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ASYMMETRIC INFORMATION, NONADDITIVE EXPECTED UTILITY, AND THE INFORMATION REVEALED BY PRICES: AN EXAMPLE*

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I develop a simple example of a model in which agents have asymmetric information, and preferences that are represented by a nonadditive expected utility function. The a priori uninformed agent, after observing the equilibrium price, has conditional beliefs that remain nonadditive. Then, even when the equilibrium price function is fully revealing (i.e., one-to-one), it may be worthwhile for an a priori uninformed agent to buy 'redundant' private information if he is more confident in that information than in that revealed by the price system.

1. INTRODUCTION

General equilibrium models of asymmetric information have the 'bad property' of yielding, in the case of a finite state space, equilibrium at which all private information is generically revealed by the price system.[2] This gives rise to well-known paradoxes, such as the Grossman–Stiglitz paradox:[3] since all the relevant information is revealed through the equilibrium price system, no agent will have an incentive to acquire it privately. This literature has explicitly adopted the expected-utility framework, in which agents' beliefs can be represented by (additive) probabilities.

This paper looks at this issue in a framework in which agents have non-necessarily additive beliefs. In such a framework, it is possible to distinguish information indirectly revealed by prices from direct information.[4] I illustrate this distinction with an example in which there are differences between learning the state through the price system or through private, direct information. The example is a two-agent, two-state economy with asymmetric information, in which an agent has, after observing the equilibrium price, nonadditive beliefs over the states of nature. Such an agent might be willing to pay to acquire (privately) a piece of information which

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[2] See, for example, Radner (1979), Allen (1981), Grossman (1981), and Allen (1984), as well as Allen (1986) and Ausubel (1990) to see how to construct nonfully-revealing equilibria when the state space is infinite.

[3] Grossman and Stiglitz (1980).

[4] Tversky and Wakker (1995, p. 1270) discuss this phenomenon of 'source dependence:' "there is evidence that people's preferences depend not only on their degree of uncertainty but also on the source of uncertainty."

is already 'contained' in the equilibrium price function. This is not possible in the standard expected-utility framework.

More precisely, the uninformed agent's conditional beliefs are supposed to be nonadditive. When the equilibrium price function is fully revealing (i.e., prices are different across states), the a priori uninformed agent is able to rule out states different from state s when he sees the price $p(s)$, associated to s. As a consequence, he puts probability zero on the complement of s. However, I assume that he does not place probability one on s. This is possible obviously only if his conditional beliefs are nonadditive. In the language adopted by Morris (1994), the agent, after seeing $p(s)$, negatively knows the event s (that is, he puts zero probability on the complement of s) but does not positively know it (that is, he does not put probability one on it). In the expected-utility framework these two notions of knowledge are equivalent, and it is inconsistent to assume negative knowledge of an event without assuming positive knowledge as well.

Such an assumption of the agent's knowledge of the state is meant to reflect that the agent does not rely on the market mechanism and hence the price system as a good way of revealing private information. In other words, he perceives that the price signal is an ambiguous one. The literature on nonadditive expected utility has shown that a way of representing the idea that decision makers are uncertainty averse and dislike ambiguity is to assume that agents have nonadditive (more precisely, super-additive) beliefs (see Schmeidler 1989, or Wakker 1990). Thus, in the present paper, nonadditive (conditional) beliefs capture the idea of to what extent the agent trusts information revealed by prices.

The nonadditivity of an agent's beliefs can be interpreted as reflecting either that agents have a misspecified model, or that they perceive only a coarse partition of the true state space. These two interpretations are provided by Gilboa and Schmeidler (1994) and Ghirardato (1994), respectively.[5] According to Gilboa and Schmeidler (1994), the beliefs' nonadditivity reflects the facts that agents have a misspecified model. They show that it is always possible to extend the state space in a way that restores beliefs' additivity. Hence, an agent, upon seeing a particular equilibrium price, acts as if he was learning of a new state, previously omitted. According to the second interpretation, nonadditivity results from the decision maker's perception that he sees only a coarse partition of the true state space. 'Adopting' a nonadditive expected utility is then a way to cope with his ignorance. In the present model this would mean that the agent does not know 'for sure' that he is not forgetting some relevant states when taking his decision. However, he is unable to refine the state space further. As a consequence, he behaves (on the perceived state space) as if he were maximizing a nonadditive expected utility.

In the standard expected-utility framework, agents fully trust the information revealed by the price system since Bayes' rule implies that if the equilibrium price function is fully revealing, agents, when they observe the price, know which state occurred. In that framework, though, there exists a way to capture the idea that agents might have various degrees of confidence in the information carried by market prices: it is to assume the presence of noise traders, that is, irrational agents

[5] See also Jaffray and Wakker (1994), and Mukerjii (1997).

who act in a random way, thus blurring the price signal. Then, rational, noninformed agents who have probabilistic beliefs on the actions of noise traders are not able to infer the state from the price any longer. In such models agents are treated as probabilistically sophisticated agents. This yields different equilibrium properties.

