiguity and a test to determine whether or not a DM considers an arbitrary event as ambiguous. As will be seen the test has one remarkable feature: it uses acts whose payoffs may be entirely subjective to the DM and not known to the observer.

The rest of the paper is organized as follows. Section 2 concentrates on some technical preliminaries along with a description of the formal setting; Section 3 explains the formal propositions; Section 4 concludes, with a discussion of the implication of the propositions. Formal proof of the propositions are to be found in the Appendix A.

2. Preliminaries

Consider a subject (DM) choosing among Savage acts $f : S \to \mathbb{R}$, where S denotes the state space and $\mathbb{R}$ the set of real payoffs. The set of all acts f is denoted by $\mathcal{F}$. It will be assumed that the DM has a well defined, complete, and continuous (i.e. whose upper contour sets are closed) preference ordering, $\succsim$, over such acts. Set $S = \{s_1, s_2, s_3, s_4\}$; it is a maintained assumption throughout the paper that S is the product space generated by the payoff relevant events of two families of acts, one of which is measurable with respect to the partition $\{\{s_1, s_2\}, \{s_3, s_4\}\}$ while the other is measurable with respect to $\{\{s_1, s_3\}, \{s_2, s_4\}\}$. The payoff vector $(a_i)_{i=1,\dots,4}$ describes an act $\mathbf{a}$ that pays a_i conditional on the event $\{s_i\}$, $i = 1, \dots, 4$; correspondingly, $\lambda\mathbf{a}$, $\lambda \in \mathbb{R}$, denotes the act that pays λa_i conditional on the event $\{s_i\}$, $i = 1, \dots, 4$. The state space S and the two families of acts will be used to model two different, hypothetical, decision theoretic "experiments". In both experiments the DM is assumed to have an initial wealth W. Since W is assumed to be constant across the contingencies, we suppress it w.l.o.g. in our notation for final payoffs obtained from any choice.

In one experiment, we think of S as the product space generated by an Ellsberg experiment. Thus, letting the two colors be red (R) and black (B), and writing first the outcome of the random drawing from the urn with known (50–50) proportion of red and black balls and next the outcome of the draw from the urn with unknown proportion, we have $s_1 = \text{RR}$, $s_2 = \text{RB}$, $s_3 = \text{BR}$, $s_4 = \text{BB}$. Hence, bets on the draw from the urn with known (respectively, unknown) proportion of balls are acts measurable with respect to $\{\{s_1, s_2\}, \{s_3, s_4\}\}$ (respectively, $\{\{s_1, s_3\}, \{s_2, s_4\}\}$). The DM's task is to express his preferences between the bets on offer.

The other experiment involves the DM making a portfolio choice with respect to an asset offered at a price p. The portfolio decision will be the amount of asset to hold, allowing for zero, negative, and fractional holdings. We will compare the DM's behavior across two "replications" of the experiment, the two replications using two different assets. One asset will have payoffs measurable with respect to $\{\{s_1, s_2\}, \{s_3, s_4\}\}$, while the other will have payoffs measurable with respect to $\{\{s_1, s_3\}, \{s_2, s_4\}\}$. Thus, in this second experiment, S is the product space generated by payoff relevant events of the two assets.

The following axiom, to be invoked in the proof of Proposition 2, will be assumed to hold for the preference ordering $\succsim$. The axiom essentially asserts that if a DM prefers a small fraction of an act f to a small fraction of an act g then the direction of the preference remains unaltered when the DM considers *small* fractions of acts $f+c$ and $g+c$ where c is a constant act (i.e. c pays off the same amount irrespective of the resolution of uncertainty).[3]

Axiom 1. Let f, g be two acts and c be a constant act and $\bar{\lambda}, \bar{\lambda}' \in \mathbb{R}_{++}$. If for all $\lambda \in (0, \bar{\lambda})$, $\lambda(f + c) \succ \lambda(g + c)$ then there exists $\bar{\lambda}'$ such that for all $\lambda' \in (0, \bar{\lambda}')$, $\lambda'(f) \succ \lambda'(g)$.

The following formal notion of ambiguity, due to Epstein and Zhang, will be applied throughout the paper.[4] At the heart of the idea is the notion that a DM's beliefs are untainted by ambiguity if his ranking of any two events A and B in terms of likelihoods (derived behavioristically, say, from the ranking of acts measurable w.r.t. the events A and B, respectively) is independent of how payoffs vary across states lying in the complement of $A \cup B$.

Definition 1 (Ambiguity). (Epstein and Zhang, 2001) Let A, B, T, T^c be subsets of S. An event T is *unambiguous* if: (a) for all disjoint subevents A, B of T^c, acts h, and

[3] It is relatively straightforward to show that the axiom holds for EU, CEU and MMEU preferences so long as the relevant utility index, u, is a smooth function. The sketch of the argument for EU is as follows. Let f, g and c be acts as in Axiom 1 and assume that $\mathbb{E}u(\lambda(f+c)) > \mathbb{E}u(\lambda(g+c))$ for all $\lambda \in (0, \bar{\lambda})$.
Then, by a Taylor expansion at 0, it is the case that $\mathbb{E}\lambda(f+c)u'(0) > \mathbb{E}\lambda(g+c)u'(0)$ and hence, since c is a constant act, $\mathbb{E}f > \mathbb{E}g$. Now, assume that there exists $\bar{\lambda}'$ such that for all $\lambda' \in (0, \bar{\lambda}')$, $\lambda'(f) \prec \lambda'(g)$, i.e. $\mathbb{E}u(\lambda' f) < \mathbb{E}u(\lambda' g)$. A similar argument, applying Taylor expansion as above, shows that $\mathbb{E}f < \mathbb{E}g$, a contradiction. The arguments for CEU and MMEU are very similar, a point that is evident once one notices that $\lambda(f+c)$ and $\lambda' f$ are collinear (and, thus, also comonotonic) acts for $\lambda, \lambda' > 0$.

