Parenthood and Gender Labour Income Gap :
The Role of Working Conditions

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Abstract

During the transition to parenthood mothers and fathers look for different job amenities to adjust to their new obligations. At the same time, the transition to parenthood increases the gender labour income gap. However, there is no consensus on the role of working conditions in this increase. Moreover, working conditions are often limited to job flexibility and autonomy. Based on a panel data from two French surveys about working conditions, this study tries to estimate and decompose the effect of parenthood on the gender labour income gap. The main contributions are the use of original propensity scores and Kitagawa-Oaxaca-Blinder decomposition methods for triple differences and the introduction of physical constraints in the study of working conditions. The results show that fathers and mothers working conditions evolve differently. However, these differences have no impact on earnings and cannot explain the observed gender labour income gap increase. On the other hand, the increasing share of unemployed and part-time working mothers can explain it entirely. This result seems partly driven by the gendered allocation of roles within the household. Indeed, mothers declare more frequently working part-time to take care of children.

Keywords: labour Economics, Inequalities, Gender, Gender labour Income Gap, Parenthood, Children, Motherhood Penalty, Working Conditions, Physical Constraints, Propensity Scores, Decomposition.

JEL classifications: J13 - J17

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1 Introduction

The birth of the first child and the transition to parenthood play a major role in people's lives on both a personal and professional level. Household tasks and childcare are time-consuming and parents need to adapt their lives to it. However, the share of household chores and childcare between men and women is generally unequally allocated and the latter assume a larger part of it. Hence, we might expect a divergence in labour market dynamics between young mothers and young fathers, contributing to the gender labour income gap. Several articles highlight the role of job amenities in parents' career paths and their effect on the gender labour income gap. However, these job amenities are not always well-defined and there is no consensus on their positive effect on the gender labour income gap.

This thesis presents new variations of the propensity score and Kitagawa-Oaxaca-Blinder decomposition methods to investigate the effect of parenthood on the gender labour income gap and the role of working conditions. These methods are similar to triple differences with two periods, two groups of interest and two groups with one receiving the treatment and one used as counterfactual. The propensity score method uses machine learning to estimate the similarity of the individuals and to emphasize the comparable ones. This is used to accurately estimate the parenthood effect. The decomposition method separates the effect of differences in characteristics and in coefficients to highlight the differences between the subgroups. This is used to estimate the contribution of differences in characteristics and coefficients to the parenthood effect.

Using a balanced panel data from two French surveys on working conditions, this thesis confirms the existence of a gap in average labour income at the expense of women. Moreover, it also confirms a higher increase of this gap between parents during the period, than between those who remain childless. The results also show differences of evolution in working conditions between fathers and mothers. The decompositions show that the effect of parenthood only happens through the motherhood penalty. labour force explains all this motherhood penalty with an increasing share of mothers unemployed or working part-time after the birth of their first child. The surveys do not provide information on the reasons of unemployment but mothers working part-time report doing it mostly to take care of their children.

Besides these results, this thesis also contributes to the literature on the effect of job amenities and working conditions on earnings and parenthood effect, with the study of physical constraints indicators. It shows that most job flexibility and physical constraints indicators does not have a significant effect on earnings, gender earning gap and

parenthood effect.

The remainder of this thesis is structured into five main sections. Section 2 presents the literature review on the gender labour income gap, the parenthood effect and the role of working conditions in a first part and the literature reviews on propensity scores and the Kitagawa-Oaxaca-Blinder decomposition in a second part. Section 3 describes the overall methodology of this thesis and details the original estimation methods used. Section 4 presents the data used. Section 5 shows the results, first on the whole population including non-employed individuals and then on the population of individuals who are employed in both periods. Finally, Section 6 discuss these results and concludes.

2 Literature Review

This section is separated in two parts, first it presents the literature review related to the subject of this thesis : the gender labour income gap and the role of parenthood. Then it presents the literature reviews related to the methods used in this thesis : the propensity score and the Kitagawa-Oaxaca-Blinder decomposition.

2.1 Gender labour Income Gap and Parenthood

The study of economic inequalities started with Becker's book (1957) *"The economics of discrimination"* about the wage gap between whites and blacks in the US. He defines economic discrimination on the labour market as the difference of treatment between individuals with identical productive characteristics. After 1964 researchers start to study economic inequalities between men and women. Currently, this question remains a hot topic and the scientific literature continues to grow.

2.1.1 The Gender labour Income Gap Evolution Across the Years

The first estimations of the gender wage gap are made by Sanborn (1964) (women earned 58% of men wage in 1949 in the US), Fuchs (1971) (60% in 1960) and Cohen (1971) (55% in 1969) using different US data sources. It is then estimated in different countries and several studies shows its evolution across. For example Gallen, Lesner and Vejlin (2019) show that in Denmark women earned 62% of men wage in 1980 for 75% in 2010. Blau and Kahn (2017) find similar results in the US with a ratio going from 60% to more than 75% during the same period.

According to Blau and Kahn (2017), the increase of women participation to the labour market, education duration and labour-market experience are the main factors

which explain this convergence. Other explanations exist such as changes in job tasks driven by technology changes (Black and Spitz-Oener 2010) or a decrease in sector and industrial segregation (Mandel and Semyonov 2014; Khitarishvili 2018). Moreover, the unexplained part also decreases over time (Blau and Khan 2006; Mandel and Semyonov 2014). According to Castagnetti, Rosti and Töpfer (2017), the anti-discrimination legislation seems to have a negligible impact compared to the effect of observable characteristics.

However, most of the authors observe that the convergence slows down (Blau and Khan 2006). This plateau is also observed by Meurs and Ponthieux (2006). Using French data they observe that the gender wage gap only decreased by 1 percentage point during a period of 10 years, going from 26.2% in 1990 to 25.3% in 2002. This is partly due to the fact that women have already catch up with men on different characteristics such as education where women have already surpassed men. This slowdown shows that the causes of the remaining gender labour income gap are not the same ones as in the middle of the 20^{th} century.

A part of the current remaining gap can be explained by different criteria. Bozio, Dormont and Garcia-Penalosa (2014) estimate that in average working women earn 24.5% less than working men in France in 2012. They decompose it and find that 10.9 percentage points can be explained by differences in time worked and part-time jobs, 7.3 percentage points can be explained by sector and job segregation or education and 6.3 percentage points remain unexplained with these variables.

With this decomposition method we can measure the impact of differences in characteristics on the differences in labour incomes. However, this method cannot explain the mechanism behind it, nor the reason why men and women have different characteristics.

2.1.2 Gender labour income Gap Evolution Across the Life

To understand the mechanism behind the gender labour income gap, we can take a look at its evolution through individuals' life. First a gender labour income gap is already observed in the early career. However, it is lower than what is measured over the whole population. For example Goldin (2014) finds an average gap of 10% for the most recent cohort born in 1978. Controlling for education, experience and job, Combet and Oesch (2019) find an unexplained gender wage gap between 3 and 6 percent in favor of men in Switzerland for the early career. They emphasize that in average it represents half a monthly labour income each year. Similar results are obtained by others researchers in different contexts (Manning and Swaffield 2008; Napari 2009; Ochsenfeld 2014).

With the life cycle approach, Goldin (2014) observes the evolution of the gender wage gap in France through the life of men and women for different cohorts. Similarly, to the previous articles, the younger cohorts have a lower wage gap and within each cohort, there is a lower wage gap in the early career. This is followed by an increase until the age of 40 and a slight decrease afterward.

By following a British cohort born in 1958, Joshi et al. (2019) try to explain this evolution of the gender wage gap. They find that working experience is the main contributor to the explained gender wage gap and is different between men and women partly because of the family formation. Their results confirm the importance of the transition to parenthood in the observed gender wage gap.

2.1.3 The Gender labour Income Gap and the Transition to Parenthood

To investigate the reason why the gender labour income gap increases with age, researchers suggest to focus on different events affecting workers' life. Using event studies, the focus is made on the transition to parenthood with the birth of the first child. Using Danish data Kleven, Landais and Søgaard (2019) find a sharp decrease of approximately 30% in women average labour income after the birth of the first child, while men's one stays constant. They partly explain this by a fall of 10% in women's participation, time worked and wages, which is not observed for men. This fall does not fully recover afterwards with a remaining gap of 20% after 20 years. This finding is consistent with previous results obtained using other methods (Waldfogel 1997; Lundberg and Rose 2000). Moreover, they estimate the effect of this transition to parenthood on the gender labour income gap and find a child-related effect between 15% and 20% relatively constant between 1980 and 2013. With a gender labour income gap of 45% in 1980, differences in education explain 15 percentage points and child-related effect explains 15 percentage points too. In 2013, with a gender labour income gap of 25%, differences in education only explain 3 percentage points while the child-related effect explains 18 percentage points. They conclude that the transition to parenthood is now the main origin of gender wage gap in Denmark. On the counterpart several authors find a relatively low wage gaps between men and women without children (Waldfogel 1998; Polachek 2004; Blau and Kahn 1992).

The event study method is accurate to estimate the impact of the transition of parenthood on the gender wage gap but the mechanisms which explain such a gap between mothers and fathers are not clearly highlighted. In a recent meta-analysis Cukrowska and Matysiak (2020) summarize the different causes of motherhood penalty.

First, the birth of the first child might imply a career break for mothers and hence a potential loss in human capital (Gangl and Ziefle 2009; Napari 2010). Then there might be job segregation depending on job amenities (Felfe 2012; Anderson, Binder and Krause 2003). Finally, employers can discriminate against mothers (Corell, Benard and Paik 2007).

The question of job amenities and mothers participation to the labour market is linked with their role in the household and the family (Misra, Budig and Boeckmann 2001; Bertrand, Kamenica and Pan 2015). Indeed, the largest part of the non-paid work done within the household is done by women and it increases with parenthood (Kim and Cheung 2019). Consequently, working mothers need job amenities to regulate their paid and non-paid work (Fuwa 2004). This can be done by joining "family friendl" firms.

2.1.4 Firms and Parenthood

To estimate the impact of firms on individual wages Abowd, Kramarz and Margolis (1999) develop a method to identify firm fixed effects. Card, Cardoso and Kline (2016) combine this method with an Oaxaca-style decomposition to differentiate the gender wage gap within and between firms. They estimate that within firms women obtain 90% of men wage premium which is interpreted as bargaining effect. They also estimate that there is a sorting effect where women go to low-surplus firms with smaller wage premium for both genders. The authors conclude that these mechanisms can explain 20% of the gender wage gap in Portugal with two thirds due to sorting and one third due to bargaining.

Coudin, Maillard and Tô (2018) reproduce this analysis in France and find a similar sorting effect which can explain 11% of the gender wage gap in the French private sector. However, the authors find no evidence of a within-firm inequality. Decomposing the gender wage gap all along workers' life cycle, they find that this sorting mechanism strengthens after the birth of the first child.

To study more precisely the connection between family friendly firms and parenthood, Hotz, Johansson and Karimi (2017) use event studies on a Swedish employer-employee data. First they observe a fall in women's labour income after the birth of their first child similar to Kleven, Landais and Søgaard (2019). Then they compute the marginal willingness to pay of mothers for different workplace attributes and determine the family friendliness of each workplace. They observe that mothers working in family friendly workplaces have a higher wage while fathers does not. They explain this by the fact that these workplaces allow mothers to work more hours. A limit of their work is

that the working conditions are not directly included in the workplace characteristics but commented afterwards using another database. Without the working conditions, the mechanism between mothers' preferences and family friendliness cannot be directly observed.

2.1.5 Working Conditions

Some studies analyse directly the connection between parenthood and job amenities. The most studied job amenities are : the schedule flexibility and autonomy, the proximity of the workplace and overtime.

Schedule flexibility

Flexibility and autonomy are often presented as important job amenities for mothers in the literature about family friendly firms. These terms encompass different aspects of the working conditions and which may have different effects. Hotz, Johansson and Karimi (2017) define the job flexibility as the extent to which the workers can organize their schedule and especially their free time. On the other hand they define job autonomy as the extent to which the workers can choose the composition and organization of their work.

