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Do sunspots matter when agents are Choquet-expected-utility maximizers?

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Abstract

We consider a two-period, complete market economy in which agents' preferences are represented by a non-additive expected utility. If agents are optimistic, i.e. if the measure according to which they compute their expected utility is subadditive, sunspots matter at equilibrium. If agents are pessimistic, i.e. if their measure is convex, and share the same beliefs, sunspots do not matter at equilibrium, and the (normalized) equilibrium price is indeterminate. In this latter case, one can even allow for different beliefs among agents and still have that sunspots do not matter. The analysis is contrasted with the case of additive beliefs studied by Cass and Shell, Journal of Political Economy 91 (1983), pp. 193–227. © 1998 Elsevier Science B.V. All rights reserved

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1. Introduction

Starting with the seminal paper Cass and Shell (1983), a whole strand of literature has explored conditions under which sunspots might have an effect on the equilibrium allocation. This paper is a contribution to this literature. Balasko (1983) first pointed out that Cass and Shell's analysis relied upon the use of von Neumann–Morgenstern utility functions, and extended their analysis to a more general setup. We take a different route and deal with agents that are not expected-utility maximizers, but rather, Choquet- (or non-additive-) expected-

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utility maximizers. If agents are pessimistic, different beliefs do not necessarily lead to sunspot equilibria. If agents are optimistic, sunspots matter even if markets are complete and participation is unrestricted. Had agents been additive expected utility maximizers with same beliefs, Cass and Shell impossibility theorem would have applied, and sunspots would not matter in this economy. On the other hand, sunspots matter with additive beliefs whenever beliefs are different.

Choquet-expected-utility theory seems useful to study an economy with sunspots, in which agents might not be completely confident in their probabilistic assessment of the sunspot occurrence. Schmeidler (1989) showed that a way to express a possible lack of confidence in one's probabilistic assessment of the future is to suppose that beliefs are represented by a non-additive measure. While we will focus on this interpretation, it must be pointed at the outset that a similar analysis can be made in the case of risk (i.e. there exists a known 'objective' probability distribution on the state space) with rank-dependent expected-utility maximizers (as introduced by Quiggin, 1982; Yaari, 1987). A RDEU decision maker 'distorts' probability in much the same way they 'distort' monetary payoffs through the use of an utility function.[1]

In any event, we show in this paper that sunspots are bound to matter if some agents are optimistic and if an equilibrium exists. This is so because optimism (represented by subadditive beliefs) induces a non-convexity in agents' preferences. That non-convexities can lead to equilibria at which sunspots matter is not new. Guesnerie and Laffont (1990) show by example that if agents' von Neumann–Morgenstern utility functions are not strictly concave, then sunspots may matter. Shell and Wright (1993) find that sunspots matter in the presence of non-convexities in the consumption set. However, the non-convexity introduced here in the agents' preferences is of a different type and rests on the non-additivity of the measure that 'represents agents' beliefs'.

On the other hand, if agents are all pessimistic (i.e., their beliefs are represented by a convex measure), and even if they have different beliefs about sunspots occurrence, then the latter need not matter at equilibrium, unlike what happens in Cass and Shell (1983), where in the presence of differences in agents' beliefs, sunspots are bound to matter. Furthermore, the equilibrium prices are indeterminate. This is due to the fact that the functional representing an agent's preferences is not differentiable at points of certainty (even if the certainty utility index is differentiable).

Section 2 presents the model, which is similar to that of Cass and Shell (1983) but for the representation of agents' preferences. Section 3 is the heart of the paper and characterizes situations in which sunspots do or do not matter, as well as a simple illustration, giving graphically the intuition behind the formal results.

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[1] That RDEU is formally a particular case of Choquet-expected-utility is shown in Wakker (1990b).