[4] Ghirardato and Marinacci (2002) provide an alternative definition of ambiguous events. A main reason why we do not adopt the alternative definition is that, while a principal point of this paper is to separate the empirical implication of non-expected utility theories that do depart from probabilistic sophistication from those theories that do not, Ghirardato and Marinacci's definition does not always allow such a separation. It would be beyond the scope of this paper to enter into an extensive discussion comparing the two definitions but we do want to note that the two definitions do agree, in all essential respects, in the Ellsberg two-color experiment. Since the setting of this paper may viewed as a generalization of the Ellsberg experiment, we believe that adopting the alternative definition would not alter our results.

outcomes x^*, x, z, z',

$$\begin{pmatrix} x^* & \text{if} & s \in A \\ x & \text{if} & s \in B \\ h(s) & \text{if} & s \in T^{\mathrm{c}} \backslash A \cup B \\ z & \text{if} & s \in T \end{pmatrix} \succsim \begin{pmatrix} x & \text{if} & s \in A \\ x^* & \text{if} & s \in B \\ h(s) & \text{if} & s \in T^{\mathrm{c}} \backslash A \cup B \\ z & \text{if} & s \in T \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x^* & \text{if} & s \in A \\ x & \text{if} & s \in B \\ h(s) & \text{if} & s \in T^{\mathrm{c}} \backslash A \cup B \\ z' & \text{if} & s \in T \end{pmatrix} \succsim \begin{pmatrix} x & \text{if} & s \in A \\ x^* & \text{if} & s \in B \\ h(s) & \text{if} & s \in T^{\mathrm{c}} \backslash A \cup B \\ z' & \text{if} & s \in T \end{pmatrix};$$

and (b) the condition obtained if T is everywhere replaced by T^{c} in (a) is also satisfied. Otherwise, T is *ambiguous*.

The set of unambiguous acts, denoted $\mathcal{F}^{\mathrm{ua}} \subseteq \mathcal{F}$, is the set of acts measurable with respect to the sub-algebra of unambiguous events in S, denoted $\mathcal{A}$. We will *assume* that there exists a unique probability measure on $\mathcal{A}$, such that the ranking of all unambiguous acts is based on this measure, i.e. the DM's preferences on $\mathcal{F}^{\mathrm{ua}}$ are probabilistically sophisticated. We will work with the following notions of ambiguity attitudes, formulated in Epstein and Zhang (2001).

Definition 2 (Ambiguity attitude). (Epstein and Zhang, 2001) $\succsim$ is ambiguity neutral on S if there is no event $E \subseteq S$, which is ambiguous given that acts defined on S are ordered in accordance with $\succsim$. $\succsim$ is ambiguity loving if there exists a probabilistically sophisticated preorder $\succsim^{\text{p.s.}}$ such that for all $f \in \mathcal{F}$ and $h \in \mathcal{F}^{\mathrm{ua}}$

$$f \succsim^{\text{p.s.}} h \Rightarrow f \succsim h$$

$\succsim$ is ambiguity averse if there exists a probabilistically sophisticated preorder $\succsim^{\text{p.s.}}$ such that for all $f \in \mathcal{F}$ and $h \in \mathcal{F}^{\mathrm{ua}}$

$$h \succsim^{\text{p.s.}} f \Rightarrow h \succsim f$$

Next, we give formal definitions of the terms portfolio inertia and no-portfolio inertia. The formal definitions, below, correspond exactly to the verbal definitions as they appear in the opening paragraph of the paper. Before stating the definitions, we introduce a piece of notation. Consider an asset $\mathbf{a}$ with payoff vector $(a_i)_{i=1,\dots,4}$ available at a price p. Then, we write $\mathbf{a} - \mathbf{p}$ for the act

$$\begin{pmatrix} a_1 - p \\ a_2 - p \\ a_3 - p \\ a_4 - p \end{pmatrix}.$$

Further, let $\mathbf{0}$ be the constant act yielding 0 in all four states.

Definition 3 (Portfolio inertia). Consider an asset **a** with payoff vector $(a_i)_{i=1,\dots,4}$ defined on S. A DM is said to exhibit portfolio inertia for **a** if there exists an interval $(\underline{p}, \bar{p})$ ("the subjective portfolio inertia interval") such that, when a unit of asset **a** is available at a price p, the following conditions (a) and (b) are satisfied:

(a) $\forall\lambda > 0$ and $\forall p \in (\underline{p}, \bar{p})$, $\mathbf{0} \succ \lambda(\mathbf{a} - \mathbf{p})$ and $\mathbf{0} \succ \lambda(\mathbf{p} - \mathbf{a})$
(b) $\exists\, \varepsilon > 0$ such that $\forall\lambda \in (0, \varepsilon)$,

$$\text{(i)}\ \lambda(\mathbf{a} - \mathbf{p}) \succ \mathbf{0} \succ \lambda(\mathbf{p} - \mathbf{a}), \quad \text{if } p < \underline{p}$$
$$\text{(ii)}\ \lambda(\mathbf{p} - \mathbf{a}) \succ \mathbf{0} \succ \lambda(\mathbf{a} - \mathbf{p}), \quad \text{if } p > \bar{p}$$

Definition 4 (No-portfolio inertia). Consider an asset **a** with payoff vector $(a_i)_{i=1,\dots,4}$ defined on S. The DM is said to exhibit no-portfolio inertia for **a** if there exists a unique p^* (the "subjective switch price") such that, when a unit of the asset is available at a price p, the following condition is satisfied: $\exists\, \varepsilon > 0$ such that $\forall\lambda \in (0, \varepsilon)$,

$$\text{(i)}\ \lambda(\mathbf{a} - \mathbf{p}) \succ \mathbf{0} \succ \lambda(\mathbf{p} - \mathbf{a}), \quad \text{if } p < p^*$$
$$\text{(ii)}\ \lambda(\mathbf{p} - \mathbf{a}) \succ \mathbf{0} \succ \lambda(\mathbf{a} - \mathbf{p}), \quad \text{if } p > p^*$$

Accordingly, in the definition for portfolio inertia, above, the condition (a) essentially captures the idea of an interval of prices, between the (subjective) "bid" and the (subjective) "ask" price, at which the DM prefers not to hold a position. The reason for including condition (b) in the definition is that we want to concentrate on the largest interval on which the DM exhibits portfolio inertia. Hence, condition (b(i)) states that at a price below the lower bound of the interval the DM strictly prefers to go long by some positive amount and condition (b(ii)) requires that if the price were above the upper bound of the interval the DM would strictly prefer a short position. No-portfolio inertia is characterized by the existence of a unique switch price, rather than an interval: by condition (i) at a price below the level of the switch price the DM strictly prefers to go long by some positive amount and by condition (ii) if the price were above the upper bound of the interval the DM would strictly prefer a short position.