The preference of mothers for job flexibility is ambiguous and depends on what is measured. According to Stockon (2018), childless women have a lower willingness to pay for schedule control than childless men. However, motherhood increases this preference for schedule control while fatherhood does not. This can be explained by the results in Boushey (2008) where the author finds that mothers who control their schedule to address their caring responsibilities, do not suffer motherhood penalty.

The counterpart of this flexibility is that it is difficult to access it. According to Golden (2001, 2009b) the schedule flexibility is more frequent in high qualified jobs or low unemployment occupations and industries. It is obtained in exchange for working longer weekly hours, unusual working hours or by switching to part-time or self-employed jobs. It is important to distinguish the different these amenities : longer weekly-hours and unusual working hours are not compatible with the non-paid work implied by parenthood, while part-time jobs are.

Coudin, Maillard and Tô (2018) study the role of job amenities in the gender wage gap. They show that there is a sorting mechanism after birth where mothers choose firms with flexible work hours, part-time jobs and closer to their home.

Some flexible job amenities might be harmful to the long-term career of workers, so they avoid it. The mechanism might be a negative signal that one is less committed to the firm (Blair-Loy 2001) or of an insufficient availability, which can decrease the probability of promotion or salary increase. Blair-Loy (2002) estimates that employees are more likely to use and benefit from these policies if they have colleagues and supervisors strong enough to avoid the long-term negative effects. The role of the workers' social context is confirmed by Kelly and Kalev (2006) who observe that the managerial discretion has a key role in the access to these policies. As noted by Herr and Wolfram (2012) these mechanisms affect high-educated women who leave the labour force, at least temporally, due to motherhood and even more if they work in non-family-friendly firms.

These results are consistent with the model developed in Goldin (2014). In this model, after a given threshold in time worked, there is a discontinuity in productivity. Hence, firms might penalize heavily flexible schedules with a lower remuneration. This can lead to shift in occupations or hierarchy or leaving the labour force. The underlying idea is that these discontinuities are larger in high-skilled occupations and more educated women would be more penalized.

Overtime

An extension of schedule flexibility is the increase of overtime. According to Lott and Chung (2016) schedule control is correlated with an increase in overtime for men and full-time working women. For part-time working women, more schedule control is not associated with an increase in overtime. Moreover, the returns from overtime are lower for full-time working women than for men, even after controlling for gender segregation. The authors argue that men use schedule control and overtime to increase their income, while women use them to meet their family demand. In this case overtime might be the counterpart to obtain more schedule control.

However, overtime and work organization in some firms might be incompatible with parenthood. Cha (2013) shows that mothers are more likely to leave male-dominated occupations if they do overwork, while men and childless women does not. Moreover, they might leave labour-force too. In a following paper Cha and Weeden (2014) study the effect on the gender wage gap. They find that overwork and its hourly returns increase between 1979 and 2009 in the US. With a greater proportion of men doing overwork, this increases the gender wage gap, offsetting the convergence due to other characteristics such as education or human capital. This highlights the importance of overwork, especially for blue collars.

The difference in motherhood penalty is developed in Anderson, Binder and Krause (2003) where the author differentiate depending on education. They show that high-school graduates suffered a larger wage loss than high-school dropouts and college graduates. Indeed, high-school dropouts delay their return to labour-force until their children is older and does not suffer any wage loss and college graduates find flexible jobs which allow them to avoid wage losses.

The overall idea of this literature starts from the fact that household chores are unequally allocated between men and women. The birth of a first child increase the time needed for childcare and household chores which are mainly handled by mothers. To accommodate to this increase of non-payed work, they compensate with less participation to the labour force or with family friendly job amenities. Hence, they switch for firms and occupations which are considered more family friendly with flexible schedules and less demanding jobs. This might imply a decrease in their labour income and accentuate the gender labour income gap.

2.2 Methodologies

To estimate the effect of the first childbirth on labour income inequalities and the role of working conditions, multiple parameters need to be considered. First those who will have a child and those who will not, might be different beforehand, which can explain differences in labour incomes. Second becoming a parent probably affects other individual characteristics which can explain differences in labour incomes. Hence, I need to control for the effect of these characteristics.

To ensure this question different methodologies exist. A popular one in different fields and particularly in medicine is the inclusion of propensity scores. Another family of methodologies widely used to study inequalities in economics is the decomposition of the average differences between groups introduced by Kitagawa (1955), Oaxaca (1973) and Blinder (1973). In the following section I present the literature of these two methods and develop the parts related to panel data analysis.

2.2.1 Propensity scores

Rosenbaum and Runbin (1983) define the propensity score as the probability for an individual to receive a treatment depending on its characteristics. As summarized in Austin (2011), there are several methods to estimate the propensity score and four possible methods to remove the effects of confounding. The estimation methods are logistic regression (Rosenbaum and Runbin 1983) which is the most frequently used,

Classification And Regression Trees methods (CART) (Lee, Lessler and Stuart 2010) or neural networks (Setoguchi et al. 2008).

Estimating propensity scores

According to Lee, Lessler and Stuart (2010), logistic regression capacity to reduce bias is better than some machine learning methods with simple models but decreases when interaction and non-linear terms are omitted. In the other hand, bagged CART, random forests and boosted CART (or Gradient Boosting Machine (GBM)) propensity score models provide excellent results in terms of covariate balance and effect estimation, independently of sample size and the extent of non-additivity or non-linearity. Moreover, the boosted CART method prevents overfitting and flatters the estimations at the extreme values 0 and 1, avoiding a too high variability in weights. Hence, as noted in Ferri-Garcia and Ruedo (2020), the GBM method provides weights which gives more stable estimates. A limit of this type of machine learning is their "black box" nature which implies that we cannot see how the estimations are made. However, in the case of propensity score estimations, these results are not crucial. Another limit noted by Ferri-Garcia and Ruedo (2020) is that the estimated weights depend partly on the tuned parameters.

Methodologies with propensity scores

The estimated propensity scores can be used in multiple ways to remove the effect of confounding. The four main ones are : propensity score matching, stratification, inverse probability of treatment weighting and covariate adjustment. In propensity score matching, individuals with similar propensity scores in the treated and untreated group are grouped, generally by pairs, and directly compared to estimate the treatment effect on the outcome variable. The matching can be done with or without replacement (Rosenbaum 2002, Hill and Reiter 2006), by randomly selecting treated individuals with greedy matching or by minimizing the total within-pair difference of the propensity score. For more detailed and technical explanations see Rosenbaum (2002) and Austin (2011). A similar methodology is the stratification on the propensity score : the individuals are ranked based on their propensity score and separated in different groups. The treatment effect is estimated within each stratum by comparing the outcome of treated and untreated individuals. Then an overall treatment effect can be estimated (Rosenbeaum 1984). A different approach is the Inverse Probability of Treatment Weighting (IPTW)

using the propensity score. In this method, the propensity scores are used to compute weights which gives the probability for an individual to obtain the other treatment. Joffe et al. (2004) explain how these weights can be used to estimate causal treatment effects with Marginal Structural Models. On the other hand Lunceford and Davidian (2004) summarize multiple weighted estimators to estimate the treatment effect. The last method is the covariate adjustment using the propensity scores. In this method the outcome variable is regressed on the treatment and on the propensity scores which is used as a control variable. Here the regression model depends on the nature of the data.

Methodologies with more than two groups

There is are possible extensions of these propensity scores in the case of multiple treatments with more than two groups. All the previous methods to estimate the propensity scores and the treatment effects are transposed in this new framework. In their introduction McCaffrey et al. (2013) provide a great overview of it. In this case the propensity scores can be estimated through multinomial logistic regressions or with GBM models. With the former method, the estimated propensity scores are relative to a chosen group, while with GBM models, a propensity score can be estimated for each group. In the case of IPTW, the computation method of weights depends on which the treatment effect we are interested in. The two possibilities are the Average Treatment Effect (ATE) which compares the entire population average estimated outcome under one treatment or another, and the Average Treatment effect among the Treated (ATT) which compares the actual average outcome of those who received a treatment and their estimated average outcome if they received another treatment. Hence, the comparison of two treatment can be done in two ways with ATT, depending on the population of reference. For more technical details see McCaffrey et al. (2013).

Propensity scores and difference in difference models

Another use of these propensity scores can be found in Ryan, Burgess and Dimmick (2015) where the authors introduce matching based on propensity in a difference in difference model to estimate the effect of a policy change. Using propensity scores based on pre-treatment characteristics in a difference in difference results in more accurate estimation than a simple difference. Stuart et al. (2014) extend it by estimating pre and post-treatment propensity score computed with multinomial logistic regressions. This methodology implies that the characteristics of the individuals are now comparable

before and after treatment for both treated and not treated groups. Hence, the treatment effect is isolated from the effect of changes in characteristics.

2.2.2 The Kinagawa-Oaxaca-Blinder decomposition

Since Kitagawa (1955), Oaxaca (1973) and Blinder (1973), the decomposition of average differences between two groups has been widely used and especially in labour economics. This decomposition splits this average difference between differences in characteristics and differences in coefficients. The endowment component measures the contribution of differences in characteristics to the gap in outcome, for a group's set of coefficients. On the other hand, the coefficient component measures the contribution of differences in coefficients for the other group's set of characteristics. The second part of this two-fold decomposition is often called the unexplained part and is interpreted as inequalities, considering that all the other causes of differences are included in the regressions.

Other twofold decompositions

As explained by Cotton (1988) this two-fold decomposition is sensible to the choice of reference group and might lead to the over-evaluation of one group and the underestimation of the other in these components. Hence, several economists propose different measures to create an intermediary group of reference instead of one of the two studied groups (Reimer 1983; Cotton 1988; Oaxaca and Ransom 1994). However, these measures are limited and can lead to over or under-estimations of the endowment component due to covariance between the observable characteristics and omitted variables such as discrimination (Neumark 1988; Oaxaca and Ransom 1994).

Three fold decomposition

Another possible decomposition is the three-fold decomposition (Winsborough and Dicinson 1971; Althauser and Wigler 1972 ; Iams and Thornton 1975; Daymont and Adrisani 1984; Jones and Kelley 1984). While the two-fold decomposition uses different reference groups for the two components, the three-fold decomposition uses the same one for the endowment component and the coefficient. It also contains an interaction component which measures the joint contribution of differences in characteristics and endowments. In this decomposition, even if the choice of reference group stays arbitrary, the interaction between characteristics and coefficients is not hidden within the two

components. However, the interpretation of this interaction component is less intuitive than the two others. In this case the choice of reference group is based on the economic interpretation and the majority group is generally chosen.

Extensions to new models

With the development of modern econometric methods, the corresponding decomposition methods are created. Fortin (2011) presents a literature review of the different extensions. For example Freeman (1980) proposes a first decomposition of the dispersion. Fairlie (1999; 2005) and Yun (2004) adapt it to logit and probit models, followed by Fitzenberg et al. (2011). Montgomery and Powel (2003) apply it to tobit models. Bauer and Sinning (2008) generalize their analyses to non-linear models such as count-data, tobit and truncated regression models. Machado and Mata (2005) and Melly (2005) propose different decompositions for quantile regressions.

Decomposition for panel data

Concerning panel data analysis, several decompositions exist but most of them are limited to the comparison of multiple cross-sectional analyses. For example in the Simple Subtraction Method, the measure of interest is the difference between components of two periods as in DeLeire (2000) with probit models. Wellington (1993) formalizes it with two components : a first one that shows the changes in outcome due to changes in the endowments and a second for changes in coefficients. Smith and Welch (1989) compare also two periods but analyse it with a four-fold decomposition : a main effect component for the evolution on differences of endowments, a group interaction component that valuates the change in characteristics for a given difference in coefficients, a time interaction component that valuates the change in coefficients for a given difference in characteristics and a group-time interaction that valuates the evolution in difference of coefficients. Makepeace et al. (1999) suggest a four-fold decomposition with a "pure" or "gap" effect measuring the difference in changes of endowments, a "price" effect measuring the change in returns of a group over time due to changes in endowments and two similar components for changes in coefficients. Kim (2010) points out the identification problem previously mentioned. Following the averaging method of Yun (2005), he proposes to normalize the estimated coefficients and creates a five-fold decomposition containing the mean and normalized values of the coefficients. Kröger and Hartman (2020) propose a decomposition in three components : endowments, coefficients and interaction. Their

improvement is to separate each of them as the difference between changes in each group. A common assumption of this panel-data literature is that the data is balanced. This gives a more straightforward interpretation of their results. Kröger and Hartman (2020) point out that all these decompositions are correct only for balanced panel-data and that the effect of time constant error terms needs to be included for unbalanced panel data.