2. The model and the representation of agents' preferences

The model is a standard two-period, sunspot, exchange economy, but for the agents' preferences. There are H consumers, $h = 1,\dots,H$, consuming C goods, $c = 1,\dots,C$ in the first period (called spot 0) and in state $s \in \boldsymbol{S} = \{1,\dots,S\}$ in the second period. Agents can trade at spot zero all possible contingent commodities. Consumption by household h in state s is denoted $x_h(s)$, for $s = 0, 1,\dots, S$. Prices for goods contingent on s are denoted $p(s)$. Household h has endowments $e_h(s)$ in spot s, and we will restrict our attention to the sunspot case, therefore assuming:

Assumption 1. $e_h(s) = e_h(s') \equiv e_h(1)$ for all h, and all $s, s' > 0$.

An agent's preferences are represented[2] by a utility function $u_h : \mathbb{R}_+^{2C} \to \mathbb{R}$ and a capacity π_h, i.e. a function from 2^S to $[0,1]$ such that $\pi_h(\emptyset) = 0$, $\pi_h(\boldsymbol{S}) = 1$ and, for all $E, F \in 2^S$, $E \subset F \Rightarrow \pi_h(E) \leq \pi_h(F)$.

A capacity π is superadditive if for all E, $F \in 2^S$ such that $E \cap F = \emptyset$, one has $\pi(E \cup F) \geq \pi(E) + \pi(F)$. π is said to be convex if for all $E, F \in 2^S$, $\pi(E \cup F) + \pi(E \cap F) \geq \pi(E) + \pi(F)$.

Wakker (1990a) has provided an interpretation of these cases, where convexity of the capacity reflects an agent's pessimism, while subadditivity reflects an agent's optimism. It is useful at this point to briefly recall his results.[3] Two acts f and g are said to be comonotonic if there do not exist states s and s' such that[4] $f(s) \succ f(s')$ and $g(s') \succ g(s)$. Say that $\succeq$ satisfies mixture-independence if for all acts f, g, h and all $\lambda \in (0,1)$

$$f \succ g \Rightarrow \lambda f + (1-\lambda)h \succ \lambda g + (1-\lambda)h.$$

Then an agent is pessimistic if mixture independence is required to hold only if g and h are comonotonic, while he is optimistic if it is required to hold only if f and h are comonotonic. Wakker (1990a) showed that the first case can be represented via a convex capacity, while the second case implies that agents' preferences must be represented by a subadditive capacity. Observe that if two acts are comonotonic, then a mixture of them does not reduce the uncertainty attached to them, while if they are not comonotonic, a mixture of them provides some hedge against uncertainty. Hence, the reduction of uncertainty through hedging that is provided when f and h are not comonotonic leads to further appreciation of the mixture $\lambda f + (1-\lambda)h$ when the agent is pessimistic. On the other hand,

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[2] Axiomatizations of such a representation of agents' preferences can be found in Schmeidler (1989) and Sarin and Wakker (1992).

[3] See also the discussion of Proposition 3 in Yaari (1987), as well as Karni and Schmeidler (1991 pp. 1805–1807) and Chateauneuf (1991).

[4] $f(s)$ denotes, with a slight abuse of notation, the lottery that gives $f(s)$ for sure.

if the agent is optimistic, he will dislike the reduction of uncertainty, and thus does not value the hedging provided by h against g.

Agents seek to maximize $\int u_h(x_h(0),.)\,\mathrm{d}\pi_h$, where the integral is the Choquet integral. Let $\mathcal{U}_h(x_h(0),x_h(1),\dots,x_h(S))$ denote this Choquet integral. In the case of a finite state space, and if we assume $u_h(x_h(0),x_h(1)) \leq u_h(x_h(0),x_h(2)) \leq \cdots \leq u_h(x_h(0),x_h(S))$, it takes the following form:

$$\mathcal{U}_h(x_h(0),x_h(1),\dots,x_h(S)) = \sum_{s=1}^{S-1} (\pi_h(s,\dots,S) - \pi_h(s+1,\dots,S)) \times u_h(x_h(0),x_h(s)) + \pi_h(S)u_h(x_h(0),x_h(S)).$$

The following assumption on u_h will be maintained throughout the paper.

Assumption 2. $u_h : \mathbb{R}_+^{2C} \to \mathbb{R}$ is strictly concave, differentiable, strictly increasing and satisfies boundary condition for all h.