3. The propositions

We first consider the two-color Ellsberg experiment (Ellsberg, 1961). Correspondingly, S is the product space generated by the bets used in the experiment, in the manner explained in the previous section. Suppose that the payoff is w if the subject has bet correctly on the outcome of a draw and z otherwise. (In the usual telling of the Ellsberg example, $w = 100$ and $z = 0$.) **A1** and **A2**, below, are restrictions on preferences that formalize the "modal" preferences observed in the Ellsberg experiment. **A1** says, assuming $w > z$, that the DM prefers to bet on red from Urn I (known to have fifty red and fifty black balls) to betting on red from Urn II (unknown proportion of red and black balls) and also prefers to bet on black from Urn I to betting on black from Urn II. **A2** says that the subject is indifferent between bets on the draw from Urn I.

A1 Let $w, z \in X \subseteq \mathbb{R}$, $w \neq z$. Then, $\begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix} \succ \begin{pmatrix} w \\ z \\ w \\ z \end{pmatrix}, \begin{pmatrix} z \\ z \\ w \\ w \end{pmatrix} \succ \begin{pmatrix} z \\ w \\ z \\ w \end{pmatrix}$.

A2 Let $w, z \in X \subseteq \mathbb{R}$, $w \neq z$.Then, $\begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix} \sim \begin{pmatrix} z \\ z \\ w \\ w \end{pmatrix}$.

The first proposition, below, shows that the modal preferences contain key information that allow us to predict how a DM with such preferences would choose if he were offered all possible portfolio positions, including the zero position, on an asset available at a price p per unit, which pays off w and z corresponding to draws of R and B, respectively, from Urn II. The proposition shows that, given two additional assumptions about $\succsim$, **A1** and **A2** imply that there exists a non-degenerate price interval on which the DM would strictly prefer to take a zero position on the asset. The two assumptions are, (1) the (complementary) events defined by the draw from the urn with known proportion of balls are unambiguous events and (2) (weak) risk aversion, that is, the subject prefers (weakly) getting the expectation of a random variable for sure rather than the random variable itself on the domain of unambiguous acts. A simple way to see the intuition behind the result would be to consider the case where the DM behaves as an (risk averse) expected utility maximizer with respect to unambiguous acts, even though expected utility is not presumed in the second assumption. To make things yet simpler, suppose that the payoffs are such that $w = -z$ and the price of the asset is 0. Given these payoffs, clearly, going long or short on the asset is the same as betting on R or B, respectively. Note that, the expected value of a bet on the known urn is 0; hence, by risk aversion, this bet is worth no more than getting 0 for sure. By modal behavior, going long (short) on a bet on the known urn is strictly preferred to going long (short) on the asset. Therefore, any non-zero position on the asset must be worth strictly less than the worth of taking a zero position on this asset. Continuity (of the preference ordering) ensures that the strict preference for the zero position holds over a neighborhood around p, thereby showing the existence of a subjective portfolio inertia interval for the asset around p.

Proposition 1. *Assume events* $\{s_1, s_2\}$ *and* $\{s_3, s_4\}$ *are unambiguous events and that* $\succsim$ *is (weakly) risk averse on* $\mathcal{F}^{\text{ua}}$. *Suppose* $\succsim$ *satisfies* **A1** *and* **A2** *for all values of* w *and* z, *i.e. for* $X = \mathbb{R}$. *Then, corresponding to a given pair* $\{w, z\}$, *there exists an interval* $N_\varepsilon(w + z/2) = (w + z/2 - \varepsilon, w + z/2 + \varepsilon), \varepsilon > 0$, *such that for all* $p \in N_\varepsilon(w + z/2)$ *and all* $\lambda > 0$:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \lambda \begin{pmatrix} w-p \\ z-p \\ w-p \\ z-p \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \lambda \begin{pmatrix} p-w \\ p-z \\ p-w \\ p-z \end{pmatrix}$$

Next, we consider the second experiment, involving two assets, mentioned in the previous section. Let $\mathbf{a}_0$ and $\mathbf{a}$ be the two assets, each with two possible payoffs, p^+ and p^-, $p^+ > p^-$. The asset $\mathbf{a}_0$ pays off p^+ and p^- at $\{s_1, s_2\}$ and $\{s_3, s_4\}$, respectively, while $\mathbf{a}$ pays off p^+ and p^- at $\{s_1, s_3\}$ and $\{s_2, s_4\}$, respectively. First, we observe the DM declaring his preferred portfolio positions on $\mathbf{a}_0$ at various prices, which we generically denote by p. We then repeat the exercise, but now with the asset $\mathbf{a}$ instead of $\mathbf{a}_0$. The starting point of Proposition 2 is that we observe that the DM exhibits a subjective portfolio inertia interval, $(\underline{p}, \bar{p})$ for asset $\mathbf{a}$. Suppose that we observe *one* of the following: (a) the DM also exhibits a subjective portfolio inertia interval, $(\underline{p}_0, \bar{p}_0)$, for $\mathbf{a}_0$, where $(\underline{p}_0, \bar{p}_0)$ lies inside $(\underline{p}, \bar{p})$, or (b) the DM *does not* exhibit portfolio inertia with respect to $\mathbf{a}_0$ and that the switch price, p^*, is such that $p^* \in (\underline{p}, \bar{p})$. Then, Proposition 2 finds, we may conclude that, with respect to acts defined on the space S, the DM's preferences satisfy **A1**. The key step in the proof involves showing, given either assumption about how the DM behaves w.r.t. each asset *individually*, how the DM would compare a short (or long) position on one asset to a corresponding position on the other asset, if both assets were offered at the same price. It is shown, given hypothesis (a), that the DM would prefer going long on $\mathbf{a}_0$ to going long on $\mathbf{a}$ if p were below the lower bound of the subjective portfolio inertia interval for $\mathbf{a}_0$ but above the lower bound of the subjective portfolio inertia interval for $\mathbf{a}$. Similarly, the DM would prefer going short on $\mathbf{a}_0$ to going long on $\mathbf{a}$ if $p \in (\bar{p}_0, \bar{p})$, the interval between the upper bounds of the two subjective portfolio inertia intervals. Given hypothesis (b), the situation is analogous, once we think of the switch price p^* as a degenerate subjective portfolio inertia interval for $\mathbf{a}_0$. This is similar to what goes on in the Ellsberg experiment where the DM strictly prefers *each* side of a bet (say, on a red draw) from the known urn to the corresponding side of the bet on the unknown urn. Effectively, the two assets take the place of the bets on the two urns. Notice, though, that the situation with the assets is actually quite a bit more complicated, as compared to that with the Ellsberg bets. Each side of $\mathbf{a}_0$ is preferred to the corresponding side of $\mathbf{a}$, but the comparison of the long sides and of the short sides take place at different prices. The same asset at different prices, technically, represent different bets. Hence here, unlike with the Ellsberg bets, we are not considering two sides of the same bet, per se. A substantial part of the argument in the proof involves maneuvering around this complication using Axiom 1.