The setup of the model is discussed in Section 2, while the equilibrium analysis is provided in Section 3. Section 4 deals with the issue of whether private information can be valuable to an a priori uninformed agent when the equilibrium price function is one-to-one (i.e., when the equilibrium is fully revealing). Section 5 discusses the interpretation of the model and compares its findings with the noise trader literature. Section 6 concludes. An Appendix gives a numerical example.

2. THE MODEL AND THE REPRESENTATION OF AGENTS' BELIEFS

There are two states of the world, α and β, and two goods in each state, x and y, whose prices are denoted respectively by $p(s)$ and $q(s)$ for $s = \alpha, \beta$. Prices are normalized state by state so that $p(s) + q(s) = 1$, $s = \alpha, \beta$. Each of the two agents ($h = 1, 2$), has constant-across-states endowments of one unit of each good, so that each agent's wealth in each state is one. Agent 1 is fully informed of the state that occurred. His state-dependent utility functions are respectively $u_1^\alpha(x_1, y_1) = a_1(\alpha)\log x_1 + b_1(\alpha)\log y_1$ in state α and $u_1^\beta(x_1, y_1) = a_1(\beta)\log x_1 + b_1(\beta)\log y_1$ in state β. Agent 2's preferences can be represented by a nonadditive measure (or capacity), together with state-dependent utility functions. More precisely, let $S = \{\alpha, \beta\}$ and assume that agent 2's beliefs are represented by a superadditive capacity π: $2^S \to [0, 1]$, that is, such that $\pi(\{\alpha\}) + \pi(\{\beta\}) < \pi(\{\alpha, \beta\}) = 1$.[6] Assume that 2's utility index depends on the realization of $s = \alpha, \beta$, and not (directly) on prices.[7] Let $u_2^\alpha(x_2, y_2) = a_2(\alpha)\log x_2 + b_2(\alpha)\log y_2$ and $u_2^\beta(x_2, y_2) = a_2(\beta)\log x_2 + b_2(\beta)\log y_2$ and assume, without loss of generality, that $a_h(s) + b_h(s) = 1$ for $h = 1, 2$ and $s = \alpha, \beta$.

We need now to define what agent 2's decision problem is upon observing, say, $p(\alpha)$. The crucial issue is: What is the agent able to infer from the observation of this price? In the standard expected-utility framework, this issue is easily solved. If $p(\alpha)$ is different from $p(\beta)$, then an agent learns that α occurred when he sees $p(\alpha)$, that is, he places probability one on state α, or, equivalently, he places probability zero on β.

This issue is not as simple in the nonadditive framework. Indeed, putting zero weight on an event does not mean that the complement of this event is of measure one. This is what allows different authors to define several notions of knowledge in this setup (see Morris 1994, and Dow and Werlang 1994). In particular, say that s is negatively known, if its complement has measure zero, while s is positively known if it is of measure one. Here, I assume that s is negatively known when $p(s)$ is

[6] To keep notation to a minimum I omit the subscript corresponding to agent 2 and write his capacity π instead of π_2. I also write from now on $\pi(\{s\}) = \pi(s)$.

[7] The axiomatization of nonadditive expected utility by, for example, Schmeidler (1989) yields a utility index that is independent of the state. Morris (1994), however, uses state-dependent utility functions in this framework. I adopt this approach here.

observed, but is not positively known. Had I assumed s to be positively known in this case, the results would have been identical to those obtained in the standard expected-utility framework. Formally, I assume that agent 2's conditional beliefs take the following form:

(i) if $p(s) \neq p(s')$, $\pi(s|p(s')) = 0$ and $\pi(s) \leq \pi(s|p(s)) < 1$ $s \neq s'$, $s, s' = \alpha, \beta$

(ii) if $p(s) = p(s')$, $\pi(s|p(s)) = \pi(s)$, $s, s' = \alpha, \beta$

These beliefs are taken as primitives of the model, as in Epstein and Wang (1994), therefore side-stepping the issue of which updating rule to use in a nonadditive environment and the related issue of dynamic consistency.[8] Observe also that I allow agent 2, when he sees $p(s)$, to choose 'at worst' not to revise his prior beliefs (in this case $\pi(s|p(s)) = \pi(s)$). However, the above specification prevents him from revising his beliefs in the 'wrong direction,' and to place actually less weight on s after seeing $p(s)$.