In particular, our results do not rest upon non-concave utility index, as in Guesnerie and Laffont (1990). Let's now define a rational expectations equilibrium in this framework.

Definition. $(p(0), p(1),\dots, p(S))$ is an equilibrium if

(i) given $(p(0), p(1),\dots, p(S))$, $(x_h(0),x_h(1),\dots,x_h(S))$ is a solution, for all h, to:

$$\begin{aligned} \max \quad & \mathcal{U}_h(x_h(0),x_h(1),\dots,x_h(S)) \\ \text{s.t.} \quad & \sum_{s=0}^{S} p(s)(x_h(s) - e_h(s)) = 0. \end{aligned}$$

(ii) $\sum_h (x_h(s) - e_h(s)) = 0 \quad s = 0, 1,\dots, S.$

As usual, sunspots are said to matter if there exists $s,s' > 0$ and h such that $x_h(s) \neq x_h(s')$, i.e. if consumption differs across states for at least one agent.

3. Equilibrium and sunspots

We first recall two results that are due to Cass and Shell (1983), and then proceed to show how they are modified in our setup.

3.1. The additive case

We suppose here that agents are expected utility maximizers, and therefore that π_h is an additive measure for all h. In this case Cass and Shell (1983) showed the following two propositions. The first concentrates on the case in which agents have identical beliefs.

Proposition 3.1. *Suppose agents all share the same (additive) beliefs, then sunspots do not matter at equilibrium.*

The second proposition, on the contrary, deals with the case in which agents have different beliefs about the probability of sunspots occurrence.

Proposition 3.2. *Suppose there exists* h *and* h' *such that* $\pi_h(s) \neq \pi_h'(s)$ *for some* s, *then sunspots matter at equilibrium.*

We stress that the first result is valid only under the assumptions that u_h is strictly concave and that market participation is unrestricted, both of which are satisfied in the present framework. We now show that these propositions are not valid if agents have non-additive beliefs.

3.2. The pessimistic case

Let $\text{Core}(\pi) = \{q \mid q$ is an additive probability measure and $q(E) \geq \pi(E)\ \forall E \in 2^S\}$. If π is convex,[5] its core is non-empty and $\int u_h(x_h(0),.)\,\mathrm{d}\pi = \min_{q\in \text{Core}(\pi)} \int u_h(x_h(0),.)\,\mathrm{d}q$.

This property allows us to show that if pessimistic agents share the same beliefs, then sunspots do not matter.

Proposition 3.3. *Suppose that all agents have the same beliefs, represented by the convex capacity* π. *Suppose there exists* $\varphi \in \text{Core}(\pi)$ *such that* $\varphi(s) > 0$ *for all* $s > 0$. *Then, sunspots do not matter at equilibrium.*

Proof. The proof is adapted from Cass and Shell (1983). Let κ be a probability measure in $\text{Core}(\pi)$ such that $\kappa(s) > 0$ for all $s > 0$. Suppose that at an equilibrium $x_{h'}(s) \neq x_{h'}(s')$ for some h' and some s, s'. Let $\bar{x}_h = \sum_{s=1}^{S} \kappa(s) x_h(s)$ for all h. This allocation is feasible.

Now, $\mathcal{U}_h(x_h(0), \bar{x}_h, \ldots, \bar{x}_h) = u_h(x_h(0), \bar{x}_h)$. By concavity of u_h, $\mathcal{U}_h(x_h(0), \bar{x}_h, \ldots, \bar{x}_h) \geq \sum_{s=1}^{S} \kappa(s) u_h(x_h(0), x_h(s))$. Since $\kappa \in \text{Core}(\pi)$, we get that

$$\mathcal{U}_h(x_h(0), \bar{x}_h, \ldots, \bar{x}_h) \geq \mathcal{U}_h(x_h(0), x_h(1), \ldots, x_h(S)) \quad \text{for all } h.$$