Proposition 2. *Consider two assets,*

$$\mathbf{a}_0 = \begin{pmatrix} p^+ \\ p^+ \\ p^- \\ p^- \end{pmatrix} \quad \text{and} \quad \mathbf{a} = \begin{pmatrix} p^+ \\ p^- \\ p^+ \\ p^- \end{pmatrix} p^+, p^- \in \mathbb{R}.$$

Suppose the DM exhibits portfolio inertia for $\mathbf{a}$ *on the interval* $(\underline{p}, \bar{p})$, *where* $p^+ > \bar{p} > \underline{p} > p^-$:

(a) *If the DM also exhibits portfolio inertia for* $\mathbf{a}_0$ *on the interval* $(\underline{p}_0, \bar{p}_0)$, *where* $\underline{p}_0 > \underline{p}$ *and* $\bar{p} > \bar{p}_0$, *then there exists a set* $X \subseteq \mathbb{R}$ *such that* **A1** *is satisfied by the DM's preference ordering* $\succsim$.

(b) *If the DM does not exhibit portfolio inertia for* $\mathbf{a}_0$ *and the switch price* p^* *lies in* $(\underline{p}, \bar{p})$, *then there exists a set* $X \subseteq \mathbb{R}$ *such that* **A1** *is satisfied by the DM's preference ordering* $\succsim$.

Are there meaningful examples of assets $\mathbf{a}_0$ and $\mathbf{a}$, whose payoffs are based on events in the "real world", to which the above proposition may apply? Next, we give examples of pairs of such assets, and the corresponding constructs of S, that will figure prominently in the discussion following in Section 4. In all the examples $\mathbf{a}_0$ is constructed the same way: it is an "artificial" asset, in the sense that the two payoff relevant events are generated artificially by spinning a "balanced" spinner. The asset is defined by dividing the dial of the spinner into two complementary sectors and associating a payoff to each sector. The actual values of the payoffs are those associated with the particular example of $\mathbf{a}$ to be considered. The DM knows the sizes of the sectors. In our first example, $\mathbf{a}$ is an asset from the "real world", which has two possible payoffs, say the possible asset prices in the next period, p^+ and p^-. It is doubtless difficult to find a real world asset with just binary payoffs. But we can always create a binary "derivative" asset, corresponding to any arbitrary real world asset, by offering (arbitrary) payoffs p^+ or p^- depending on whether or not the realization of the market price of the original asset at a given future date were above a given level. Such a derivative asset is our second example of $\mathbf{a}$. Finally, we go a step further and take any arbitrary, well defined real world event, e.g. "Bush (presently the president of the US) will win a second term," and create an asset by offering payoffs p^+ or p^- depending on whether or not the event occurs. This is our third example of $\mathbf{a}$. In each of these examples, we may take S to be the product space generated by taking the product of the pairs of payoff relevant events affecting $\mathbf{a}_0$ and $\mathbf{a}$, respectively.

A1, we recall, encapsulates the essence of the Ellsberg pattern of preferences, i.e. a DM preferring either side of a bet on one event to the corresponding sides of the same bet on another event. The first point of our final proposition shows that this feature, *in itself*, is immediate proof that the DM considers some event(s) to be ambiguous. To see the intuition here notice that, while the first preference order in **A1** suggests that the DM (assuming $w > z$) thinks $\{s_2\}$ is more likely than $\{s_3\}$, the second preference order suggests just the opposite. Notice, the change in the direction of the likelihood ranking occurred because of a change in the payoffs on events in the complement of $\{s_2\} \cup \{s_3\}$; hence, it would follow from the definition of ambiguous events that at least some event(s) in the complement of $\{s_2\} \cup \{s_3\}$ is (are) ambiguous. We can go further and pin down the "real culprit" if we were to assume (a priori) that $\{s_1, s_2\}$ and $\{s_3, s_4\}$ were unambiguous events (these events correspond to the urn with the *known* number of balls). Armed with this we can show that the events in the partition $\{s_1, s_3\}$ and $\{s_2, s_4\}$ (events corresponding to the urn with the *unknown* number of balls) are ambiguous. The same assumption also implies that the preference pattern **A1** is inconsistent with ambiguity love but consistent with ambiguity aversion. Finally, we note that an immediate corollary of Proposition 3 (a) is that $\succsim$, assuming that it satisfies **A1**, is incompatible with any parametric preference form in which choices are based on a monotonic transformation of a probability measure, e.g. the functional forms that incorporate first-order risk aversion while maintaining probabilistic sophistication. In particular, $\succsim$ is not consistent with RDEU or betweenness or disappointment aversion or the weighted utility model.