Because the approach taken here amounts to directly assuming the nonadditivity of conditional beliefs, justifications for it can be found in the different axiomatizations of nonadditive expected utility.[9] As mentioned in the Introduction, superadditivity of beliefs represents the fact that the decision maker perceives a situation as being ambiguous and that he is averse to such an ambiguity. In the present context it represents the fact that agent 2 does not think that the quality of the signal $p(\alpha)$ is sufficiently good to be entirely trusted, that is, state α cannot be given probability one when the price is $p(\alpha)$. Therefore, I assume that an agent, when he sees a given price, is able to rule out some states, but is not totally confident (possibly because he believes other traders act in some irrational manner) that the state that occurred is the one that remains after eliminating these states: he knows s, but only negatively. At a purely intuitive level, it seems that looking for 'trustworthy' information to supplement or confirm the one contained in the price system is indeed a real-life activity: when buying a good or a stock, people tend to look for information in specialized magazines even though this information is supposedly already contained in the price system. This can be seen as spending resources to move from a state of negative knowledge to a state of positive knowledge.

Formally, agent 2's nonadditive expected utility conditional on the price he observes (say, $p(\alpha)$) is given by:

$$E_{\pi(\cdot|p(\alpha))}\tilde{u}(x,y) = u^{\beta}(x,y) + \pi(\alpha|p(\alpha)) \cdot \left[u^{\alpha}(x,y) - u^{\beta}(x,y)\right] \quad \text{if } u^{\alpha}(x,y) \geq u^{\beta}(x,y)$$

[8] See Gilboa and Schmeidler (1993), Epstein and LeBreton (1993) and Eichberger and Kelsey (1996).

[9] Section 5 comes back on the interpretation of the beliefs' nonadditivity.

$$E_{\pi(\cdot|p(\alpha))}\tilde{u}(x,y) = u^{\alpha}(x,y) + \pi(\beta|p(\alpha))$$
$$\cdot\left[u^{\beta}(x,y) - u^{\alpha}(x,y)\right] \text{ if } u^{\beta}(x,y) \geq u^{\alpha}(x,y)$$

where $\tilde{u}(x,y)$ represents the random variable taking values $u^{\alpha}(x,y)$ and $u^{\beta}(x,y)$.[10]

At this stage, I have a full description of how agent 2 will behave after he observes a particular price and can move on to the computation of an equilibrium.

3. EQUILIBRIUM ANALYSIS

I use the usual rational expectations equilibrium notion. An equilibrium is a price vector $(p(\alpha), p(\beta))$, such that agents maximize their utility conditionally on their private information as well as the information revealed by the price system (as a consequence, demands are measurable with respect to prices), and markets clear. Here, I focus on fully revealing equilibria, that is, equilibria such that $p(\alpha) \neq p(\beta)$. In that case, agent 2's demand is measurable with respect to states as well.

DEFINITION. A price vector $p = (p(\alpha), p(\beta))$ and an allocation (x, y) is a fully revealing equilibrium if:

(i) $p(\alpha) \neq p(\beta)$

(ii) Given p, $(x_1(s), y_1(s))$ is a solution to:

$$\max u_1(x_1(s), y_1(s)) \text{ s.t. } p(s)x_1(s) + (1-p(s))y_1(s) = 1 \quad s = \alpha, \beta$$

(iii) Given p, $(x_2(s), y_2(s))$ is a solution to:

$$\max E_{\pi(\cdot|p(s))}\tilde{u}_2(x_2(s), y_2(s)) \text{ s.t. } p(s)x_2(s) + (1-p(s))y_2(s) = 1 \quad s = \alpha, \beta$$

(iv) $x_1(s) + x_2(s) = 2$ and $y_1(s) + y_2(s) = 2 \quad s = \alpha, \beta$

In view of the specification of agent 2's conditional beliefs, it is clear that the notion of revelation entailed in this definition is slightly different from the usual one. Here, a state is revealed by the equilibrium price system if agents, when they observe the associated price, put probability zero on the complement of that state. In the additive framework this is equivalent to assuming they put probability one on the state. However, as previously mentioned, these two notions of revelation are now different. If I had adopted a stronger notion of knowledge (positive knowledge), I would have obtained the usual result that an agent will not pay to acquire private information when the price system is fully revealing.

[10] The expected value with respect to a capacity v of a random variable f defined over a discrete space $\Omega = \{1, \dots, \Omega\}$ and such that $f(1) \geq f(2) \geq \dots \geq f(\Omega)$ is equal to:

$$E_v f = \sum_{\omega=1}^{\Omega-1} (f(\omega) - f(\omega+1))v(\{1,\dots,\omega\}) + f(\Omega)$$

Observe that it is necessary to order outcomes to compute this expected value.