Variance of the decomposition

A last part of the literature is about the estimation of standard errors. To estimate the significance of the previous decompositions' components, different methodologies exists. Yun (2005) assumes that the characteristics are constants and suggests test statistics asymptotically normally distributed. Jackson and Lindley (1997) use a Chow-type test to estimate the significance of the aggregated coefficient component. With this method they cannot include the influence of characteristics and cannot estimate the variations in the detailed decomposition. Jann (2008) suggests studying the components' variance as the product of two random variables, respectively the characteristics and the coefficients. Considering the characteristics as random variables implies that their variance are also included. However, this also means that the covariance between the coefficients and the characteristics values and variances needs to be included. Due to the complexity of estimating these, the author suggest assuming that characteristics and coefficients are independent. In the same article, Jann (2008) estimates the variance-covariance matrix of all the used statistics as in Weisse (2000) and apply the delta method. The last and most commonly used method is the bootstrap method introduced in Efron (1992). Following this method, Sinning et al. (2008), Halvak (2014) and Kröger and Hartmann (2020) resample their data and estimate their components multiple times. The standard deviation of these estimations is used as the standard errors.

3 Methodology

In this section I present the methodology of this master thesis. The first part introduces the general framework. The second part describes the descriptive statistics and comparisons used with multiple groups. The third part explains the method used to estimate propensity scores and treatment effects, introducing an original use of triple difference. The fourth describes the first difference estimations performed for each group. The last

part develop an original five-fold decomposition for triple differences.

3.1 The general framework

In this analysis I use a balanced panel data of $i \in \{1; n\}$ individuals observed at $t \in \{0; 1\}$ periods, summarized as t_0 and t_1 in the text. Following Fortin et al. (2011) notations I separate this population between two mutually exclusive groups $g \in \{A, B\}$. In the decomposition literature $y^g_{i,t}$ is the outcome variable of individual i in group g at the period t. Depending on his group, individual i can have two potential outcomes : $y^A_{i,t}$ or $y^B_{i,t}$. However, we can observe only one of these two outcomes. The idea of the following work is to investigate what would be the distribution of outcomes Y^g for individuals in one group is they were in the other one. In this master thesis I am not only interested in the difference between two groups but also in the evolution of these differences through a given treatment[1]. I distinguish the treated and the non-treated individuals noted with the index p equal to 1 for treated and 0 for non-treated. Hence, there are four possible subgroups depending on group g and treatment p, and they are observed during two periods t_0 and t_1.

3.2 Descriptive statistics

The first part of this analysis is a descriptive statistic analysis. This includes a summary of the average outcomes and characteristics at period t_0 and t_1 and a summary of the evolution between t_0 and t_1 for each subgroup. These summaries include the means for quantitative variables and the frequencies for categorical variables.

First I compare the average characteristics in t_0 of those who are treated in t_1, to the average characteristics of those who are not treated in t_1. Similarly, I compare the characteristics in t_0 according to gender and according to both treatment and gender. This gives a preliminary idea of the "selection bias" between the different groups[2].

Then I compare the average characteristics in t_1 for the different subgroups and comment the differences. This is completed by the average evolution between both period for each subgroup. Similarly, I compare the evolution depending on treatments, gender and both. These comparisons give a general idea of what happens through parenthood. These information are a base to interpret further results.

[1] In this master thesis, a treated individual is someone who have a first child between periods t_0 and t_1.

[2] Selection bias is the standard term used in the literature concerning the allocation of treatments.

3.3 Propensity scores

To refine the comparison of outcomes between the subgroups, I use propensity scores to control for differences in characteristics. Rosenbaum and Runbin (1983) defined the propensity scores e^p as the probability of individual i to receive a treatment j depending on its characteristics x_i :

$$e_i^j(x_{i,t}) = \mathbf{P}(z = j|x_{i,t}) \tag{1}$$

In this study, the treatment j is not limited to two groups but is extended to four subgroups. To choose the right method to estimate these propensity scores, I note that in labour economics and more specifically for Mincer like equations, the part of the variations explained by the variables is around a half. Moreover, interactions and non-linear terms are often significant in these models and might be omitted in the estimation of propensity scores. Hence, in the following analysis, the propensity scores are estimated with GBM models which are less sensible to these limitations according to Ferri-Garcia and Ruedo (2020).

The estimation relies on Friedman (2001) algorithm for GBM models, implemented by Greenwell et al. (2019) in the statistical software R. The general idea of this algorithm is to estimate the probability of being in a subgroup depending on individual characteristics. For this the probabilities are computed with a chain of decision trees, each tree using the results of the previous ones. After numerous iterations, the results converge to accurate approximations of the probabilities. For more technical details see Friedman (2001). The result of this algorithm is a matrix of vector :

$$\hat{e}^j(x_{i,t}) = \mathbf{P}(z = j|x_{i,t}) \tag{2}$$

In this matrix each individual has an estimated probability to be in each subgroup. Then there are two possibilities : weighting individuals according to pre-treatment characteristics as in Ryan, Burgess and Dimmick (2015) or weighting individuals according to pre and post-treatment characteristics as in Stuart et al. (2014). For the first possibility I compute the weights for each subgroup :

$$w_i^{j^*} = \frac{\hat{e}^{j^*}(x_{i,t_0})}{\hat{e}^j(x_{i,t_0})} \tag{3}$$

Where j^* is the subgroup of reference. Here each individual has a unique weight computed with his characteristics in the first period. Using Rosenbaum and Runbin (1983)

consistent estimator I obtain :

$$\mathbf{E}_{X_{t_0}|j=j^*,t=0}[Y(j,t)|X,j,t] = \frac{\sum_{i\in j} Y_{i,t} w_i^{j^*}}{\sum_{i\in j} w_{i,t}^{j^*}} \tag{4}$$

This gives the expected outcome for each subgroup at each period if they had the same characteristics as the reference group during the first period. It emphasizes individuals with similar characteristics and plays down individuals who are too different. From there I can compute the difference in difference between the treated and the non-treated individuals if $j = p$:

$$\begin{aligned}\Delta_{j=j^*} = &\Big(\mathbf{E}_{X_{t_0}|j=j^*,t=0}[Y(p=1,t=1)|X,j,t] - \mathbf{E}_{X_{t_0}|j=j^*,t=0}[Y(p=1,t=0)|X,j,t]\Big) \\ &- \Big(\mathbf{E}_{X_{t_0}|j=j^*,t=0}[Y(p=0,t=1)|X,j,t] - \mathbf{E}_{X_{t_0}|j=j^*,t=0}[Y(p=0,t=0)|X,j,t]\Big)\end{aligned} \tag{5}$$

This can be similarly done for a triple difference if j represent each subgroup defined by treatment p and group g. Note that the expected outcome depends on the characteristics in the first period, which means that the characteristics are not weighted to be comparable during the second period. Under the assumption of similar trends, the difference of changes in characteristics is due to the treatment.

For the latter possibility the subgroups are differentiated according to the period. I compute the weights for each subgroup at each period :

$$w_{i,t}^{j^*} = \frac{\hat{e}^{j^*}(x_{i,t})}{\hat{e}^{j}(x_{i,t})} \tag{6}$$

An individual have a different weight for each period. Using the same estimator I obtain :

$$\mathbf{E}_{X|j=j^*}[Y(j,t)|X,j,t] = \frac{\sum_{(i,t)\in j} Y_{i,t} w_{i,t}^{j^*}}{\sum_{(i,t)\in j} w_{i,t}^{j^*}} \tag{7}$$

The difference in difference in this case is :

$$\begin{aligned}\Delta_{j=j^*} = &\Big(\mathbf{E}_{X|j=j^*}[Y(p=1,t=1)|X,j,t] - \mathbf{E}_{X|j=j^*}[Y(p=1,t=0)|X,j,t]\Big) \\ &- \Big(\mathbf{E}_{X|j=j^*}[Y(p=0,t=1)|X,j,t] - \mathbf{E}_{X|j=j^*}[Y(p=0,t=0)|X,j,t]\Big)\end{aligned} \tag{8}$$

The difference with the previous estimator is that the characteristics in the second period are now weighted such that they are comparable to the ones of the reference group during the first period. Individuals who are similar to the reference subgroup in the first period

might not be comparable to it in the second period and vice versa. This estimator measures the direct effect of treatment on the outcome.

Under the similar trend assumption, the difference between the two estimators can be interpreted as the effect treatment on the outcome through variations of characteristics. If the estimation changes depending on the method, it means that individual who are similar in the first period are not similar in the second period and that those who are similar have a different outcome. However, it is impossible to develop on the effect these characteristics with this method.

3.4 Linear regressions with panel data

To develop on the effect of each characteristic, the decomposition methodologies are more relevant and the first part of is to estimate the effect of the different variables. To detail the different issues in this method I develop a framework similar to Fortin et al. (2011). The first assumption is that the outcome of each group can be explained by observable characteristics $X_{i,t}$ and unobservable ones $u_{i,t}$. Noting $m_{g,p}$ the outcome structure for individuals of group g with treatment p, this gives :

$$y_{i,t}^{g,p} = m_{g,p}(X_{i,t}, u_{i,t}) \tag{9}$$

In this specification, three sources of differences in outcomes can be identified : the labour income structure $m_{g,p}$, the observable characteristics $X_{i,t}$ and the unobservable ones $u_{i,t}$. These characteristics can be separated between the ones unique to an individual i, α_i, and the ones that can change over time, $x_{i,t}$ for observable characteristics and $\epsilon_{i,t}$ for unobservable characteristics. Assuming a linear structure this can be written :

$$y_{i,t}^{g,p} = \alpha_i + x_{i,t}^{g,p}\beta^{g,p} + \epsilon_{i,t}^{g,p} \tag{10}$$

Where α_i is the individual fixed effect of individual i, $x_{i,t}^{g,p}$ is a vector of individual i's k observable characteristics at period t, $\beta^{g,p}$ is the corresponding vector of k coefficients and $\epsilon_{i,t}$ is the error term. Then following Kröger and Hartmann (2020), I apply a first difference model :

$$y_{i,1}^{g,p} - y_{i,0}^{g,p} = \left(x_{i,1}^{g,p} - x_{i,0}^{g,p}\right)\beta^{g,p} + \epsilon_{i,1}^{g,p} - \epsilon_{i,0}^{g,p} \tag{11}$$

In this model, the individual fixed effects disappear. To simplify the notations let's define $Y_i^{g,p} = y_{i,1}^{g,p} - y_{i,0}^{g,p}$, $X_i^{g,p} = x_{i,1}^{g,p} - x_{i,0}^{g,p}$ and $\varepsilon_i^{g,p} = \epsilon_{i,1}^{g,p} - \epsilon_{i,0}^{g,p}$. Let's also define $Y^{g,p} = (Y_i^{g,p})_i$ a $n \times 1$ matrix, $X^{g,p} = (X_i^{g,p})_i$ a $n \times k$ matrix and $\varepsilon^{g,p} = (\varepsilon_i^{g,p})_i$ a $n \times 1$

matrix. This gives the summarized equation :

$$Y^{g,p} = X^{g,p}\beta^{g,p} + \varepsilon^{g,p} \tag{12}$$

From here I assume that $Cov(X^{g,p}, \varepsilon^{g,p}) = 0$. This assumption is necessary to identify the $\beta^{g,p}$ coefficients. The estimated coefficients are :

$$\hat{\beta}^{g,p} = (X^{g,p\,\prime}\, X^{g,p})^{-1}\, X^{g,p\,\prime}\, Y^{g,p} \tag{13}$$

Under the assumption of homoscedasticity, $V(\varepsilon_i^{g,p}) = \sigma^2\,,\ \forall\, i$, and of auto-correlation, $\mathbf{E}[\varepsilon_i^{g,p}\varepsilon_j^{g,p}] = 0\,,\ \forall\, i,\, j\ ,\ i \neq j$, the variance of this estimator is :

$$\mathbf{V}(\hat{\beta}^{g,p}|X^{g,p}) = \sigma^2(X^{g,p\,\prime}\, X^{g,p})^{-1} \tag{14}$$

This gives a set of estimated coefficients for each subgroup which can be compared similarly as the evolution of characteristics in the descriptive statistics part. Moreover, it is possible to estimate the part of the variance explicated by the variables within each subgroup using the R^2 coefficient.