Furthermore, the above inequality is strict for h' by strict concavity of $u_{h'}$ and because $x_{h'}(s) \neq x_{h'}(s')$ (together with the fact that $\kappa(s) > 0$ for all $s > 0$). Thus, the equilibrium allocation is Pareto dominated by the allocation $\bar{x}$, a

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[5] See e.g. Gilboa and Schmeidler (1994).

contradiction to the first welfare theorem, which holds in the present economy. Therefore, sunspots do not matter. □

The restriction that agents hold the same beliefs is not necessary. Indeed, one can let agents somewhat disagree in their beliefs about sunspots occurrence and still find that they do not matter at equilibrium. To show this, observe that the argument in the proof of Proposition 3.3 holds under the weaker assumption that there exists a measure κ that belongs to the intersection of the different capacities π_h's cores.

Proposition 3.4. Suppose that π_h *is convex for all* h *and that there exists* $\varphi \in \cap_h \mathrm{Core}(\pi_h)$ *such that* $\varphi(s) > 0$ *for all* $s > 0$. *Then, sunspots do not matter at equilibrium.*

Hence, if agents' beliefs are not too different, sunspots do not matter in this economy.

$\mathscr{U}_h$ being a minimum of functions, it is not necessarily differentiable at some points: in particular, the non-differentiability of $\mathscr{U}_h$ at points of certainty leads to equilibrium price indeterminacy.[6]

Proposition 3.5. Suppose π_h *is convex for all* h, *and* $\cap_h$ *Core*(π_h) *contains two measures,* ϕ *and* ψ *such that* $\phi(s) > 0$ *and* $\psi(s) > 0$ *for all* $s > 0$. *Then, there is a continuum of (normalized) equilibrium prices.*

Proof. In the proof, prices are taken to be in the $((S+1)C-1)$-dimensional simplex.

Suppose all agents have the same additive prior, $\phi \in \cap_h \mathrm{Core}(\pi_h)$, with $\phi \gg 0$ (call this economy the 'ϕ-economy'). Then, sunspots do not matter at equilibrium and $x_h(s) = x_h(s')$. From first-order conditions, one gets

$$\frac{\phi(s)}{\phi(s')} p(s') = p(s).$$

For a different common additive prior ψ (such that $\psi \gg 0$), equilibrium prices satisfy

$$\frac{\psi(s)}{\psi(s')} \tilde{p}(s') = \tilde{p}(s)$$

and therefore $p \neq \tilde{p}$ since there exist states s, s' such that $\phi(s)/\phi(s') \neq \psi(s)/\psi(s')$.

Hence, different common additive priors lead to different equilibrium prices. We now show that an equilibrium of the ϕ-economy is also an equilibrium of the original economy.

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[6] This result rests on the same intuition as in the model of Epstein and Wang (1994), who show the possible indeterminacy of asset prices in an infinite horizon, representative agent type of model.

Let $(p, \bar{x})$ be an equilibrium of the ϕ-economy. By construction, $\bar{x}_h$ is affordable at price p for household h. Furthermore, $\bar{x}(s) = \bar{x}(s')$. Let x'_h be another affordable bundle and suppose that $\mathscr{U}_h(x'_h) > \mathscr{U}_h(\bar{x}_h)$. Then

$$\min_{\psi \in \mathrm{Core}(\pi_h)} \sum_s \psi(s) u_h(x'_h(0), x'_h(s)) > \min_{\psi \in \mathrm{Core}(\pi_h)} \sum_s \psi(s) u_h(\bar{x}_h(0), \bar{x}_h(s))$$
$$= u_h(\bar{x}_h(0), \bar{x}_h(s))$$

and in particular $\sum_s \phi(s) u_h(x'_h(0), x'_h(s)) > u_h(\bar{x}_h(0), \bar{x}_h(s)) = \sum_s \phi(s) u_h(\bar{x}_h(0), \bar{x}_h(s))$, a contradiction to the fact that $\bar{x}$ is an equilibrium allocation at price p in the ϕ-economy. Therefore, $(p, \bar{x})$ is an equilibrium of the original economy.