Agent 1 being fully informed solves a different problem, depending on whether α or β occurred. This leads to the demand functions:

$$x_1(s,p) = \frac{a_1(s)}{p} \quad \text{and} \quad y_1(s,p) = \frac{b_1(s)}{1-p} \quad s = \alpha, \beta$$

Assuming for the moment that $p(\alpha) \neq p(\beta)$ (this condition will be shown to hold at equilibrium), agent 2 solves the following program, if he observes say, $p(\alpha)$ and if his utility in state β is less than in state α (this condition will also be shown to hold at equilibrium):

$$\max_{x,y} u_2^{\beta}(x,y) + \pi(\alpha | p(\alpha))\big(u_2^{\alpha}(x,y) - u_2^{\beta}(x,y)\big)$$

$$\text{s.t. } p(\alpha)x + (1 - p(\alpha))y = 1$$

Demand functions are then given by:

$$x_2(p(\alpha)) = \frac{(1 - \pi(\alpha | p(\alpha)))a_2(\beta) + \pi(\alpha | p(\alpha))a_2(\alpha)}{p(\alpha)}$$

$$y_2(p(\alpha)) = \frac{(1 - \pi(\alpha | p(\alpha)))b_2(\beta) + \pi(\alpha | p(\alpha))b_2(\alpha)}{1 - p(\alpha)}$$

Conditionally on $p(\alpha)$ being different from $p(\beta)$, the first component of the equilibrium price vector $(p(\alpha), p(\beta))$, is therefore equal to:

$$p^*(\alpha) = \frac{a_1(\alpha) + (1 - \pi(\alpha | p^*(\alpha)))a_2(\beta) + \pi(\alpha | p^*(\alpha))a_2(\alpha)}{2}$$

It remains to be checked that the utilities of agent 2 are indeed ranked as I assumed, that is, $u_2^{\alpha}[x(\alpha, p^*(\alpha)), y(\alpha, p^*(\alpha))] > u_2^{\beta}[x(\alpha, p^*(\alpha)), y(\alpha, p^*(\alpha))]$. This condition is equivalent to:

$$(1) \qquad (a_2(\alpha) - a_2(\beta))\big[(1 - \pi(\alpha | p^*(\alpha)))a_2(\beta) + \pi(\alpha | p^*(\alpha))a_2(\alpha) - a_1(\alpha)\big] > 0$$

Similarly, one can compute the equilibrium price in state β,

$$p^*(\beta) = \frac{a_1(\beta) + (1 - \pi(\beta | p^*(\beta)))a_2(\alpha) + \pi(\beta | p^*(\beta))a_2(\beta)}{2}$$

conditional on the fact that $u_2^{\beta}[x(\beta, p^*(\beta)), y(\beta, p^*(\beta))] > u_2^{\alpha}[x(\beta, p^*(\beta)), y(\beta, p^*(\beta))]$. This condition is equivalent to:

$$(2) \qquad (a_2(\beta) - a_2(\alpha))\big[(1 - \pi(\beta | p^*(\beta)))a_2(\alpha) + \pi(\beta | p^*(\beta))a_2(\beta) - a_1(\beta)\big] > 0$$

It is easy to check that both conditions 1 and 2 can hold simultaneously. Suppose for example that $a_1(\alpha) = b_1(\beta) = 0$ and that $b_1(\alpha) = a_1(\beta) = 1$. Then these two conditions hold for any choice of $a_2(\alpha) > a_2(\beta)$. Therefore, by continuity, these conditions hold for an open set of parameters around these values.

If these conditions were not satisfied, one would have to look for an equilibrium with a different ranking of agent 2's utility levels in the two states. It is tedious but straightforward to show that an equilibrium exists for almost all parameter configurations. Existence (at least generic existence) is therefore not an issue in this parametric example. However, the properties of this equilibrium do depend on the ranking of 2's utility levels. As a matter of fact, conditions 1 and 2 cannot hold simultaneously if $a_1(s) = a_2(s)$, $s = \alpha, \beta$, that is, if agents are identical but for their beliefs. In this case the equilibrium is the no-trade equilibrium. This simply comes from the fact that I assumed $\pi(\beta | p(\alpha)) = 0$ and reflects agent 2's pessimism (or uncertainty aversion): he is not totally sure that α occurred, but the 'worst' he can think of is that same state α (since if he were in state β his utility would be higher). As a consequence, he acts as if he were sure α occurred, even though his 'true motivation' is simply his pessimism. Finally, one can remark that $p^*(\alpha)$ and $p^*(\beta)$ are different for almost all choices of the parameters, thus ensuring that the equilibrium is fully revealing.

4. IS IT WORTH ACQUIRING PRIVATE INFORMATION?

Having computed equilibrium prices, and observed that they are revealing in the sense that prices are different for different realizations of $s = \alpha, \beta$, one can now ask whether agent 2 has, in the present framework, an incentive to acquire private information concerning which state occurred. Suppose agent 2 has access to another source of information, modeled by a signal $\sigma^\alpha, \sigma^\beta$. Assume this signal is not 'ambiguous' in the sense that the agent's preferences can now be represented by an additive probability measure $\tilde{\pi}$ and the same state-dependent utility function as above. In particular, one has: $\tilde{\pi}(\{\alpha, \sigma^\alpha\}) + \tilde{\pi}(\{\beta, \sigma^\beta\}) = 1$. This yields a conditional probability (computed according to Bayes' rule) having the following properties:

$$\tilde{\pi}(s|\sigma^s) = \frac{\tilde{\pi}(\{s, \sigma^s\})}{\tilde{\pi}(\{s, \sigma^s\} \cup \{s', \sigma^s\})} = 1 \quad \text{for } s \neq s' \quad \text{and} \quad \tilde{\pi}(s'|\sigma^s) = 0 \quad \text{for } s \neq s'$$