Proposition 3. *Suppose there exists a non-empty set $X \subseteq \mathbb{R}$ such that* **A1** *holds.*

(a) $\succsim$ *is not ambiguity neutral on S: in particular, at least one of* $\{s_1\}$ *and* $\{s_4\}$ *is an ambiguous event.*

(b) *Suppose, also, that* $\{s_1, s_2\}$ *and* $\{s_3, s_4\}$ *are unambiguous events, given* $\succsim$. *Then*:

   (i) *The events* $\{s_1, s_3\}$ *and* $\{s_2, s_4\}$ *are ambiguous.*

   (ii) *Condition* **A1** *violates the conditions required for* $\succsim$ *to be ambiguity loving, but is consistent with the conditions required for* $\succsim$ *to be ambiguity averse.*

4. Discussion

Propositions 2 and 3, taken together, lead to some interesting conclusions which we now discuss. Suppose a DM exhibits portfolio inertia with respect to an asset like, say, one of the three examples of **a** discussed at the end of Section 3. The propositions give two kinds of (alternative) sufficient conditions, which, if satisfied, would allow one to infer, from this observation, that the DM considers (some of) the payoff relevant events to be ambiguous. One kind of sufficient condition is that the DM behaves like an expected utility maximizer with respect to any lottery (i.e. risky acts, with "objective" probabilities) or, if at the least, w.r.t. any act whose outcomes are generated by spinning some "balanced" spinner, the DM behaves as if he were approximately risk neutral when the stakes are very small. In either case the implication would be that we could define an artificial asset $\mathbf{a}_0$ that would satisfy the conditions required in Proposition 2(b): the DM would show no-portfolio inertia behavior w.r.t. $\mathbf{a}_0$ and its switch price will lie within the portfolio inertia interval for **a** (the latter can be ensured by choosing the outcomes p^+ and p^- to obtain with appropriate probabilities). Hence, portfolio inertia (alternatively, kinkedness of the indifference curves) which is manifest *on some* domain of events (i.e. a partition of the state space) but *not all*, is distinctively due to ambiguity.

However, it may be that the DM does not behave as if he were approximately risk neutral when choosing between risky lotteries with very small stakes. This could be because his risk preferences involve first-order risk aversion arising from, say, disappointment aversion or rank dependence type of effects. Such first-order risk aversion would imply that the DM exhibits portfolio inertia for any bet, including a bet such as $\mathbf{a}_0$. Could ambiguity attitudes affect, in a distinctive way, portfolio inertia of even those DMs whose preferences are affected by first-order risk aversion? Could the effect of ambiguity be disentangled and detected in these circumstances? Proposition 2(a) shows ambiguity can be detected in portfolio inertia behavior even in the presence of first-order risk aversion. The DM may exhibit a portfolio inertia interval for $\mathbf{a}_0$, but nevertheless, we may infer that ambiguity is at work if the subjective portfolio inertia interval for **a** contains the interval[5] for $\mathbf{a}_0$.

So what can we say of the various empirical phenomena which have been linked to portfolio inertia (alternatively, to kinkedness of the indifference curve); when may the link

[5] This is not hard to check (in principle!) so long as the DM's behavior is probabilistically sophisticated on a domain of events with known probabilities (such as that generated by a balanced spinner). Then, the location of the portfolio inertia interval for $\mathbf{a}_0$ will be determined by the ratio $prob(p^+)/prob(p^-)$. Hence, by adjusting the probabilities appropriately, we can check if the portfolio inertia interval for $\mathbf{a}_0$ could lie inside that for **a**.

be justifiably made to ambiguity but less justifiably to the other theories? What the foregoing analysis suggests is that ambiguity can be seen as a contributing factor in those cases where portfolio inertia intervals exist only selectively (or at least, more pronouncedly) across certain partitions of the state space. The same point informs us of an intuition about what type of capacities (in the CEU model) distinctively model ambiguity attitudes, rather than departures from expected utility that are consistent with probabilistic sophistication. Larry Epstein, in personal communication, has conjectured that a CEU model with a product capacity which is the product of a probability and a convex capacity, can be construed as a model of ambiguity. Proposition 2 provides us a simple behavioral intuition for the conjecture. To see this, observe that, in a CEU framework, a portfolio inertia interval for $\mathbf{a}$ is possible only by having a (strictly) convex capacity on the partition $\{\{s_1, s_3\}, \{s_2, s_4\}\}$ while a no-portfolio inertia for $\mathbf{a}_0$ requires a probability on the partition $\{\{s_1, s_2\}, \{s_3, s_4\}\}$.

If, indeed, there were even one rich enough partition of the state space (e.g. a balanced spinner) over which the DM's behavior was like that of an EU maximizer, we would have a very simple way of testing whether an event was ambiguous (to the DM). Whatever the event, call it E, we offer the DM an asset that pays a certain amount if E occurs but pays nothing if E does not occur. If the DM shows portfolio inertia for the asset, we know that E is ambiguous.[6,7] The test is very simple in one significant way: the observer/experimenter does not actually have to know what the contingent payoffs are; the payoff (for E) could be a banana whose value to the DM could be entirely subjective and unknown to the observer. All that has to be observed is whether or not portfolio inertia obtains. The point is significant because it suggests the possibility of applying the idea, in future research, to develop a criterion for determining whether a particular *subjective state* (Dekel et al., 2001) were ambiguous. We close with an example which, it is hoped, will illustrate the possibility more vividly. Suppose, following the spirit of the examples in Dekel et al., we are in a world with just one objective/exogenous, future, contingent state and two commodities b and b'. Consider the contracts, $\{b, b'\}$, $\{b\}$ and $\{b'\}$, available today. The contract $\{b, b'\}$ would allow the DM to choose, when the future (objective) contingency arises, between the delivery of a unit of either commodity. However, the contracts $\{b\}$ and $\{b'\}$ do *not* allow a choice when the future arrives: $\{b\}$ delivers a unit of b and $\{b'\}$ delivers b'. Suppose the DM's preferences exhibit a preference for flexibility, i.e. $\{b, b'\} \succ \{b\}$ and $\{b, b'\} \succ \{b'\}$, revealing that the DM perceives the objective state to be partitioned into two subjective contingencies, one where he prefers b and another where he prefers b'. Hence, the contract $\{b\}$ has two possible (subjective) payoffs for the DM; one payoff corresponding to the state where b is preferred to b' and the other where it is not. Suppose $\{b\}$ were available, at a price, exactly in manner of an asset (i.e. we allow the DM to take short and long positions). Let us also assume, in the manner of some of the analysis in Dekel et al., that the DM behaves as an EU maximizer w.r.t. lotteries. Now, if we observe the DM exhibiting portfolio inertia

[6] Notice that, we are comparing, albeit indirectly, acts across two different partitions of the state space; the partition $\{\{s_1, s_3\}, \{s_2, s_4\}\}$ and the partition $\{\{s_1, s_2\}, \{s_3, s_4\}\}$. This comparison across different partitions is what is needed to make the effects of ambiguity evident from behavior, a point noted earlier in Epstein and Zhang (2001).