3.5 The decomposition

The last step of this methodology is to decompose the observed difference in outcomes between the observed characteristics and the estimated coefficients. To compare the groups I assume that $\varepsilon^{g,p}$ is similarly distributed across the groups, for a given distribution of observable characteristics. This implies that changes in unobservable characteristics are not specific to a group. This is known as the conditional independence or ignorability assumption in the literature.

Using the previous framework, I can write my measure of interest as :

$$\begin{aligned}\Delta^2(Y) =&(\mathbf{E}[Y|X, g = A, p = 1] - \mathbf{E}[Y|X, g = B, p = 1]) \\ &- (\mathbf{E}[Y|X, g = A, p = 0] - \mathbf{E}[Y|X, g = B, p = 0])\end{aligned} \tag{15}$$

The underlying assumption behind is the common trend assumption : $E[Y^{g,1}|X, g, p = 1] = E[Y^{g,0}|X, g, p = 1]$, which means that the treated and non-treated individuals of the same group should have the same evolution between the two period if they received the same treatment. Using the mean to estimate these values I obtain :

$$\hat{\Delta}^2(Y) = (\bar{Y}^{A,1} - \bar{Y}^{B,1}) - (\bar{Y}^{A,0} - \bar{Y}^{B,0}) \tag{16}$$

Using the $\hat{\beta}^{g,p}$ coefficients previously estimated, this can be written as :

$$\hat{\Delta}^2(Y) = (\bar{X}^{A,1}\,\hat{\beta}^{A,1} - \bar{X}^{B,1}\,\hat{\beta}^{B,1}) - (\bar{X}^{A,0}\,\hat{\beta}^{A,0} - \bar{X}^{B,0}\,\hat{\beta}^{B,0}) \tag{17}$$

3.5.1 The fivefold decomposition

As proven in Equation 23 in Appendix, Equation 17 can be decomposed in five components :

$$\begin{aligned}
\hat{\Delta}^2(Y) = \quad & \underbrace{\overbrace{(\bar{X}^{A,1} - \bar{X}^{B,1})\,(\hat{\beta}^{B,1} - \hat{\beta}^{B,0})}^{\Delta(PE)} + \overbrace{(\bar{X}^{B,1} - \bar{X}^{B,0})\,(\hat{\beta}^{A,1} - \hat{\beta}^{B,1})}^{\Delta(PC)}}_{\Delta(P)} \\
& + \underbrace{[(\bar{X}^{A,1} - \bar{X}^{B,1}) - (\bar{X}^{A,0} - \bar{X}^{B,0})]\,\hat{\beta}^{A,0}}_{\Delta(E)} \\
& + \underbrace{\bar{X}^{A,0}\,[(\hat{\beta}^{A,1} - \hat{\beta}^{B,1}) - (\hat{\beta}^{A,0} - \hat{\beta}^{B,0})]}_{\Delta(C)} \\
& + \underbrace{[(\bar{X}^{A,1} - \bar{X}^{B,1}) - (\bar{X}^{A,0} - \bar{X}^{B,0})]\,[(\hat{\beta}^{A,1} - \hat{\beta}^{B,1}) - (\hat{\beta}^{A,0} - \hat{\beta}^{B,0})]}_{\Delta(I)}
\end{aligned} \tag{18}$$

In this decomposition the reference subgroup is $(A, 0)$. The first component $\Delta(P)$ measures what would be the average difference of output between groups due to treatment if we consider the disparities with treatment and the impact of treatment on group B. This can be interpreted as the "projection" of differences with treatment, through the treatment effect on group B. This can be split between $\Delta(PE)$ the "projection" of differences in endowments and $\Delta(PC)$ the "projection" of differences in coefficients. The second component $\Delta(E)$ measures the expected difference on output due to the growing or the shrinking difference of endowments. The third component $\Delta(C)$ is the counterpart for the coefficients. The fourth component $\Delta(I)$ measures the expected difference in outcome due to the simultaneous evolution of differences in endowments and coefficients.

The $\Delta(E)$ and $\Delta(C)$ components can be split similarly as in Kröger and Hartman

(2020) between the difference of evolution in each group due to treatment.

$$\begin{aligned}
\Delta(E) =& (\bar{X}^{A,1} - \bar{X}^{A,0})\,\hat{\beta}^{A,0} - (\bar{X}^{B,1} - \bar{X}^{B,0})\,\hat{\beta}^{A,0} \\
=& (\bar{X}^{A,1} - \bar{X}^{B,1})\,\hat{\beta}^{A,0} - (\bar{X}^{A,0} - \bar{X}^{B,0})\,\hat{\beta}^{A,0} \\
=& \bar{X}^{A,1}\,\hat{\beta}^{A,0} - \bar{X}^{B,1}\,\hat{\beta}^{A,0} - \bar{X}^{A,0}\,\hat{\beta}^{A,0} + \bar{X}^{B,0}\,\hat{\beta}^{A,0} \\
\Delta(C) =& \bar{X}^{A,0}\,(\hat{\beta}^{A,1} - \hat{\beta}^{A,0}) - \bar{X}^{A,0}\,(\hat{\beta}^{B,1} - \hat{\beta}^{B,0}) \\
=& \bar{X}^{A,0}\,(\hat{\beta}^{A,1} - \hat{\beta}^{B,1}) - \bar{X}^{A,0}\,(\hat{\beta}^{A,0} - \hat{\beta}^{B,0}) \\
=& \bar{X}^{A,0}\,\hat{\beta}^{A,1} - \bar{X}^{A,0}\,\hat{\beta}^{B,1} - \bar{X}^{A,0}\,\hat{\beta}^{A,0} + \bar{X}^{A,0}\,\hat{\beta}^{B,0}
\end{aligned} \tag{19}$$

This gives a more precise comprehension of the evolution of disparities with the difference of evolution for a given group or a given treatment status.

This decomposition is sensible to the choice of the reference group. However, as explained in Fortin et al. (2011) there is no real solutions to this problem and what is important is the economic reasoning behind this choice. The choice of subgroup (A,0) as the reference group is based on the general choice and logical in the literature. Group B is the minority group and is compared to the group A which is considered as the normal situation. On the other hand, the treated group is compared to the non-treated one which describe the normal situation.

3.5.2 The variance of the decomposition

As noted in Jann (2008) a wild problem in the decomposition literature is the lack of attention paid to the issue of statistical inference. In many of the previous mentioned studies, the authors don't test the significance of their decomposition results. Starting from Equation 12 the sampling variance can be written as :

$$V(\bar{Y}^{g,p}) = V(\bar{X}^{g,p}\hat{\beta}^{g,p}) \tag{20}$$

Noting $\mathbf{E}(\bar{X}) = \mu_X$, $V(\bar{X}) = \Sigma_X$, $\mathbf{E}(\hat{\beta}) = \mu_\beta$ and $V(\hat{\beta}) = \Sigma_\beta$, Equation 6 in Appendix proves that it equals :

$$\begin{aligned}
V(\bar{Y}) =& \mu_X'\,\Sigma_\beta\,\mu_X + \mu_\beta'\,\Sigma_X\,\mu_\beta + tr(\Sigma_X\,\Sigma_\beta) \\
& - tr(Cov(\bar{X}\,\bar{X}', \hat{\beta}\,\hat{\beta}')) - 2\mu_X\mu_\beta' Cov(\bar{X}, \hat{\beta}) - Cov(\bar{X}, \hat{\beta})^2
\end{aligned} \tag{21}$$

Some components of this result are too difficult to estimate. With the assumption that $Cov(X^{g,p}, \varepsilon^{g,p}) = 0$, $\bar{X}^{g,p}$ and $\hat{\varepsilon}^{g,p}$ are independent. Moreover, assuming that $\bar{X}$ and

$\hat{\beta}$ are independent, this gives as proven in Jane (2005b) :

$$V(\bar{Y}^{g,p}) = (\bar{X}^{g,p})' V(\hat{\beta}^{g,p}) (\bar{X}^{g,p}) + (\hat{\beta}^{g,p})' V(\bar{X}^{g,p\prime})(\hat{\beta}^{g,p}) + tr\left(V(\bar{X}^{g,p}) V(\hat{\beta}^{g,p})\right) \quad (22)$$

The problem from here is to test the significance of these statistics. These statistics are the product of two random variables which are asymptotically normal. There are no simple test statistics existing for this kind of statistic and developing it is beyond the purpose of the present thesis. Hence, following Sinning et al. (2008), Halvak (2014) and Kröger and Hartmann (2020) I will use bootstrap to estimate it. The standard deviation of the different estimations is used as the standard error of the results.

4 Data

The present analysis uses two surveys : *Conditions de travail 2013*[3] and *Enquête Conditions de travail et Risques psychosociaux 2016*[4] conducted by the *Direction de l'Animation de la recherche, des Études et des Statistiques* (DARES) from the French Minister of Labour.

For the 2013 survey[5], 34,000 individuals were interviewed at home by pollsters from the *Institut national de la statistique et des études économiques* (INSEE) between October 2012 and March 2013 and a supplementary survey was posted to employers and collected between November 2012 and October 2013. A part of the personal in home survey was self-administrated. Concerning the supplementary survey, there were two different samples : one of firms with at least 10 employees and another one of 7000 firms in France with at least 1 employee, selected from the *Système national d'identification et du répertoire des entreprises et de leurs établissements* (SIRENE) and representative of the French economy. This questionnaire differs depending on the type of facility[6].

[3]Enquête Conditions de travail - 2013, DARES - Ministère du Travail et de l'Emploi, INSEE, DREES - Ministère de la Santé, DGAFP - Ministère de la Fonction Publique (producteurs), ADISP (diffuseur)

[4]Enquête Conditions de travail et Risques psychosociaux - Volet Individus - 2016, DARES - Ministère du Travail et de l'Emploi, INSEE, DREES - Ministère de la Santé, DGAFP - Ministère de la Fonction Publique (producteurs), ADISP (diffuseur)

[5]The 2013 personal in home surveys are available online :
http://travail-emploi.gouv.fr/IMG/pdf/Questionnaire_personnes_en_emploi.pdf

[6]The different 2013 questionnaires are available online :

- For administrative establishment :
  http://travail-emploi.gouv.fr/IMG/pdf/Questionnaire_Fonction_publique.pdf
- For the hospital sector :
  http://travail-emploi.gouv.fr/IMG/pdf/Questionnaire_Secteur_hospitalier.pdf
- For all the other establishments :
  http://travail-emploi.gouv.fr/IMG/pdf/Questionnaire_Secteur_marchand_et_associatif.pdf

4.1 Data presentation

For the 2016 survey, 27,700 individuals answered the survey between October 2015 and June 2016. Within them 2,970 individuals were not considered active, 4,758 were not surveyed in 2013 and 19,882 are considered active and were surveyed[7]. Similarly, to the 2013 survey, the survey contains a personal in home survey with a self-administrated questionnaire[8] and a supplementary survey posted to the employer with the same method[9].

4.2 Data cleaning

Using the individual index provided for both surveys, I obtain a panel data of 45,704 observations with 22,852 individuals observed in 2013 and 2016.

In this surveys the respondent can answer that he does not know the answer or that he refuses to answer to several questions. I consider these answers as missing values. For other questions, I suspect that the respondent answers wrongly and that the pollsters miss-encoded for one of the years. For example, some of them are over one hundred years old while the survey concerns active workers. In this case I assume that they are one hundred years too old, which is consistent with the age reported during the other survey. However, 40 of them have a different gender reported in each survey. In this case I choose to remove them from the panel.

Concerning their monthly income, 34,354 answered, 3,048 didn't know, 1,057 refused to answer and 4,181 didn't answer. In these cases, the pollsters asked for an estimation of their incomes with intervals[10]. I replace the missing values by the median of the reported intervals to obtain 7,463 approximations and 3807 missing values which I remove. For those who does not have any earnings, I choose to report their labour income as 1€ instead of 0€ to obtain a logarithm of the labour income equal to 0.