The signal σ is consequently more 'informative' than the price system, not that it truly reveals more information, but rather because the agent is more confident in his private information than in the information revealed by the price system. Indeed, the ratio $\tilde{\pi}(s|\sigma^s)/\tilde{\pi}(s'|\sigma^s)$ 'is equal to' the ratio $\pi(s|p(s))/\pi(s'|p(s))$ and is infinite: the agent believes s infinitely more probable than s' when he sees σ^s or $p(s)$. However, he trusts the private signal more and therefore $\pi(s|p(s)) < \tilde{\pi}(s|\sigma^s)$. This notion of confidence cannot be expressed in the traditional, additive, framework. There, if the previous ratios were equal, the probabilities themselves would also be equal.

It is easy to compute the equilibrium in the economy where both agents are fully informed of the state and place probability one on the state that occurred and zero on the other one. It is given by the price system:

$$\bar{p}(\alpha) = \frac{a_1(\alpha) + a_2(\alpha)}{2} \quad \text{and} \quad \bar{p}(\beta) = \frac{a_1(\beta) + a_2(\beta)}{2}$$

Observe that $p^*(s) \to \bar{p}(s)$ if $\pi(s|p^*(s)) \to 1$. In other words, as the nonadditivity of beliefs become smaller, the fully revealing equilibrium tends to the equilibrium of the full information economy. Hence, as agent 2 becomes more confident in the ability of the market to pool private information, the 'gain' (whether positive or negative) from acquiring private information becomes smaller.

What remains is to see whether agent 2 has an incentive to use (buy) this private information. He will do so if the ex-post utility he gets at equilibrium, in each state, is larger when he has access to a private source of information than when he learns which state occurred through the price system, that is, if:

$$u_2^s[x_2(\bar{p}(s)), y_2(\bar{p}(s))] > u_2^s[x_2(p^*(s)), y_2(p^*(s))] \quad \text{for } s = \alpha, \beta$$

These conditions have no simple expression. However, it can be shown that they can hold for a configuration of the parameters satisfying the previous restrictions. I give such an (numerical) example in the Appendix. Therefore, there exists an open set of parameters for which the uninformed agent has an incentive to acquire some private information, despite the fact that the equilibrium price system is fully revealing.[11] In other words, if one was to give him the additional information in the form of the signal σ^s he would not tend to disregard it, as in the additive case where private information is redundant with respect to the one revealed by the price system.[12] Observe that the agent is willing to buy the information no matter what it is. In both states he is better off (ex post) having bought the information. To sum up this discussion, the agent will be willing to pay a positive amount to move from the situation of negative knowledge of the state to that of positive knowledge of the state.

It is also interesting to study the welfare of the a priori informed agent. A simple calculation shows that if $a_1(\alpha)$ is close to zero, while $a_1(\beta)$ is close to one, and if $a_2(\alpha) > a_2(\beta)$, then ex post utility level in both states is higher in the full information economy compared with the one obtained at the revealing equilibrium:

$$u_1^s[x_1(\bar{p}(s)), y_1(\bar{p}(s))] > u_1^s[x_1(p^*(s)), y_1(p^*(s))] \quad \text{for } s = \alpha, \beta$$

This inequality holds in our numerical example. Thus, the informed agent might gain from the fact that the a priori uninformed agent acquires, privately, some

[11] One can observe that this example is also robust to the assumption of log utilities. Indeed, the result would still hold if utility functions were perturbed.

[12] It is clear from the previous analysis that this result depends on the parameters and is not true for all parameter values. In particular, it does not hold in our example if agents have identical utility functions; a case that is often looked at in finance.

information: it could be in the informed agent's interest to convince, even at a cost, the other agent that a particular state occurred. In other words, it might pay for agent 1 to spend some resources to convince agent 2 that the information he inferred from the equilibrium price system can actually be trusted. The reduction in uncertainty attached to the fact that each agent now fully trusts the information he had already received through the price system raises everybody's welfare. Even if the a priori uninformed agent does not change his relative assessment of the occurrence of α and β, the mere fact that he trusts it more (that he now has an additive measure) can ex post Pareto-improve the situation.