[7] It is worth pointing out that observing a portfolio inertia involves observing choice behavior at different prices. As was pointed out in Epstein (2000), it is necessary to observe behavior at different prices to infer the lack of probabilistic sophistication.

w.r.t. to the "asset" $\{b\}$, then we may conclude, given the analysis in this paper, that the DM perceives the subjective contingencies to be ambiguous.

Acknowledgements

We thank Michèle Cohen, Larry Epstein, Tzachi Gilboa, an anonymous referee, and a guest editor for helpful comments. Mukerji gratefully acknowledges financial support from the ESRC Research Fellowship Award R000 27 1065. Part of this work was done while Tallon was at the Department of Applied Mathematics, Universita' Ca' Foscari, Venezia, Italy.

Appendix

Proof of Proposition 1. Fix $w \neq z \in \mathbb{R}$. Since **A1** holds for $X = \mathbb{R}$, we have, for all $p > 0$ and the given w and z:

$$\begin{pmatrix} w-p \\ w-p \\ z-p \\ z-p \end{pmatrix} \succ \begin{pmatrix} w-p \\ z-p \\ w-p \\ z-p \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} z-p \\ z-p \\ w-p \\ w-p \end{pmatrix} \succ \begin{pmatrix} z-p \\ w-p \\ z-p \\ w-p \end{pmatrix} \tag{A.1}$$

Take $p = (w+z)/2$; then (A.1) implies,

$$\begin{pmatrix} \frac{1}{2}(w-z) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(z-w) \end{pmatrix} \succ \begin{pmatrix} \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \frac{1}{2}(z-w) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(w-z) \end{pmatrix} \succ \begin{pmatrix} \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \end{pmatrix} \tag{A.2}$$

Now, $\{s_1, s_2\}$ and $\{s_3, s_4\}$ are unambiguous events and therefore

$$\begin{pmatrix} \frac{1}{2}(w-z) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(z-w) \end{pmatrix}$$

and

$$\begin{pmatrix} \frac{1}{2}(z-w) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(w-z) \end{pmatrix}$$

are unambiguous acts. Let π be a probability measure such that $\succsim$ is probabilistically sophisticated with π on the set of unambiguous acts. Then **A2** implies $\pi(\{s_1, s_2\}) = \pi(\{s_3, s_4\}) = 1/2$, since $\{s_1, s_2\} \cup \{s_3, s_4\} = S$. Therefore these two acts have zero expected *value* under the probability π. Hence, by (weak) risk aversion, it is the case that

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succsim \begin{pmatrix} \frac{1}{2}(w-z) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(z-w) \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succsim \begin{pmatrix} \frac{1}{2}(z-w) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(w-z) \end{pmatrix} \tag{A.3}$$

Thus, taking (2) and (3) together,

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \begin{pmatrix} \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \begin{pmatrix} \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \\ \frac{1}{2}(z-w) \\ \frac{1}{2}(w-z) \end{pmatrix}$$

Hence, by continuity of $\succsim$, there exists an open neighborhood around $(w+z)/2$, $N_\varepsilon((w+z)/2) = ((w+z)/2-\varepsilon, (w+z)/2+\varepsilon)$, $\varepsilon > 0$, such that for all $p \in N_\varepsilon((w+z)/2)$,

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \begin{pmatrix} w-p \\ z-p \\ w-p \\ z-p \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \begin{pmatrix} p-w \\ p-z \\ p-w \\ p-z \end{pmatrix}$$

Finally, since **A1** holds for all $w \neq z \in \mathbb{R}$, it follows that for all $\lambda > 0$ and the given w and z,

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \lambda \begin{pmatrix} w-p \\ z-p \\ w-p \\ z-p \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \succ \lambda \begin{pmatrix} p-w \\ p-z \\ p-w \\ p-z \end{pmatrix} \qquad \square$$

Proof of Proposition 2.

(a) From the hypothesis we know that, if both assets are available at a price $p \in (\bar{p}_0, \bar{p})$, then the DM would strictly prefer a zero position to taking any non-zero position on **a** and will strictly prefer to go short on some amount of $\mathbf{a}_0$ rather than a zero position (see, parts (a) and (b)(ii) of the definition of portfolio inertia). Hence, for $p \in (\bar{p}_0, \bar{p}) \exists$ $\varepsilon > 0$ such that $\forall \lambda \in (0, \varepsilon)$,

$$\lambda(\mathbf{p} - \mathbf{a}) \succ \mathbf{0} \succ \lambda(\mathbf{p} - \mathbf{a}_0) \tag{A.4}$$

Fix p^*, a point in the interval $(\underline{p}_0, \bar{p}_0)$. Then, there exists $\alpha > 0$ such that $p \equiv p^* + \alpha$, and, with obvious notation, we may rewrite (A.4) as below:

$$\lambda(\mathbf{p}^* + \alpha - a) \succ \mathbf{0} \succ \lambda(\mathbf{p}^* + \alpha - a_0) \quad \text{(A.5)}$$

From Axiom 1, we may then conclude that there exists $\bar{\lambda}'$ such that for all $\lambda' \in (0, \bar{\lambda}')$

$$\lambda'(\mathbf{p}^* - \mathbf{a}) \succ \mathbf{0} \succ \lambda'(\mathbf{p}^* - \mathbf{a}_0) \quad \text{(A.6)}$$