Now, $\bigcap_h \mathrm{Core}(\pi_h)$ is convex. Hence, if it contains two points, it contains a continuum of points. To each of these additive priors correspond a different equilibrium price vector, and there exists a continuum of equilibrium prices. □

This proposition therefore shows that a given equilibrium allocation can be supported by a continuum of price vectors. However, it does not give any information on the number of equilibrium allocations. Indeed, the next result shows that equilibrium allocations are locally unique.

Proposition 3.6. Suppose π_h *is convex for all* h, *and* $\bigcap_h \mathrm{Core}(\pi_h)$ *contains a measure* φ *such that* $\varphi(s) > 0$ *for all* $s > 0$. *Then, equilibrium allocations are locally unique.*

Proof. The proof is standard and we omit the formal argument. Informally, since sunspots do not matter, the equilibrium allocation is an equilibrium of the related 'certain' economy. In this latter economy, equilibrium allocations are locally unique (Debreu, 1970). □

Remark. The results obtained here can be cast in a different framework,[7] where the objective distribution of the sunspot is known to all, and agents conform to the theory of rank-dependent-expected-utility (Quiggin, 1982; Yaari, 1987). More specifically, the results of this section can be applied to the case of strongly risk-averse[8] rank-dependent-expected-utility maximizers. It is easy to show that, for such decision makers, the intersection of the cores is non-empty (it always contains the objective probability distribution). It is of interest to note that sunspots do not matter in such a model (with agents transforming differently the objective probability distribution) under the weaker assumption of weak risk aversion (see Cohen (1995) for a definition and characterization in the RDEU model).

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[7] See Tallon (1995) for more details on this.

[8] See Cohen (1995).

3.3. The optimistic case

We now turn to the case of optimistic agents, i.e. agents whose beliefs are represented by a subadditive capacity. The next result shows that optimism is a sufficient condition for sunspots to matter. The key to the result is to observe that, if agents are Choquet-expected-utility maximizers, and if the capacity that represents, together with their utility index, their preferences is subadditive, then the overall utility function is not quasi-concave.

Proposition 3.7. Suppose an equilibrium exists. If there exists h such that π_h is subadditive (and π_h is not additive) then, sunspots matter at equilibrium.

Proof. Suppose by contradiction that $x_h(s)=x_h$ for all s for the 'subadditive agent'. In the rest of the proof we deal with this agent and drop the subscript h.

We first show that there exists a state, say state 1, such that $1-\pi(\{2,\dots,S\})<\pi(1)$. Suppose this were not the case, i.e., since π is subadditive, that $1-\pi(\{2,\dots,S\})=\pi(1)$. Then it must be the case (by subadditivity of π) that $1\geq\pi(1)+\pi(2)+\cdots+\pi(S)$. But this is a contradiction to the fact that π is subadditive and not additive, which implies that $1=\pi(\{1,\dots,S\})<\sum_{s=1}^{S}\pi(s)$.

Let $x'(1)=x+\varepsilon$, $x'(S)=x-[p^1(1)/p^1(S)]\varepsilon$ and $x'(s)=x$ for $s\neq 1,S$, with $\varepsilon=(\varepsilon,0,\dots,0)$. The consumption vector x' is affordable at price p. p being an equilibrium, one has

$$\mathscr{U}[x(0),x'(1),\dots,x'(S)]\leq\mathscr{U}[x(0),x,\dots,x]\quad\text{for all }\varepsilon.$$

In the case $\varepsilon<0$, we have, by definition of the Choquet integral

$$\begin{aligned}\mathscr{U}[x(0),x'(1),\dots,x'(S)]&=(1-\pi(\{2,\dots,S\}))\,u[x(0),x'(1)]\\&\quad+\pi(S)u[x(0),x'(S)]\\&\quad+(\pi(\{2,\dots,S\})-\pi(S))\,u[x(0),x],\end{aligned}$$

since $x'(s)=x'(s')=x$ for $s,s'\neq 1,S$. Hence,

$$\begin{aligned}&\mathscr{U}[x(0),x,\dots,x]-\mathscr{U}[x(0),x'(1),\dots,x'(S)]\\&\quad=\pi(S)(u[x(0),x]-u[x(0),x'(S)])\\&\qquad+(1-\pi(\{2,\dots,S\}))(u[x(0),x]-u[x(0),x'(1)]).\end{aligned}$$