This phenomenon can be simply illustrated in the extreme case of total ignorance of agent 2. This is modeled by saying that he puts zero probability on each state occurrence, even after he observed the equilibrium price. Thus, upon seeing $p(\alpha)$, he 'negatively knows' that β did not occur ($\pi(\beta \mid p(\alpha)) = 0$), but he does not trust the market at all and affect probability zero to α as well ($\pi(\alpha \mid p(\alpha)) = 0$). Then, he acts as if to maximize the minimum of his utility, and therefore ends up demanding an equal amount of good x and good $y: x_2(p) = y_2(p)$ for all p. The equilibrium price in this case is such that $x_1(s, p) = y_1(s, p)$, which implies $p^*(s) = a_1(s)$, and therefore $x_1(s, p^*) = y_1(s, p^*) = 1$. The equilibrium allocation entails no trade. This 'autarchic' equilibrium is in general not ex post Pareto efficient since it is clear that if agents value the two goods differently, that is, $a_h(s) \neq a_{h'}(s)$, then they would both benefit from consuming different amounts of the two goods. Thus, agent 1 has an interest in leading agent 2 to trade, and in order to do this, he needs to somehow convince him of which state occurred.[13] Therefore, the 'Hirshleifer effect,' that a change in the information structure leads to changes in equilibrium prices that affect welfare in equilibrium, is also at work in this model.

5. DISCUSSION AND RELATIONSHIP WITH THE NOISE TRADER APPROACH

One could wonder if (and how) the story told in this example could be told in the usual, additive framework. In this section, two models with additive beliefs are detailed. In these models, agents have additive beliefs over an 'enlarged state space.' These two constructions are meant to replicate the results of the previous section, and therefore will serve as a basis to evaluate the contribution of a nonadditive approach. A comparison with the noise trader approach is then developed.

5.1. *Perceived State Space Versus True State Space.* Consider the following model, in which there are now four states. Endowments are constant across states, equal to one unit of each good for each consumer.

(i) In state ♣, agent 1 has utility function u_1^α, and agent 2 has utility function u_2^α.
(ii) In state ♦, agent 1 has utility function u_1^β, and agent 2 has utility function u_2^α.

[13] This result raises the question of how agent 1 could convince agent 2. In a perfectly competitive setup, this is not obvious, and one should probably move to some kind of costly signalling model.

(iii) In state ♥, agent 1 has utility function u_1^{α}, and agent 2 has utility function u_2^{β}.

(iv) In state ♠, agent 1 has utility function u_1^{β}, and agent 2 has utility function u_2^{β}.

Agent 1 is perfectly informed and agent 2 completely uninformed, with an additive prior on the four states. An interpretation of these four states could be that, in state ♦, α occurs but agent 1 is irrational and behaves as if β occurred, and conversely for state ♥, while in the two dark suits states, agent 1 is rational. It is then easy to see that there exists a partially revealing equilibrium in which agent 2 is not able to distinguish between ♥ and ♣, and between ♠ and ♦. As a consequence, he might be willing to pay to get some private information. Hence, this example shows that this result can be obtained in an additive setup. However, the additive example is very peculiar, and of course nongeneric: agent 1's utility function is the same in states ♣ and ♥, and similarly, is the same in state ♠ and in state ♦. Thus, the additive example has no robustness at all, something we already knew from the generic full-revelation result (Radner 1979). Furthermore, the construction above is only valid for the equilibrium I constructed, that is, when $u_2^{\alpha}[x(\alpha, p^*(\alpha)), y(\alpha, p^*(\alpha))] > u_2^{\beta}[x(\alpha, p^*(\alpha)), y(\alpha, p^*(\alpha))]$ and $u_2^{\beta}[x(\beta, p^*(\beta)), y(\beta, p^*(\beta))] > u_2^{\alpha}[x(\beta, p^*(\beta)), y(\beta, p^*(\beta))]$. If these conditions did not hold, then one would have to change the state space in the construction above to reproduce the equilibrium.

Instead, in the setup developed in the previous sections, agent 2 might know he has only a coarse representation of the true state space, but is unable to specify it further, and behaves in a pessimistic fashion with regard to this situation of ignorance. Indeed, in this interpretation, the 'perceived state space' (see Ghirardato 1994) of agent 2 after he observes a given price, $\{\alpha, \beta\}$, is a partition of the 'true state space' {♣, ♦, ♥, ♠}.[14] Nonadditive conditional beliefs therefore model a situation in which agent 2 ignores states ♦ and ♥ and knows he is ignoring something. The particular specification of states ♦ and ♥ is irrelevant since the agent ignores them (and is aware he is ignoring *something*). Said differently, agent 2 does not have to specify what he is omitting. This explains why u_2^{β} appears in the computation of 2's utility if he sees $p(\alpha)$: it is the only known outcome on which he can base an evaluation of what might happen if an unspecified (ignored) event occurred. Finally, this outcome is relevant to the agent's decision only if it gives him less utility than consumption in state α, that is, if $u_2^{\beta} < u_2^{\alpha}$. This is because I assumed agent 2 is pessimistic: he expects for this ignored state the worst outcome he can think of. Indeed, 2 is willing to pay for private information, only for some parameters value and conversely would not pay for some extra information if u_2^{β} were greater than u_2^{α} at equilibrium.