Similarly, from the hypothesis we know that, if both assets were available at a price $p \in (\underline{p}, \underline{p}_0)$, then the DM would strictly prefer a zero position to taking any non-zero position on **a** and would strictly prefer to go long on some amount of $\mathbf{a}_0$ rather than a zero position (see, part (a) and (b)(i) of the definition of portfolio inertia). Hence, for $p \in (\underline{p}, \underline{p}_0)$, $\exists\, \varepsilon > 0$ such that $\forall \lambda \in (0, \varepsilon)$,

$$\lambda(\mathbf{a} - \mathbf{p}) \succ \mathbf{0} \succ \lambda(\mathbf{a}_0 - \mathbf{p}) \Rightarrow \lambda(\mathbf{a} - \mathbf{p}) \succ \lambda(\mathbf{a}_0 - \mathbf{p}) \quad \text{(A.7)}$$

Hence, writing $p \equiv p^* - \beta$, $\beta > 0$ and applying Axiom 1 as in the argument leading to (6), we conclude that there exists $\bar{\lambda}''$ such that for all $\lambda'' \in (0, \bar{\lambda}'')$:

$$\lambda''(\mathbf{a} - \mathbf{p}^*) \succ \lambda''(\mathbf{a}_0 - \mathbf{p}^*) \quad \text{(A.8)}$$

To conclude the argument, rewrite (A.6) as

$$\lambda' \begin{pmatrix} p^- - p^* + c \\ p^- - p^* + c \\ p^+ - p^* + c \\ p^+ - p^* + c \end{pmatrix} \succ \lambda' \begin{pmatrix} p^- - p^* + c \\ p^+ - p^* + c \\ p^- - p^* + c \\ p^+ - p^* + c \end{pmatrix}$$

with $c = 2p^* - p^+ - p^-$. Then by Axiom 1, there exists $\bar{\mu}'$ such that for all $\mu' \in (0, \bar{\mu}')$,

$$\mu' \begin{pmatrix} p^- - p^* \\ p^- - p^* \\ p^+ - p^* \\ p^+ - p^* \end{pmatrix} \succ \mu' \begin{pmatrix} p^- - p^* \\ p^+ - p^* \\ p^- - p^* \\ p^+ - p^* \end{pmatrix} \quad \text{(A.9)}$$

Finally, to get the desired result, pick any μ in $(0, \min(\bar{\mu}', \bar{\lambda}''))$ and observe that (A.8) and (A.9) yield the same pattern of preferences as in **A1** with $X = \{\mu(p^+ - p^*), \mu(p^- - p^*)\}$, e.g. $w = \mu(p^+ - p^*)$ and $z = \mu(p^- - p^*)$

(b) The proof for this part is very similar to that in part (a); essentially, the role of the interval $(\underline{p}_0, \bar{p}_0)$ is replaced by its (degenerate) counterpart, the switch price p^*. First, from the hypothesis we know that, if both assets are available at a price p in $(p^*, \bar{p})$, then the DM would strictly prefer a zero position to taking any non-zero position on **a** and will strictly prefer to go short on some amount of $\mathbf{a}_0$ rather than a zero position. Hence, for $p \in (p^*, \bar{p})$, $\exists\, \varepsilon > 0$ such that $\forall \lambda \in (0, \varepsilon)$, (A.4) holds and writing $p = p^* + \alpha$ we may then proceed to (6), just as in part (a). Secondly, from the hypothesis we also know that, if both assets are available at a price p in $(\underline{p}, p^*)$, then the DM would strictly prefer a zero position to taking any non-zero position on **a** and would strictly prefer to go long on some amount of $\mathbf{a}_0$ rather than a zero position. Hence, for $p \in (\underline{p}, p^*)$, $\exists\, \varepsilon > 0$ such that $\forall \lambda \in (0, \varepsilon)$, (A.7) holds and writing $p = p^* - \beta$ we may then proceed to (8), just as in part (a). Rest of the proof proceeds exactly as in part (a). □

Proof of Proposition 3.

(a) Assume to the contrary that both $\{s_1\}$ and $\{s_4\}$ are unambiguous events. Then,

$$\begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix} \succ \begin{pmatrix} w \\ z \\ w \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} w \\ w \\ z \\ w \end{pmatrix} \succ \begin{pmatrix} w \\ z \\ w \\ w \end{pmatrix} \text{since } \{s_4\} \text{ is unambiguous}$$

$$\Rightarrow \begin{pmatrix} z \\ w \\ z \\ w \end{pmatrix} \succ \begin{pmatrix} z \\ z \\ w \\ w \end{pmatrix} \text{since } \{s_1\} \text{ is unambiguous.}$$

A contradiction.

(b.i) Recall if an event T is unambiguous, so must be T^{c}. Thus, if we were to assume to the contrary that at least one of the two events $\{s_1, s_3\}$ and $\{s_2, s_4\}$ is unambiguous, it would imply that both events are unambiguous. Thus,

$$\{\{s_1, s_2\}, \{s_3, s_4\}, \{s_1, s_3\}, \{s_2, s_4\}\} \subset \mathcal{A}$$

Hence, there exists a probability distribution p on $\mathcal{A}$ such that the DM is probabilistically sophisticated (p.s.) w.r.t. that probability for events that are measurable w.r.t. $\mathcal{A}$. Notice that all the acts mentioned in **A1** are measurable w.r.t. $\mathcal{A}$.

Define $\succsim_l$ to be the likelihood relation associated with $\succsim$. Under **A1**, assuming w.l.o.g. that $w > z$, we get that $\{s_1, s_2\} \succ_l \{s_1, s_3\}$, i.e. the DM prefers to bet on $\{s_1, s_2\}$ rather than on $\{s_1, s_3\}$ and similarly $\{s_3, s_4\} \succ_l \{s_2, s_4\}$. Since the DM is p.s. w.r.t. to acts measurable w.r.t. $\mathcal{A}$, this implies that Epstein and Zhang (2001)

$$p(\{s_1, s_2\}) > p(\{s_1, s_3\}) \text{and} p(\{s_3, s_4\}) > p(\{s_2, s_4\})$$

and hence $p(\{s_1, s_2\}) + p(\{s_3, s_4\}) > 1$, a contradiction to the fact that $p(\{s_1, s_2, s_3, s_4\}) = 1$. Observe that if in **A1** $w < z$, then a similar reasoning (with the inequalities reversed) shows that $p(\{s_1, s_2\}) + p(\{s_3, s_4\}) < 1$, a contradiction.