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[7] The sum of these sub-samples equals 27,610 which means that 90 individuals answered but are not in the tables.

[8] The 2016 personal in home surveys are available online :
http://dares.travail-emploi.gouv.fr/IMG/pdf/questionnaire_rps2016_cartouche.pdf

[9] The different 2016 questionnaires are available online :

- For administrative establishment :
  http://dares.travail-emploi.gouv.fr/IMG/pdf/quest_rps_fpublic.pdf
- For the hospital sector :
  http://dares.travail-emploi.gouv.fr/IMG/pdf/quest_rps_shospitalier.pdf
- For all the other establishments :
  http://dares.travail-emploi.gouv.fr/IMG/pdf/quest_rps_smarchand.pdf

[10] 0€, 400 €, 600 €, 800 €, 1 000 €, 1 200 €, 1 500 €, 1 800 €, 2 000 €, 2 500 €, 3 000 €, 4 000 €, 6 000 €, 10 000 € and above

Finally I can identify parents and non parents with the number of children in each survey. Hence, I can differentiate four categories : those who don't have children in both years (8368 individuals), those who become parent between the two surveys (953 individuals), those who are parent in both surveys (12,202 individuals) and those who were parents in the 2013 surveys but answered that they don't have in the 2016 survey (1,289 individuals). In this study I will focus on the two first categories with the new parents as my population of interest and the non-parents as the counterfactual population. Hence, I obtain a final data-set of 15,558 observations with 7,779 individuals.

4.2.1 Data summary

Detailed descriptive statistics are presented latter in the results for the different groups of interest. However, in this part I present some general statistics to globally understand the data used in this master thesis.

The first remark is that this database is not representative of the French labour force. This is partly due to the surveys' methodology. The DARES provides documents that discuss the representativeness of the data for the Metropolitan France[11] and the French Oversees Territories and Department[12] in 2013.

The first reason is the choice to add a focus on the public sector with the over-representation of individuals working in State, territorial and hospital administrations. Individuals working in the private hospital sector are also over-represented. The second issue is the selection bias due to the non-response of the households. Table 4.2.1 summarizes this with the frequency of each job status. In this database 37% of the individuals work in the public sector while they are less than 20% in the French active population.

Concerning the different subgroups of interest the share of parents is lower than the share of non-parents in terms of observations. There are more women than men in the whole population and within the parent and non-parent groups.

Table 4.2.1 summarizes the working conditions for the whole population. Most of the working conditions variables are rated on ordinal scales with 2 to 4 possibilities. The answers are re-scaled to be between 0 and 1 with 0 the lowest frequency or level of agreement and 1 the highest frequency or level of agreement.

[11] https://dares.travail-emploi.gouv.fr/IMG/pdf/Conditions$_d e_t ravail_2 013_M etropole_R edressement_e t_c orrection_d e_l a_n on$

[12] https://dares.travail-emploi.gouv.fr/IMG/pdf/Conditions$_d e_t ravail_2 013_R edressement_e t_c orrection_d e_l a_n on_r eponse_d a$

Table 1: Status Frequency of the whole population

	Frequency
State administration	0.15
Independent	0.01
Territorial administration	0.13
Public Hospital	0.08
Private health sector	0.05
Public social health care	0.01
Firm artisan or association employee	0.44
Private individual's employee	0.03
Help a member of the family	0.00
Employed director CEO associate	0.00
Unknown Status	0.10
Private sector	0.53
Public sector	0.37
Unknown Sector	0.10

	N	Frequency
Fathers	756	0.05
Mother	916	0.06
Childless Men	6170	0.40
Childless Women	7716	0.50
Total	15,558	1.00

5 Results

In the following analysis I differentiate the potential causes of the motherhood penalty as : the impact of participation to the labour market, the impact of productive characteristics and the impact of working conditions. Because the observation of productive characteristics and working conditions is conditional to the participation to the labour market, I need to separate the analyses. The first part use the whole population including individuals who are not working in 2016. The second part focuses on working individuals and explores the impact of the usual productive characteristics and working conditions.

5.1 The whole population

In this section I analyse the impact of participation to the labour market on monthly earnings, depending on gender and parenthood for the whole population.

Table 2: Working Condition Summary

	N	Mean	Sd	Median	Min	Max
Physical Constraints						
Accident	13978	0.04	0.14	0.00	0	1
Stand	13975	0.51	0.50	1.00	0	1
Displacement	13977	0.37	0.48	0.00	0	1
Heavy	13979	0.42	0.49	0.00	0	1
Movement	13975	0.40	0.49	0.00	0	1
Posture	13975	0.36	0.48	0.00	0	1
Vibrations	13978	0.16	0.37	0.00	0	1
View	13963	0.42	0.49	0.00	0	1
Flexibility						
Convenient	13948	0.74	0.26	0.67	0	1
Prevision	13520	0.39	0.16	0.33	0	1
Modify	12762	0.23	0.42	0.00	0	1
Overwork	13929	0.38	0.28	0.33	0	1
Reachable	13254	0.46	0.50	0.00	0	1
Days worked	13886	4.81	0.77	5.00	0.5	7
Work Interruption	13954	0.69	0.46	1.00	0	1
Morning	13982	0.23	0.38	0.00	0	1
Evening	13982	0.24	0.37	0.00	0	1
Night	13981	0.11	0.28	0.00	0	1
Saturday	13980	0.37	0.42	0.00	0	1
Sunday	13980	0.23	0.37	0.00	0	1

5.1.1 Descriptive Statistics

Table 3 presents some demographic characteristics, the labour incomes and the employment situation of individuals in 2013. In this population, all individuals are employed in 2013 because the first survey concerns active workers. Moreover, by construction, none of them have children in 2013, so the groups depend on their parenthood status in 2016. In average the childless individuals are older and have a more seniority than future parents. Concerning earnings, future parents have lower labour incomes than those who remain childless and women have lower labour incomes than men. These results confirm that there are differences between future parents and non-parents before the transition to parenthood. I describe the other characteristics in the next parts, without individuals who are not employed in 2016.

When we look at the logarithm of the monthly labour income, after computing the exponential we found lower values respectively equal to 1685.81, 1380.22, 1737.15 and 1436.55. These results are the geometric means of labour incomes and are less sensible

to high values. This highlights the presence of outliers in the groups.

The average labour income gap between future fathers and mothers equals 387.16 euros while it equals 343.01 euros for the others. This means that future mothers earn 20.02% less than future fathers, while other women earn 16.84% less than childless men. In terms of log labour incomes, this is equivalent to gaps of 0.20 and 0.19.

Table 3: Data summary of the whole population in 2013

	Fathers	Mothers	Childless Men	Childless Women
Monthly Labour Income	1933.51	1546.35	2036.29	1693.28
Log Monthly Labour Income	7.43	7.23	7.46	7.27
Age	35.05	33.81	45.09	48.02
Seniority	111.38	98.59	195.68	220.94
Potential Experience	15.61	14.03	27.05	29.92
Employed	1.00	1.00	1.00	1.00
Observations	378	458	3085	3858

Table 4 presents the means and frequencies of the same variables but in 2016. We observe the same patterns as in 2013 concerning the differences in age and tenure and women have a lower monthly labour income than men in average for a given parenthood status. However, in 2016 parents earn more than childless individuals and the employment rates drop which is normal because the survey follows workers who were active in 2013.

If we look at the geometric means of labour incomes we obtain 1064.22, 544.57, 372.41 and 311.06. This is far lower than the arithmetic means and shows a huge disparity in labour incomes, probably due to the increasing share of non-employed individuals. However, the comparisons are consistent with the arithmetic means.

In 2016 in average mothers earn 26.12% less than fathers while non-mothers earn 12.32% less than non-fathers. For parents, the log labour income difference equals 0.67 and cannot be interpreted as a relative difference, while it equals 0.18 for non parents. Compared to the initial situation in 2013, the labour income gap between parents has increased, while the labour income gap between non-parents has decreased.

Table 4: Data summary of the whole population in 2016

	Fathers	Mothers	Childless Men	Childless Women
Monthly Labour Income	1924.86	1419.75	1654.80	1403.87
Log Monthly Labour Income	6.97	6.30	5.92	5.74
Age	38.07	36.86	48.12	51.05
Seniority	121.79	118.54	196.74	225.40
Potential Experience	18.61	17.03	30.05	32.92
Employed	0.92	0.86	0.79	0.78
Apprentice or trainee	0.00	0.00	0.00	0.00
Student	0.00	0.01	0.00	0.00
Unemployed	0.04	0.08	0.06	0.04
Retired	0.02	0.02	0.14	0.15
House-wife/husband	0.00	0.02	0.00	0.00
Inactive	0.00	0.00	0.01	0.01
Other situations	0.00	0.01	0.01	0.01
Observations	378	458	3085	3858

Table 5 presents the evolution between the two periods. The fall in monthly labour income is low for fathers and wide for childless men. These changes seem to be driven by variations in employment status with a decrease of employment above 20% for childless individual, 8% for fathers and 14% for mothers. This difference between parents and non-parents is probably due to the age gap which leads a larger share of non-parents to retire. Retirement concerns around 2% of parents while it concerns around 15% of non-parents. The difference in employment situation between fathers and mothers is due to a larger share of unemployed and house-wives mothers.

The decrease of labour incomes for fathers equals a 0.45% change when we compare monthly labour incomes but seems wilder when we compare the log labour incomes. This can be interpreted as the effect of outliers who have a large increase in labour incomes and compensate the decreasing labour income of non-employed fathers. With an initial gap of 20.02% and a final gap of 26.12%, there is an increase of 6.10 percentage points in the gender labour income gap during the transition to parenthood. On the other hand, non-mothers earn 16.84% than non-fathers in 2013 and only 12.32% in 2016. This means that during the same period the labour income gap decreased by 4.52 percentage points for those who remain childless. This convergence is due to a larger fall in non-fathers' labour incomes. If we expect similar trends between parents and non-parents, this means

that the effect of parenthood would be an increase of the gender labour income gap of 10.62 percentage points. However, this hypothesis is unrealistic due to differences in age and retirement.

Similarly, the log of monthly labour incomes decreased less for fathers and more for non-parents. The gender labour income gap increased by 0.47 for parents and decreased by 0.01 for non parents. Then the triple difference equals 0.48. This cannot be interpreted as relative variations but it is consistent with previous results in terms of signs.

Table 5: Data summary of the evolution between 2013 and 2016 for the whole population

	Fathers	Mothers	Childless Men	Childless Women
Monthly Labour Income	-8.65	-126.60	-381.49	-289.41
Log Monthly Labour Income	-0.46	-0.93	-1.54	-1.53
Seniority	10.41	19.95	1.06	4.46
Employed	-0.08	-0.14	-0.21	-0.22
Apprentice or trainee	0.00	0.00	0.00	0.00
Student	0.00	0.01	0.00	0.00
Unemployed	0.04	0.08	0.06	0.04
Retired	0.02	0.02	0.14	0.15
House-wife/husband	0.00	0.02	0.00	0.00
Inactive	0.00	0.00	0.01	0.01
Other situations	0.00	0.01	0.01	0.01
Observations	378	458	3085	3858

In this case we can observe an increase in the gender labour income gap but the most striking results are the fall in labour incomes for all the groups except fathers probably driven by changes in employment situations.

5.1.2 Propensity Scores

To estimate the motherhood penalty, I control for individual characteristics to estimate the motherhood effect. For this I predict the probability of being in one group depending on characteristics as describe in Equation 2.

First I compute propensity scores for each individual depending on their characteristics in 2013. I control for their age, education, relationship, marital status and potential

experience. I can control for work-related characteristics but it would be inconsistent with the other analyses due to individuals non-employed in 2016. The propensity scores are computed using a Gradient Boosted Model previously described. Using these propensity scores I compute the weights described in Equation 3. Detailed results concerning these propensity scores and weights are provided in Appendix. From these weights I compute the consistent estimator described in Equation 4.

Table **??** provides the weighted means of labour incomes and log labour incomes for the different groups for both years. After weighting, we can see that if women had characteristics similar to fathers in 2013, they would have a higher labour income, while childless men would have lower labour incomes. When we look at 2016, we can see that all groups would have higher labour incomes with a reduced decrease between the two periods.