Dividing by ε and taking limits, one then gets that

$$\pi(S)\frac{p^1(1)}{p^1(S)}\frac{\partial u(x(0),x)}{\partial x^1}-(1-\pi(\{2,\dots,S\}))\frac{\partial u(x(0),x)}{\partial x^1}\leq 0.$$

Therefore,

$$\frac{p^1(1)}{p^1(S)}\leq\frac{1-\pi(\{2,\dots,S\})}{\pi(S)}.$$

Similarly, for $\varepsilon > 0$,

$$\frac{p^1(1)}{p^1(S)} \geq \frac{\pi(1)}{1 - \pi(\{1,\ldots,S-1\})}.$$

From the above two inequalities, it follows that

$$\frac{1 - \pi(\{2,\ldots,S\})}{\pi(S)} \geq \frac{\pi(1)}{1 - \pi(\{1,\ldots,S-1\})},$$

which contradicts the facts that $\pi(1) > 1 - \pi(\{2,\ldots,S\})$ and $1 - \pi(\{1,\ldots,S-1\}) \leq \pi(S)$. Hence, sunspots matter at equilibrium. □

The mechanism through which sunspots matter when agents are optimistic is easy to understand. The subadditivity of the capacity representing h's beliefs implies that $\mathcal{U}_h$ is not quasi-concave (see Fig. 2 for an intuition). As a result, an optimistic agent prefers to take bets rather than to get a sure lottery. Guesnerie and Laffont (1990) show the possibility of sunspots in the additive framework if agents do not have strictly concave von Neumann–Morgenstern utility functions. Our result has nothing to do with non-strict concavity of u, but rests on the non-additivity of the capacity which is at the source of the non-strict quasi-concavity of the overall utility function (our $\mathcal{U}$). The advantage of the present construction is to allow a separation between the attitude towards uncertainty (captured by the non-additivity of the capacity) and the concavity of the utility index. Observe also that Guesnerie and Laffont (1990) need to impose in their example that u is convex on a neighborhood of the endowment point. That the non-quasi-concavity of $\mathcal{U}$ arises precisely at the certainty point is a result in the present framework, rather than an assumption.

Proposition 3.7 says nothing about existence of an equilibrium. Indeed, in this non-convex environment, an equilibrium need not exist. It is however possible to apply the usual argument (see, e.g., Hildenbrand and Kirman, 1988) to show that an equilibrium exists if we let the number of agents go to infinity. Note also that Proposition 3.7 establishes the fact that there is no non-sunspot equilibrium in our framework, if some agents are optimistic.

Finally, note that, as in Guesnerie and Laffont (1990), the sunspot equilibrium Pareto dominates the initial endowments. However, initial endowments are not an equilibrium allocation of the model. Furthermore, the first welfare theorem applies and the sunspot equilibrium is Pareto optimal.

3.4. An illustration

We illustrate here the above analysis. Assume there is one good, two states (α, and β), no consumption in the first period. Choquet-expected utility takes a particularly simple form. Let $A_h = \pi_h(\alpha)u_h(x_h(0), x_h(\alpha)) + (1 - \pi_h(\alpha))u_h(x_h(0),$

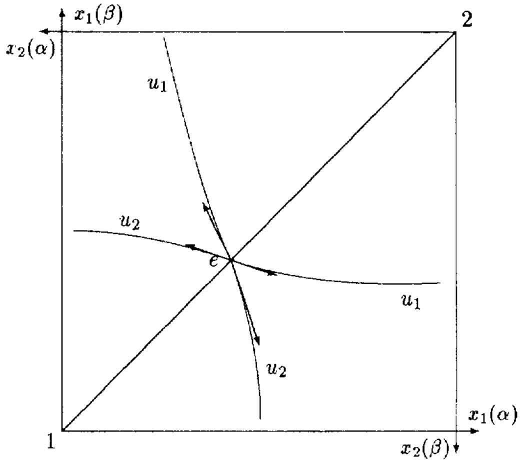

Fig. 1.