Therefore the fact that agent 2 might be willing to pay to get more information is an artifact of the construction if one adopts the usual additive setup (indeed, the example is nongeneric), whereas it is a way to cope with his ignorance in (this interpretation of) the nonadditive setup.

[14] In the example, $\alpha = \{♣\} \cup \{♦\}$ and $\beta = \{♠\} \cup \{♥\}$. Agent 2 knows that his utility function can take two possible functional forms, but is unable to really identify the states, that is, agent 1's utility function.

5.2. *Misspecified Model.* The second additive model is based on Gilboa and Schmeidler (1994), who show that the nonadditive model is in fact equivalent to an additive model with an enlarged state space. A possible completion of the model, very similar to the one just done would go along the following lines: expand the state space by adding two more states γ and δ. Agent 2's conditional beliefs become additive if one sets $\pi_2(\gamma | p(\alpha)) = 1 - \pi_2(\alpha | p(\alpha))$ and $\pi_2(\delta | p(\beta)) = 1 - \pi_2(\beta | p(\beta))$. The utility function in state γ and δ has to be of a special form: $u_2^\gamma(x, y) = u_2^\delta(x, y) = \min\{u_2^\alpha(x, y), u_2^\beta(x, y)\}$, while $u_1^\gamma = u_1^\alpha$ and $u_1^\delta = u_1^\beta$. The expected value of a consumption bundle is the same whether computed on this enlarged state space with the additive prior, or with the nonadditive measure on the smaller state space so that the two models are indeed equivalent. Actually, it is straightforward to see that the equilibrium computed in the nonadditive model is also an equilibrium of this expanded model, where $p(\alpha) = p(\gamma)$ and $p(\beta) = p(\delta)$. Hence, agent 2 is unable to distinguish between states α and γ, and it is possible that he might be willing to pay to acquire some 'extra' information. However, the construction is also rather contrived, and the specification of the utility functions very peculiar.

Although rather particular, the example in its additive form does provide an alternative interpretation of the nonadditivity of 2's conditional beliefs: when he sees $p(\alpha)$ (resp. $p(\beta)$), agent 2 behaves as if he learned of a new state γ (resp. δ). The reason for the nonadditivity is, in this interpretation, the lack of information on which beliefs are based. Thus, it seems reasonable that an agent might be willing to pay to acquire reliable information, if it increases the confidence he has of his probabilistic assessment of the future.

At the end of this discussion, I hope I have convinced the reader that the nonadditive approach is more natural to model a situation in which an agent is not confident in his interpretation of an endogenous (equilibrium) variable, that is, is not confident he has the right model of the economy in mind. It precisely avoids to specify a given model: he only perceives some facts, is aware that this is possibly not the full picture, and acts in a pessimistic way upon what he perceives. As shown here, this can be replaced by an expected utility agent, with a very accurate view of what a possibly irrational agent would do (more precisely, that agent 1 in state α could behave as if he had utility function u^β, as represented in state ♦ above). Vague beliefs, representing agents' awareness of their ignorance, are, in my opinion, a better way to represent this situation.

5.3. *The Noise Trader Approach.* As mentioned in the Introduction, a way to obtain nonfully revealing rational expectations equilibrium, frequently used in finance, is to suppose that the observed price is contaminated by the action of irrational agents, or noise traders. Fully detailed in the context of the present model, this approach is basically the one discussed above: agent 2 believes agent 1 can be irrational, and has additive beliefs about it. As argued above, the nonrevelation result is nongeneric in that setup. Consequently, I will focus here on the often used 'reduced form' of such a model, where the noise is simply added to the price function, and can take on an uncountable number of values. Hence, if $p^*(s)$ is the equilibrium price in state s of the noiseless model, the observed equilibrium price is equal to $p^*(s) + \varepsilon$, where ε is some exogenous noise (whose law is, however, assumed to be known by the agents). If $p^*(s) + \varepsilon = p^*(s') + \varepsilon'$, for some values $\varepsilon, \varepsilon'$, rational traders cannot tell for sure if the state is s or s', even though they might revise their prior upon seeing a given price. The ad hoc presence of noise hence captures the intuition that traders are never completely sure of the map from states to prices (as in the construction above, where the 'noise' was explicitly related to some specified 'irrational' behavior). Since it is the same intuition that is captured through the notion of uncertainty aversion, one could wonder how these two assumptions can be distinguished.