With the difference in difference estimator described in Equation 5 I compare the evolution the gender labour income gap between fathers and mothers. With an initial gap of 19.27%, the final gap equals 26.55%. Hence, if mothers and fathers were comparable in characteristics beforehand, parenthood would be correlated with an increase of 7.28 percentage points in the gender labour income gap. On the other hand, if non-fathers and non mothers had fathers' characteristics in 2013, there would be a gap of 16.92% in 2013 and of 15.52% in 2016, which means a decrease of 1.40 percentage points. With a triple difference I obtain that the transition to motherhood would imply a change 8.68 percentage points. However, all these results are obtained by assuming similar characteristics in 2013.

Regarding the log of monthly labour incomes, there is an increase of the gap between parents of 0.52 and an increase of 0.02 for non-parents. The triple difference gives an increase of 0.50 through parenthood which is similar to what we obtained without controlling for initial characteristics.

Table 6: Estimated labour incomes with men 2013 characteristics for the whole population

	Fathers	Mothers	Childless Men	Childless Women
2013				
Monthly Labour Income	1933.51	1560.95	1897.76	1576.65
Log Monthly Labour Income	7.43	7.24	7.43	7.21
2016				
Monthly Labour Income	1924.86	1413.88	1783.02	1506.35
Log Monthly Labour Income	6.97	6.26	6.65	6.41

I repeat the same computation but controlling for characteristics in 2013 and 2016. This implies that for each individual I compute eight different propensity score depending on parenthood, gender and year. In this configuration I can also control for employment status in 2016. The propensity scores are computed as before and the weights are estimated as in Equation 6.

In Table 6 we can see the estimated weighted means with this method. The estimations in 2013 are similar to the ones obtained with the previous method. However, in 2016, the estimated average labour income of mothers is extremely lower, even more than without controls for the characteristics. On the other hand childless individuals have higher labour incomes in average and fathers' one is slightly lower.

Mothers earn 22.63% less than fathers in 2013 and 33.24% less in 2016. With a difference in difference the labour income gap between fathers and mothers seems to have increased by 10.61 percentage points between 2013 and 2016. Childless women earn 17.51% less than men in 2013 and 14.23% less in 2016. During the same period the labour income gap seems to have decreased by 3.28 percentage points for non-parents. With a triple difference I estimate that the transition to parenthood is correlated with an increase of the gender labour income gap of 13.89 percentage points.

Regarding the log of monthly labour incomes, there is an increase in the gap between parents of 1.21 and a decrease of 0.12 for non-parents. This leads to an increase of the gap of 1.33 through parenthood. This is wider than the previous estimations made with the log of monthly labour incomes.

Table 7: Estimated labour incomes with men 2013 characteristics for the whole population

	Fathers	Mothers	Childless Men	Childless Women
2013				
Monthly Labour Income	1933.51	1495.92	1857.86	1532.58
Log Monthly Labour Income	7.43	7.20	7.41	7.18
2016				
Monthly Labour Income	1902.08	1269.81	1888.17	1619.49
Log Monthly Labour Income	7.28	5.84	7.31	7.21

To conclude this part on propensity scores, my previous results wildly differ depending on the control of changes in characteristics. To understand it, we need to remind how these weighted average are computed. To estimate the treatment effect, the non-treated individuals who are more likely to be treated according to their characteristics are over-represented with the weights, while the other ones are under-represented. This implies that if we consider future mothers who have characteristics similar to men's ones before the birth of their first child, they have a reasonable decrease in wages. However, if we over-represent mothers that have characteristics in 2016 similar to fathers' ones in 2013, the average labour income of mothers falls.

It seems that changes in employment situation and demographic characteristics are responsible for important variations in mothers' labour income compared to other groups. However, the opacity of the propensity score methods limit the understanding of these results and the discussion of the estimated parenthood effect. Moreover, there are no methods to estimate directly the significance of these results.

5.1.3 First Difference Analysis

To complete the previous results, Table 8 summarizes the first difference regressions of log labour incomes on employment situation for each possible group. I control for their age, education, relationship, marital status and potential experience. Each of these regressions seems to explain more between 84.2% and 89.0% of the log labour incomes variations. With a log labour income equal to 0 in many cases of non-employment, these variations are wider for those who leave employment. Without individuals in some categories for fathers and mothers, it is impossible to estimate the related coefficients. However, we can compare to the childless individuals and observe that most of these coefficients are significant and are similar between the groups. We can see that mothers

have higher penalties for going back to studies and lower penalties for being house-wives.

Concerning the differences in coefficients significance, unemployment, inactivity and other non-employment situations coefficients are significantly lower for childless men than for childless women. Similarly, the unemployment coefficient is significantly lower for mothers than for childless women. However, the difference in difference with all subgroups is significant for none of the variables.

Table 8: Log labour incomes evolution and employment situation changes

	Dependent variable: Log Monthly Labour Income			
	Fathers	Mothers	Childless Men	Childless Women
Apprentice or trainee	−0.190		−0.430	−0.299
	(0.788)		(1.044)	(0.851)
Student		−6.994***	−2.318*	−5.702***
		(0.792)	(1.042)	(0.326)
Unemployed	−6.902***	−6.987***	−6.958***	−6.165***
	(0.198)	(0.187)	(0.084)	(0.096)
Retired	−7.464***	−6.807***	−7.216***	−6.975***
	(0.328)	(0.317)	(0.068)	(0.062)
Inactive	−7.548***		−7.014***	−6.558***
	(0.628)		(0.188)	(0.198)
House-wife/husband		−6.056***	−7.300***	−6.447***
		(0.382)	(1.041)	(0.323)
Other situations		−7.061***	−7.100***	−5.422***
		(0.587)	(0.241)	(0.218)
(Intercept)	−0.128	−0.011	0.129	−0.065
	(0.314)	(0.407)	(0.145)	(0.112)
Observations	370	450	3,027	3,797
R^2	0.869	0.851	0.891	0.856
Adjusted R^2	0.860	0.842	0.890	0.855
F Statistic	95.675***	93.105***	787.332***	698.148***
df	(24; 345)	(26; 423)	(31; 2995)	(32; 3764)
Note:				*p<0.05; **p<0.01; ***p<0.001

These results show that leaving employment have a large and significant impact on labour incomes and that it explains most of the labour income variations within groups. However, these regressions cannot explain differences between groups' outcomes.

5.1.4 Decomposition

To compare the differences of earning evolution between groups I use the decomposition method described in Equation 18 and comment it with the previous results. The significance of the different components is estimated with bootstrap. Due to the low number of individuals in some subgroups, collinearity appears during the bootstrapping process and I cannot control for education, marital status and potential experience. Hence, the decomposition look at the effect of employment status on the logarithm of monthly labour incomes, controlling for age and relationship status. However, none of the control has a significant effect at a level of 5%.

Table 9 summarises the decomposition results depending on the different component defined in Equation 18 and 19. First, we can see that multiple estimations are not possible due to the absence of fathers and mothers in some categories. Without observations for one of them, we cannot compare their coefficients and estimate the related components.

Reminding that the triple difference of log monthly labour incomes equals 0.48, the difference of endowments evolution seems to be the main reason of the increase of gender labour income gap trough parenthood. More precisely, the difference comes from the larger share of unemployed mothers we observed in Table 5. On the other hand, differences of coefficients does not seem to have a significant impact on the gender labour income gap through parenthood. Note that due to rounding, the sum of the different estimations cannot be perfectly equal to 0.48.

To summarise, with this method the motherhood penalty can be explained by an increase of unemployed mothers after the birth of their first and the remaining unexplained part, generally interpreted as the inequality is non-significant.

However there an important limit to this method is the impossibility to compare differences of coefficients if one of the groups is not represented in this category. The roles of house-husband and house-wives are probably important in the transition to parenthood and we cannot compare the penalties.

Table 9: Decomposition estimation for the whole population

	PE	PC	E	C	I
Aggregated	0.05 *	0.08	0.35 **	0.07	-0.05
(Intercept)	0	0	0	-0.38	0
Age	0	0	0	0.49 .	0
Relationship	0.01	-0.01	0	0	-0.01
Apprentice or trainee			0		
Student	0.01		0.01		
Unemployed	0.02 .	-0.01	0.33 ***	0.03	-0.03
Retired	0	0.1	-0.11	-0.07	-0.01
House-wife/husband	-0.01		0.11		
Inactive			-0.03		
Other situations	0.01		0.04		
Note:	.p<0.1;*p<0.05; **p<0.01; ***p<0.001				

To develop on the effects of differences in endowments evolution, Table 10 separate the effect of parenthood on labour income depending on gender. The first column shows that there is a fatherhood premium mostly due to the high retirement of childless men. Similarly, for women, the higher retirement rate of childless women creates a mother premium. However, these two premium are similar for both groups so there is no significant overall effect. Similarly, the larger share of inactive childless women and childless men with other employment situations creates a difference based on parenthood status within each gender group but the overall effect is not significant. Finally, only the larger share of unemployed mothers have a significant effect. Hence, the effect of parenthood on the gender labour income gap is mostly driven by the increasing share of unemployed mothers during their transition to parenthood.

Table 10: Employment situation effect depending on gender

	Men	Women
Aggregated	1 ***	0.65 ***
(Intercept)	0	0
Age	0	0
Relationship	0.01	0
Apprentice or trainee	0	0
Student	0	-0.01
Unemployed	0.08	-0.25 ***
Retired	0.84 ***	0.94 ***
House-wife/husband	0	-0.1
Inactive	0.03	0.06 ***
Other situations	0.04 ***	0.01
Note:	.p<0.1;*p<0.05; **p<0.01; ***p<0.001	

To summarise this analysis on the whole population, we observe a labour income gap between men and women in the first period. We also see that parent and non-parent individuals are initially different, especially in terms of age and working experience. This implies difference in labour income and retirement during the studied period. Between 2013 and 2016, the gender labour income gap between the young parents increases while it decreases for those who remained childless. This can be interpreted as the effect of parenthood. Leaving the labour-force can explain most of the labour income variations within each group. Moreover, differences in employment status between groups explains most of parenthood effect. More precisely, the increasing share of unemployed mothers creates a significantly larger difference in average labour income between parents than between non-parents.

These results highlight the importance of the labour-force participation to understand the motherhood penalty. However, studying those who stays in the labour force allows to estimate the effect of work-related characteristics.

5.2 The working population

In this part I focus on active workers and study the work related characteristics, including working conditions. Similarly, to the previous part, I first describe the active population, then estimate the parenthood effect with propensity score, estimate the effect of the different variable on the log of labour incomes variations within each population and

finally decompose the labour income difference across groups depending on the different variables.

5.2.1 Descriptive Statistics

The descriptive statistics are separated in two parts : the classical work-related variables and the working conditions.

Standard Variables

Table 11 summarizes the standard work-related characteristics for each subgroup in 2013. As in the previous part we can see that childless individuals are older than future parents. They also have more seniority and potential experience. Women work fewer hours per week than men and have more part-time jobs and work more frequently in the public sector. However, childless individuals work more frequently in the public sector than future parents. We can note the share of childless mothers working part-time is high compared to the other subgroups.

Regarding differences in average monthly labour incomes, future mothers earn 18,23% less than future fathers in 2013, while childless women earn 14,24% less than childless men. When we look at the difference in the average log monthly labour incomes, it seems that for both groups there is a difference of 0.16.

Table 11: Data summary of the active population in 2013

Standard Variables

	Fathers	Mothers	Childless Men	Childless Women
Monthly Labour Income	1947.09	1592.11	2001.45	1716.39
Log Monthly Labour Income	7.43	7.27	7.46	7.30
Age	34.36	33.36	43.07	46.20
Seniority	107.27	96.26	176.05	205.99
Potential Experience	14.75	13.32	24.72	27.71
Hours per Week	38.98	35.46	38.55	35.27
Full-time	0.96	0.83	0.93	0.77
Public	0.25	0.44	0.36	0.49
Observations	349	396	2431	3001

Table 12 summarizes the standard work-related characteristics for each subgroup in 2016. In 2016, we can see that a large share of individuals interrupt their career between

2013 and 2016, especially parents. For fathers this is mostly because 21% of them change firms. On the other hand, 31% of mothers have a career interruption while only 16% of them change firms. Concerning the other characteristics, the comparisons are similar to those in 2013. The evolution is discussed with the next table.