(b.ii) Let $\succsim^{\text{p.s.}}$ be any p.s. preorder. We first start by establishing that

$$\begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix} \succsim^{\text{p.s.}} \begin{pmatrix} w \\ z \\ w \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} z \\ w \\ z \\ w \end{pmatrix} \succsim^{\text{p.s.}} \begin{pmatrix} z \\ z \\ w \\ w \end{pmatrix} \tag{A.10}$$

Define $\succsim_l^{\text{p.s.}}$ to be the likelihood relation associated with $\succsim^{\text{p.s.}}$. Then

$$\begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix} \succsim^{\text{p.s.}} \begin{pmatrix} w \\ z \\ w \\ z \end{pmatrix} \Leftrightarrow \{s_1, s_2\} \succsim_l^{\text{p.s.}} \{s_1, s_3\} \Leftrightarrow p(s_1) + p(s_2) \geq p(s_1) + p(s_3) \Leftrightarrow p(s_4) + p(s_2) \geq p(s_4) + p(s_3) \Leftrightarrow \{s_2, s_4\} \succsim_l^{\text{p.s.}} \{s_3, s_4\} \Leftrightarrow \begin{pmatrix} z \\ w \\ z \\ w \end{pmatrix} \succsim^{\text{p.s.}} \begin{pmatrix} z \\ z \\ w \\ w \end{pmatrix}$$

Now, to show that $\succsim$ cannot be ambiguity loving, pick $w, z \in X$ satisfying assumption **A1** and assume first that $\begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix} \succsim^{\text{p.s.}} \begin{pmatrix} w \\ z \\ w \\ z \end{pmatrix}$. Then, by (10), $\begin{pmatrix} z \\ w \\ z \\ w \end{pmatrix} \succsim^{\text{p.s.}} \begin{pmatrix} z \\ z \\ w \\ w \end{pmatrix}$. Take $f = \begin{pmatrix} z \\ w \\ z \\ w \end{pmatrix}$ and $h = \begin{pmatrix} z \\ z \\ w \\ w \end{pmatrix}$. Observe that h so defined is an unambiguous act (since by assumption, the events $\{s_1, s_2\}$ and $\{s_3, s_4\}$ are unambiguous events). Further, by assumption **A1**, $h \succ f$. Hence, $f \succsim^{\text{p.s.}} h$ and $h \succ f$, which shows that $\succsim$ cannot be ambiguity loving in this case.

Assume, $\begin{pmatrix} w \\ z \\ w \\ z \end{pmatrix} \succsim^{\text{p.s.}} \begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix}$. Then, taking $f = \begin{pmatrix} w \\ z \\ w \\ z \end{pmatrix}$ and $h = \begin{pmatrix} w \\ w \\ z \\ z \end{pmatrix}$ yields the desired result. □

References

Chew, S.H., 1989. Axiomatic utility theories with the betweenness property. Annals of Operations Research 19, 273–298.

Chew, S.H., Epstein, L.G., Wakker, P., 1993. A unifying approach to axiomatic non-expected utility theories. Journal of Economic Theory 59, 183–188.

Dekel, E., 1986. An axiomatic characterization of preferences under uncertainty: weakening the independence axiom. Journal of Economic Theory 40, 304–318.

Dekel, E., Lipman, B., Rustichini, A., 2001. Representing preferences with a unique subjective state space. Econometrica 69, 891–934.

Dow, J., Werlang, S., 1992. Uncertainty aversion, risk aversion, and the optimal choice of portfolio. Econometrica 60 (1), 197–204.

Ellsberg, D., 1961. Risk, ambiguity, and the Savage axioms, quarterly. Journal of Economics 75, 643–669.

Epstein, L., 1992. Behavior under risk: recent development in theory and applications. In: Laffont, J.-J. (Ed.), Advances in Economic Theory, vol. II. Cambridge University Press, Cambridge, Chapter 1, pp. 1–63.

Epstein, L., 2000. Are probabilities used in markets. Journal of Economic Theory 91, 86–90.

Epstein, L., Wang, T., 1994. Intertemporal asset pricing under Knightian uncertainty. Econometrica 62 (3), 283–322.

Epstein, L.G., Wang, T., 1995. Uncertainty, risk-neutral measures and security price booms and crashes. Journal of Economic Theory 67, 40–82.

Epstein, L., Zhang, J., 2001. Subjective probabilities on subjectively unambiguous events. Econometrica 69, 265–306.

Ghirardato, P., Katz, J.H., 2000. Indecision Theory: Explaining Selective Abstention in Multiple Elections, Discussion Paper 1106. California Institute of Technology, Pasadena.

Ghirardato, P., Marinacci, M., 2002. Ambiguity made precise: a comparative foundation. Journal of Economic Theory 102, 251–289.

Gilboa, I., 1987. Expected utility with purely subjective non-additive probabilities. Journal of Mathematical Economics 16, 65–88.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with a non-unique prior. Journal of Mathematical Economics 18, 141–153.

Gul, F., 1991. A theory of disappointment aversion. Econometrica 59, 667–686.

Machina, M., 2000. Payoff Kinks in Preferences over Lotteries, Discussion Paper 2000.22. University of California, San Diego.

Mukerji, S., 1998. Ambiguity aversion and incompleteness of contractual form. American Economic Review 88 (5), 1207–1231.

Mukerji, S., Tallon, J.M., 2000. Ambiguity Aversion and the Absence of Indexed Debt, Discussion Paper 2000.28. Cahiers de la MSE, Université Paris I, Paris.

Mukerji, S., Tallon, J.M., 2001. Ambiguity aversion and incompleteness of financial markets. Review of Economic Studies 68 (4), 883–904.

Quiggin, J., 1982. A theory of anticipated utility. Journal of Economic Behavior and Organization 3, 323–343.

Schmeidler, D., 1989. Subjective probability and expected utility without additivity. Econometrica 57 (3), 571–587.

Segal, U., Spivak, A., 1990. First order versus second order risk aversion. Journal of Economic Theory 51, 111–125.

Wakker, P., 1990. Under stochastic dominance, Choquet-expected utility and anticipated utility are identical. Theory and Decision 29, 119–132.