First, uncertainty-averse traders' preferences rest on a sound axiomatic basis. Thus, even though these agents are not Bayesian rational, their preferences obey a certain number of rather appealing axioms.[15] On the contrary, simply assuming noise in the model has no justification in terms of basic economic principles. Second, the nonadditive expected utility model yields implications that separate it from noisy models. It enables one to introduce some 'asymmetry' in what agents can learn. Indeed, it is possible that the a priori uninformed agent can associate state α with $p(\alpha)$ 'for sure,' while he will not place probability one on β when seeing $p(\beta)$ (even though he puts probability zero on α). In that case, $\pi(\alpha | p(\alpha)) = 1$, but $\pi(\beta | p(\beta)) < 1$. In other words, the agent has negative knowledge of β when he sees $p(\beta)$, and positive knowledge of α when he sees $p(\alpha)$. Observe that this cannot arise in noisy models. In these models an agent reasons only by comparing equilibrium prices in different states. Thus, if $p^*(s) + \varepsilon = p^*(s') + \varepsilon'$, for some realization $\varepsilon, \varepsilon'$, agents cannot distinguish between s and s', a relation that is obviously symmetric. Note finally that agents in the noisy model are 'probabilistically sophisticated,' and are able to derive additive beliefs on which state occurred after observing the (noisy) price. Thus, one could directly test the relevance of the present model: in the nonadditive case, agents' beliefs would exhibit Ellsberg-type paradoxes, and agents could potentially be subject to Dutch books,[16] whereas these two phenomena have no room in the noisy model.

6. CONCLUDING REMARKS

The application of nonadditive expected utility theory to the case of asymmetric information reported in this paper provided an example in which an uncertainty-averse trader might be willing to pay to acquire information already 'objectively' contained in the price system. At equilibrium, even though the price system is fully revealing (the price function is one-to-one), agents might find it valuable to buy private information. Thus, one is led to distinguish between fully revealing equilibrium and an equilibrium of the full information economy. In the present framework, a priori uninformed agents do infer some information from market prices, but do not see this as perfectly substitutable with other (privately obtained) pieces of information, even though the content is 'objectively' the same. The explanation of this phenomenon is that agents do not completely trust market prices as a vector of information, the lack of confidence in the market being captured by the nonadditivity of the measure representing their conditional beliefs.

[15] Schmeidler (1989), Sarin and Wakker (1992), and Ghirardato (1994).

[16] See Kelsey (1995), in the related case of the multiple priors approach.

Several extensions to this exercise can be contemplated. One could study a set of private information structures that vary in ambiguity and the impact of acquiring a more or less ambiguous piece of information on the equilibrium, and define an ambiguity premium attached to a particular structure. Similarly, the example also begs for extensions generalizing the result in relation to the a priori given information structure. Finally, as in much of the rational expectations literature, this paper does not look at the issue of how agents arrive at an equilibrium. This is especially important in this framework, for it was not revealed how agent 2 decides that the state space he perceives is only a coarse partition of the true space, or, alternatively, why he behaves as if he had a misspecified model of the economy. This issue would lead to considering the problem of 'state space' revision and learning in a nonadditive setup.

APPENDIX

I demonstrate a numerical example that has all the features (and more) discussed in the text.

(i) Agent 1: $a_1(\alpha) = 0.5$, $a_1(\beta) = 0.45$. I specify also 1's prior when he is not informed (something I did not need in the text): $\pi_1(\alpha) = \pi_1(\beta) = 0.5$

(ii) Agent 2: $a_2(\alpha) = 0.65$, $a_2(\beta) = 0.4$, $\pi_2(\alpha) = 0.4$, $\pi_2(\beta) = 0.3$, $\pi_2(\alpha \mid p(\alpha)) = 0.6$, $\pi_2(\beta \mid p(\beta)) = 0.85$, $\pi_2(\alpha \mid p(\beta)) = \pi_2(\beta \mid p(\alpha)) = 0$

I computed the equilibrium in four cases: 1 and 2 a priori not informed, 1 informed and 2 not informed, 1 not informed and 2 informed, 1 and 2 informed, under the assumption (which can be checked to hold at equilibrium) that condition 1 and 2 hold and that equilibrium prices are different in each state. The (ex post) welfare of each agent in each state is reported in the following table. If agent 2 is informed, then 1 can infer all the information from the price system, and it therefore does not make a difference whether 1 is a priori informed or not. Thus, these four situations reduce to only three.

It is readily seen in Table 1 that agent 2 has an incentive to acquire private information although 1 is already informed and the price system revealing.

Second, agent 1 also gains from 2 getting private information. Finally, if one assumes that the probability of occurrence of each state is indeed 0.5, then one can check that, ex ante, agent 1 has an incentive to acquire information if 2 is uninformed (his expected utility is 0.00033 if he is uninformed, versus 0.00066 if he is informed).

To sum up, when nobody is informed, both agents gain from acquiring private information; if 1 is informed, 2 gains from acquiring private information and 1 benefits from it as well.

TABLE 1

	1 Not informed 2 Not informed		1 Informed 2 Not informed		1 Informed 2 Informed	
	α	β	α	β	α	β
Agent 1	−0.00094	0.0016	0.0013	0.00008	0.0114	0.0013
Agent 2	0.0078	−0.0047	0.0113	0.0010	0.0117	0.0013

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