In 2016 the average labour income gap between fathers and mothers equals 21.21% for parents and 14.24% for non-parents. With the log monthly labour incomes the differences are respectively 0.26 and 0.18.

Table 12: Data summary of the active population in 2016

Standard Variables

	Fathers	Mothers	Childless Men	Childless Women
Monthly Labour Income	2078.07	1637.28	2086.90	1789.64
Log Monthly Labour Income	7.51	7.25	7.44	7.26
Age	37.38	36.41	46.10	49.24
Career Interruption	0.28	0.31	0.21	0.20
Firm Change	0.21	0.16	0.14	0.13
Seniority	122.42	118.83	197.45	226.94
Potential Experience	17.75	16.32	27.72	30.71
Hours per Week	38.92	34.90	38.61	35.25
Full-time	0.95	0.71	0.92	0.76
Public	0.24	0.41	0.34	0.48
Observations	349	396	2431	3001

Table 13 presents the evolution of the standard work-related characteristics between 2013 and 2016 for each subgroup. The firm changes and career interruptions have already been discussed previously. However, we can note that the average seniority of fathers increases less than in other subgroup, probably due to the larger share of fathers who change firms. We can see that the number of hours worked per week and the share of full-time workers decreases for mothers.

Regarding labour incomes evolution, the average labour income increases for all subgroups, especially for fathers and less for mothers. However, if we look at the log monthly labour income evolution, it seems that it increases only for fathers. This might be explained by the presence of outliers in the other groups.

The average labour income gap increase by 1.98 percentage points for parents and stays perfectly constant for non-parents. With the log monthly labour income the evolution of the labour income gap are respectively an increase of 0.10 for parents and an

increase of 0.02 for non-parents. The overall triple difference is hence equal to 1.98 percentage points with the monthly labour income and 0.08 with the log of the monthly labour income.

Table 13: Data summary of the active population evolution between 2013 and 2016

Standard Variables

	Fathers	Mothers	Childless Men	Childless Women
Monthly Labour Income	130.98	45.17	85.45	73.25
Log Monthly Labour Income	0.08	-0.02	-0.02	-0.04
Career Interruption	0.28	0.31	0.21	0.20
Firm Change	0.21	0.16	0.14	0.13
Seniority	15.15	22.57	21.40	20.95
Hours per Week	-0.06	-0.56	0.06	-0.02
Full-time	-0.01	-0.12	-0.01	-0.01
Public	-0.01	-0.03	-0.02	-0.01
Observations	349	396	2431	3001

Working Conditions

Table 14 presents the different indicators of working conditions for each subgroup in 2013. The first part gather the physical constraints and the second part the flexibility of the job. A general observation can be that women are less exposed to physical constraints than men. The four groups are comparable in terms of flexibility, except that parents are less reachable and find their schedule less convenient with their personal obligations, women work fewer days, less frequently during mornings and nights and can anticipate their schedule more frequently.

Table 14: Data summary of the active population in 2013

Working Conditions Indicators	Fathers	Mothers	Childless Men	Childless Women
Physical Constraints				
Accident	0.06	0.04	0.05	0.03
Stand	0.53	0.51	0.54	0.49
Displacement	0.41	0.37	0.40	0.33
Heavy	0.46	0.41	0.45	0.40
Movement	0.38	0.35	0.41	0.40
Posture	0.34	0.31	0.36	0.35
Vibrations	0.28	0.05	0.26	0.07
View	0.45	0.40	0.43	0.38
Flexibility				
Convenient	0.70	0.70	0.75	0.75
Prevision Schedule	0.42	0.38	0.41	0.39
Modify Schedule	0.22	0.24	0.23	0.23
Overwork	0.40	0.42	0.38	0.38
Reachable	0.50	0.50	0.44	0.43
Days Worked	4.94	4.75	4.92	4.74
Work Interruption	0.76	0.58	0.74	0.64
Morning	0.28	0.22	0.28	0.20
Evening	0.28	0.29	0.25	0.23
Night	0.17	0.12	0.14	0.08
Saturday	0.36	0.44	0.37	0.37
Sunday	0.22	0.29	0.22	0.24
Observations	349	396	2431	3001

Table 15 displays the same indicators for each group in 2016. As in 2013, women seems to be less exposed to physical constraints. However, in 2016, mothers seem less exposed than childless women except in terms of long and frequent displacement by foot and having to keep a look at their work. In terms of flexibility, parents are more reachable, work more frequently during the weekend and do more overwork. Women work less frequently during morning, evening and night and fewer days per week, have more difficulty to predict their schedule and interrupt their work when they want. However, they work more frequently the Sunday. Finally, fathers have more difficulty to mod-

ify their schedule and mothers find their schedule less convenient with their social and familial commitments.

Table 15: Data summary of the active population in 2016

Working Conditions Indicators

	Fathers	Mothers	Childless Men	Childless Women
Physical Constraints				
Accident	0.06	0.03	0.05	0.03
Stand	0.54	0.45	0.53	0.48
Displacement	0.44	0.35	0.42	0.34
Heavy	0.47	0.41	0.43	0.41
Movement	0.42	0.32	0.40	0.41
Posture	0.32	0.26	0.38	0.35
Vibrations	0.27	0.06	0.27	0.08
View	0.46	0.44	0.47	0.41
Flexibility				
Convenient	0.71	0.67	0.75	0.74
Prevision Schedule	0.42	0.37	0.40	0.38
Modify Schedule	0.18	0.22	0.22	0.23
Overwork	0.43	0.40	0.37	0.38
Reachable	0.61	0.55	0.48	0.48
Days Worked	4.88	4.61	4.94	4.75
Work Interruption	0.76	0.62	0.75	0.67
Morning	0.29	0.18	0.27	0.20
Evening	0.30	0.25	0.24	0.22
Night	0.16	0.10	0.13	0.08
Saturday	0.40	0.40	0.36	0.37
Sunday	0.25	0.26	0.22	0.24
Observations	349	396	2431	3001

Table 16 summarizes the evolution of the working condition indicators between 2013 and 2016 for each subgroup. Regarding physical constraints only mothers have a clear decreasing trend, except for the two last indicators. In terms of flexibility, they find their schedules less convenient and loose flexibility regarding schedule. However, they gain in the counterparts : they can more freely interrupt their work, they have less unconventional hours with fewer days worked, less overwork and less work during morning,

evening, night, Saturday and Sunday.

Concerning the other groups, non-parents seems to remain quite constant apart that they are more reachable. Finally, fathers seems to lose in schedule flexibility and work fewer days like mothers, but they tend to do more overwork and have more unconventional hours.

Table 16: Data summary of the active population evolution between 2013 and 2016

Working Conditions Indicators				
	Fathers	Mothers	Childless Men	Childless Women
Physical Constraints				
Accident	0.00	-0.01	0.00	0.00
Stand	0.01	-0.06	-0.01	-0.01
Displacement	0.03	-0.02	0.02	0.01
Heavy	0.01	0.00	-0.02	0.01
Movement	0.04	-0.03	-0.01	0.01
Posture	-0.02	-0.05	0.02	0.00
Vibrations	-0.01	0.01	0.01	0.01
View	0.01	0.04	0.04	0.03
Flexibility				
Convenient	0.01	-0.03	0.00	-0.01
Prevision Schedule	0.00	-0.01	-0.01	-0.01
Modify Schedule	-0.04	-0.02	-0.01	0.00
Overwork	0.03	-0.02	-0.01	0.00
Reachable	0.11	0.05	0.04	0.05
Days Worked	-0.06	-0.14	0.02	0.01
Work Interruption	0.00	0.04	0.01	0.03
Morning	0.01	-0.04	-0.01	0.00
Evening	0.02	-0.04	-0.01	-0.01
Night	-0.01	-0.02	-0.01	0.00
Saturday	0.04	-0.04	-0.01	0.00
Sunday	0.03	-0.03	0.00	0.00
Observations	349	396	2431	3001

While non-parents seems to have stable working conditions, fathers and mothers seems to have opposite trends in unconventional hours. On the contrary, the fact that fathers find their schedule more convenient and mothers find their less convenient seems

puzzling. A possible interpretation is that the less demanding work is insufficient to compensate the increase in non-paid work done in the household for mothers, while fathers accommodate more of their role of "bread-winner" where they accept more unconventional hours in exchange of a decrease in hours worked.

5.2.2 Propensity Scores

Similarly to the first part with the whole population, I compute the propensity scores with a Gradient Boosted Model based on 2013 characteristics first and on 2013 and 2016 characteristics then. Then I compute the related weights and estimators.

The estimation of propensity scores based on 2013 characteristics uses the all the variables presented in the descriptive statistics, except monthly labour income, log monthly labour income, career interruption and firm change, to predict the probability of being in one subgroup.

Table 17 presents the weighted averages for each subgroup. According to these results, the labour income gap between parents equals 10.41% in 2013 and 14.14% in 2016, which leads to an increase of 3.73 percentage points between the two periods. For non-parents, there is a gap of 1.84% in 2013 and -3.73% in 2016, which means that there is a decrease of 3.17 percentage points between the two periods. This implies that if we emphasize childless individuals similar to future fathers in 2013, then in this group women have a higher labour income than men in 2016. The triple difference shows an increase of 6.91 percentage points in the gender labour income gap through parenthood.

Regarding the log monthly labour income, there is a gap of 0.05 between parents in 2013 and 0.18 in 2016, which gives an increase of 0.13. These results are larger than the previous ones due to outsiders in the mothers subgroup who raise the average labour income. For non-parents, there is a gap of 0.02 in 2013 and 0.01 in 2016, which gives a decrease of 0.01. This is consistent with the previous results. The triple difference equals an increase of 0.14 through parenthood.

Table 17: Estimated labour incomes with fathers' 2013 characteristics for the employed population in 2013

	Fathers	Mothers	Childless Men	Childless Women
2013				
Monthly Labour Income	1947.10	1744.37	1876.70	1842.19
Log Monthly Labour Income	7.43	7.38	7.43	7.41
2016				
Monthly Labour Income	2078.07	1784.13	2001.11	2027.84
Log Monthly Labour Income	7.51	7.33	7.46	7.45

To estimate the propensity scores based on 2013 and 2016 characteristics, the computations uses all the previous variables, plus the career interruption and firm changes.

Table 18 presents the resulting weighted means. For parents the labour income gap equals 9.32% in 2013 and 7.07% in 2016. Hence, there is a decrease of 2.25 percentage points between the two periods. On the other hand the labour income gap between non-parents goes from 1.65% to 5.65%, which gives an increase of 4 percentage points. The triple difference equals -6.25 percentage points, which means that, for individuals similar to father in 2013 (before the transition to parenthood), the transition to parenthood seems to decrease the gender labour income gap by 6.25 percentage points.

With the log monthly labour income, the gap between parents equals 0.03 in 2013 and 0.17 in 2016, while it equals 0.02 in 2013 and 0.09 in 2016 for non-parents. This gives respectively an increase of 0.14 and 0.07. The triple difference with log monthly labour income equals 0.07.

The difference in results might be driven by the presence of outsiders in the mothers and childless women subgroups where there is an increase in average monthly labour incomes and a decreases in average log monthly labour incomes.

Table 18: Estimated labour incomes with fathers' 2013 characteristics for the employed population in 2013 and 2016

	Fathers	Mothers	Childless Men	Childless Women
2013				
Monthly Labour Income	1947.10	1765.62	1893.10	1861.91
Log Monthly Labour Income	7.43	7.40	7.45	7.43
2013				
Monthly Labour Income	1968.00	1828.81	1950.90	1840.61
Log Monthly Labour Income	7.51	7.34	7.48	7.39

With these results we expect that parenthood increases the gender labour income gap within the employed population. However, parenthood is correlated with a much lower decrease than within the whole population and seems to impact mothers heterogeneously. Indeed, mothers who are similar to fathers in 2013 seems to have a decrease in labour incomes most of the time, except for some outsiders who have a large increase in labour incomes.