$x_h(\beta))$ and $B_h = \pi_h(\beta)u_h(x_h(0),x_h(\beta)) + (1-\pi_h(\beta))u_h(x_h(0),x_h(\alpha))$. Then,

$$\mathscr{U}_h((x_h(0),x_h(\alpha),x_h(\beta)) = \begin{cases} \min(A_h,B_h) & \text{if } \pi_h(\alpha)+\pi_h(\beta) \leq 1, \\ \max(A_h,B_h) & \text{if } \pi_h(\alpha)+\pi_h(\beta) \geq 1. \end{cases}$$

If the measure is convex, the agent takes the minimum of the possible cases (i.e., he expects uncertainty to turn out unfavorably), while if the measure is subadditive, he takes the maximum (i.e., he expects uncertainty to turn out favorably).

Concentrate first on the pessimistic case, and suppose that $\pi_1(\alpha)/\pi_1(\beta) \neq \pi_2(\alpha)/\pi_2(\beta)$, that is, agents have different beliefs about the sunspot occurrence.[9] The initial endowments is the only equilibrium of this economy, as shown in Fig. 1, and therefore sunspots do not matter (compare with the analysis in Cass and Shell, 1983, pp. 210–211).

Furthermore, equilibrium prices are indeterminate, as shown on Fig. 1: the equilibrium relative price $p(\alpha)/p(\beta)$ can take any value $[\max_h(\pi_h(\alpha)/(1-\pi_h(\alpha)), \min_h((1-\pi_h(\beta))/\pi_h(\beta))]$. If the previous interval is empty, then sunspots do matter at equilibrium. Thus, difference of opinions about the relative likelihood of sunspots can lead, if they are sufficiently large, to equilibrium at which sunspots matter (recall that in Cass and Shell (1983) *any* difference in agents' beliefs leads to an equilibrium at which sunspots matter).

The optimistic case can also be illustrated easily. In the case of optimistic agents, the non-differentiability already noted induces a non-convexity as well: if $(1-\pi(\beta))/\pi(\beta) < \pi(\alpha)/(1-\pi(\alpha))$, then indifference curves are not convex along

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[9] In Fig. 1, $\pi_1(\alpha)=\frac{1}{4}$, $\pi_1(\beta)=\frac{1}{3}$, while $\pi_2(\alpha)=\pi_2(\beta)=\frac{1}{4}$.

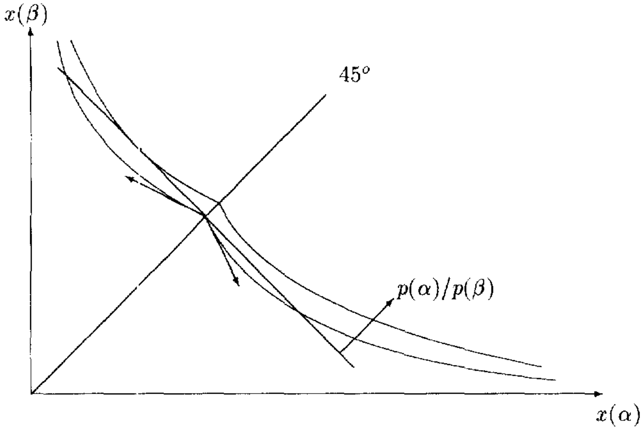

Fig. 2.

the diagonal, as shown in Fig. 2. It is readily seen that this condition is equivalent to π being subadditive, i.e. to the agent being optimistic.

In the situation represented in Fig. 2, sunspots are bound to matter, for the endowment point, which rests on the diagonal, will never be a solution to the agent's maximization problem, no matter what the prices are. Indeed, as shown on the figure, if $p(\alpha)$ and $p(\beta)$ are equilibrium prices, the agent will consume more in state β than in state α. The non-quasi-concavity in $\mathscr{U}$ introduced when an agent is optimistic is crucial in explaining how sunspots could matter in this otherwise well-behaved economy.

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