The limit of this method is once again the opacity of the computation and the resulting impossibility to estimate the impact of each variable.

5.2.3 First Difference

This section looks at the effect of working conditions on wages evolution and separate the results in two tables : one for the standard work-related variables and one for the working condition indicators. Table 19 presents the effect of the control variables. Most of the coefficients are not significant and these variables explain between 3.5% to 8.8% of the variations according to the adjusted R^2. Most of the variables does not have a significant impact on labour income variations. The most interesting is part-time which has a negative effect on all the subgroups except fathers and that mothers are the most penalized by part-time work. On the other hand we can note that the number of worked hours has a positive impact on childless individuals' labour income evolution but a negative impact on fathers' labour income evolution. This counter-intuitive results might be explained by some control variables such as overwork which has a higher positive impact on fathers' labour income evolution, as we can see in Table 20. Regarding the other variables, changing firm seems to have a positive impact for childless mothers and seniority seems to benefit childless men. The estimated coefficients related to workers' function within firms are not displayed in this table but most of them

are not significant, except for those who are working in health-care jobs who seem to be penalized compared to other workers, especially fathers.

Table 19: Log labour incomes evolution and general characteristics

	Dependent variable: Log Monthly Labour Income			
	Fathers	Mothers	Childless Men	Childless Women
(Intercept)	−0.089	−0.225	−0.032	0.021
	(0.339)	(0.375)	(0.079)	(0.041)
Age	0.051	0.077	0.019	0.007
	(0.110)	(0.122)	(0.026)	(0.013)
Relationship	−0.052	−0.050	0.011	0.034
	(0.110)	(0.133)	(0.029)	(0.021)
Firm Change	0.157	−0.102	0.098	0.108**
	(0.202)	(0.166)	(0.052)	(0.036)
Career Interruption	−0.034	0.139	−0.038	−0.024
	(0.185)	(0.134)	(0.045)	(0.030)
Seniority	−0.001	0.001	0.001*	0.0002
	(0.001)	(0.001)	(0.0002)	(0.0002)
Hours per Week	−0.023**	0.005	0.004*	0.008***
	(0.008)	(0.008)	(0.002)	(0.001)
Part-time	−0.208	−0.452***	−0.332***	−0.196***
	(0.236)	(0.119)	(0.056)	(0.032)
Public	−0.003	−0.167	−0.012	0.060
	(0.172)	(0.179)	(0.047)	(0.033)
Function	-	-	-	-
Observations	268	313	1,864	2,261
R^2	0.203	0.168	0.055	0.103
Adjusted R^2	0.070	0.056	0.035	0.088
F Statistic	1.531*	1.504*	2.782***	6.730***
df	(38; 229)	(37; 275)	(38; 1825)	(38; 2222)
Note:				*p<0.05; **p<0.01; ***p<0.001

In Table 20 we can see that most of the working conditions does not have any significant effect. Only overwork has a positive effect on men's labour incomes, especially for fathers as explained before. Being reachable and working Sunday seems positively

effect with childless mothers' labour income evolution, while having a predictable schedule seems negatively correlated. For fathers working while standing up is positively correlated with an increase in labour incomes and for childless men working in an uncomfortable posture is correlated with a decrease in labour incomes. These results are not really convincing statistically and theoretically, except for overwork which has a positive significant effect two subgroups which can be explained through the literature.

Table 20: Log Labour Income evolution and general characteristics

	Dependent variable: Log Monthly Labour Income			
	Fathers	Mothers	Childless Men	Childless Women
Physical Constraints				
Accident	0.153	0.014	−0.029	0.011
	(0.197)	(0.291)	(0.054)	(0.045)
Stand	0.290*	−0.160	−0.006	0.021
	(0.126)	(0.137)	(0.031)	(0.025)
Posture	−0.006	−0.044	−0.056*	−0.005
	(0.095)	(0.100)	(0.022)	(0.015)
Displacement	−0.082	0.006	−0.015	−0.002
	(0.095)	(0.103)	(0.021)	(0.017)
Heavy	0.179	−0.232	0.020	0.020
	(0.122)	(0.124)	(0.026)	(0.019)
Movement	0.092	0.207	−0.009	0.026
	(0.104)	(0.116)	(0.026)	(0.019)
Vibrations	0.016	0.035	−0.002	−0.006
	(0.108)	(0.182)	(0.029)	(0.028)
View	−0.125	−0.081	−0.015	−0.021
	(0.086)	(0.084)	(0.020)	(0.014)
Flexibility				
Convenient	0.214	0.034	0.010	−0.015
	(0.140)	(0.159)	(0.039)	(0.028)
Prevision Schedule	−0.258	−0.059	−0.060	−0.126*
	(0.207)	(0.355)	(0.057)	(0.049)
Modify Schedule	−0.023	−0.053	−0.015	−0.007
	(0.091)	(0.098)	(0.022)	(0.017)
Overwork	0.449*	0.031	0.075*	0.016
	(0.174)	(0.154)	(0.038)	(0.027)
Reachable	0.019	0.047	0.028	0.037*
	(0.084)	(0.084)	(0.021)	(0.014)
Days Worked	−0.094	−0.033	0.021	0.015
	(0.084)	(0.069)	(0.021)	(0.012)
Work Interruption	0.041	0.00000	−0.022	0.014
	(0.098)	(0.088)	(0.023)	(0.016)
Morning	−0.013	−0.072	0.004	−0.012
	(0.159)	(0.205)	(0.035)	(0.031)
Evening	0.104	0.027	−0.015	−0.009
	(0.155)	(0.150)	(0.040)	(0.029)
Night	0.187	0.120	−0.041	0.050
	(0.204)	(0.210)	(0.051)	(0.050)
Saturday	−0.064	−0.130	0.028	0.004
	(0.168)	(0.157)	(0.042)	(0.032)
Sunday	0.062	−0.073	0.069	0.086*
	(0.200)	(0.179)	(0.050)	(0.037)
Constant	−0.089	−0.225	−0.032	0.021
	(0.339)	(0.375)	(0.079)	(0.041)
Observations	268	313	1,864	2,261
R^2	0.203	0.168	0.055	0.103
Adjusted R^2	0.070	0.056	0.035	0.088
F Statistic	1.531*	1.504*	2.782***	6.730***
df	(38; 229)	(37; 275)	(38; 1825)	(38; 2222)

Note: *p<0.05; **p<0.01; ***p<0.001

This section highlights that most of the working conditions does not have significant impact on labour incomes with controls. Moreover, part-time is the most important variable to explain labour income variations within each group, accounting for the low explanatory capacity of this model with many variables. However, note that for all the previous coefficients, the only significant difference at a level of five percent is between childless men and childless women for the coefficient of part-time working. Moreover, the difference in difference between the four subgroups is not significant for any variable.

5.2.4 Decomposition

The different effects are separated with the decomposition method previously used on the whole population and summarized in Equation 18. Table 21 presents the results of this decomposition. Most of the variations are explained by changes in characteristics. More precisely by differences in the share of individuals working part-time. Indeed, as noted previously in Table 13 the share of individuals working in full-time jobs decreases more in the mother subgroup compared to others. Moreover, as noted in Table 19, working part-time has a significant negative impact on labour income evolution. This difference of part-time workers results in a significant increase of the gender labour income gap of 5 percentage points through parenthood.

On the other hand even if, as in Table 16, differences in working condition evolution are observed between the subgroups, the low effect of these working conditions on labour incomes observed in Table 20 implies that the overall effect of these differences on the gender labour income gap evolution is not significant. Similarly, the low differences in coefficients coupled with low variations in characteristics are not sufficient to impact significantly the gender labour income gap evolution.

The effects related to the different work functions are not displayed in the table but none of them has a significant effect. The rest of the aggregated endowment component of 7 percentage points comes from none significant endowments effects of work function variables. Hence, the most important result is the 5 percentage point effect related to differences in part-time work. We can also note that the coefficient component has no significant effect at a five percent level, even on the intercept.

Table 21: Decomposition estimation for active population

	PE	PC	E	C	I
Aggregated	0	-0.01	0.07 ***	0	0.03
(Intercept)	0	0	0	0.19	0
Age	0	0	0	-0.12	0
Relationship	-0.01	0	0	0	0
Firm Change	-0.01	0.01	0	0.04	0.01
Career Interruption	0	-0.02	0	-0.03	0
Tenure	-0.01	0	0	-0.06 .	0.02
Hours per Week	0	0.03	0	0	-0.01
Part-time	0.04	0.04	0.05 ***	0	-0.06
Public	0	0	0	0	0
Function					
Flexibility					
Convenient	0	-0.01	0	0	0.01
Modify	0	0	0	0	0
Overwork	0	-0.01	0	0	0.02
Work Interruption	0	0	0	0	0
Reachable	0	0	0	0	0
Days Worked	0	0.01	0	0	-0.01
Prevision	0	0	0	0	0
Morning	0	0	0	0	0
Evening	0	0	0	0	0
Night	0	0	0	0	0
Saturday	-0.01	0	0	0	0
Sunday	-0.01	0	0	0	0.01
Physical Constraints					
Accident	0	0	0	0	0
Stand	-0.01	-0.02	0	-0.01	0.02
Displacement	0	0	0	0	0
Heavy	0	0	0	-0.01	0.01
Movement	0.01	0	0	0	-0.01
Posture	0	0	0	0	0
Vibrations	0	0	0	0	0
View	0.01	0	0	0	0

Note: .p<0.1;*p<0.05; **p<0.01; ***p<0.001

Table 22 present the detailed version of the decomposition presented in Equation 19, focusing on the effect of part-time on the characteristics depending on gender status. It shows that the parenthood effect on the gender labour income gap is only driven by the increase of mothers working part-time, resulting in a motherhood penalty, while no fatherhood premium can be observed.

Table 22: Part-time effect depending on gender

	Men	Women
Aggregated	0.01	-0.06 ***
Part-time	0	-0.05 ***
Note:	.p<0.1;*p<0.05; **p<0.01; ***p<0.001	

According to this decomposition method, most of the variations in the gender labour income gap due to parenthood can be explained by changes in the share of mothers working part-time. Moreover, all the other variables does not have a significant effect, even variables such has career interruption or overwork which are frequently cited in the literature. Finally, this highlights the absence of direct effect of working conditions on the gender labour income gap evolution due to parenthood.

5.2.5 Reasons to work Part-Time

These surveys contain a question about the reasons of working part-time. All the part-time workers answer this question in both years. Table 23 presents the reasons to work part-time for the different subgroups in 2013. The main reason seems is the incapacity to find a full-time job. For women, the second reason is to have more leisure time, while for men it is another reason not specified in the possibilities. However, for future-fathers the results are a bit weak due to the low share of them who are working part-time, which gives only 14 individuals from a total sub-population of 349 individuals.

Table 23: The reason to work part-time in 2013

	Fathers	Mothers	Childless Men	Childless Women
Part-Time	0.04	0.17	0.06	0.22
To have a complementary job	0.07	0.08	0.07	0.04
To study	0.07	0.02	0.11	0.02
For health reasons	0.07	0.09	0.12	0.12
Cannot find Full-Time job	0.29	0.53	0.43	0.37
To care for children	0.00	0.03	0.01	0.02
To care for a family member	0.14	0.03	0.01	0.05
To do household chores	0.00	0.02	0.01	0.01
For leisure	0.14	0.18	0.07	0.27
Other reasons	0.21	0.03	0.18	0.10
Observations	14	66	159	667

Table 24 presents the distribution of these reasons in 2016. For fathers and childless individuals, the main reason to work part-time is the incapacity to find a job. For fathers the second reason is child care while for childless men it is health reasons and for childless women it is to have more leisure time. Concerning mothers, the main reason is childcare, far before the incapacity to find a full-time job.

Table 24: The reason to work part-time in 2016

	Fathers	Mothers	Childless Men	Childless Women
Part-Time	0.05	0.28	0.06	0.22
To have a complementary job	0.17	0.01	0.06	0.07
To study	0.00	0.00	0.03	0.00
For health reasons	0.17	0.04	0.22	0.16
Cannot find Full-Time job	0.28	0.18	0.35	0.30
To care for children	0.22	0.62	0.00	0.01
To care for a family member	0.00	0.01	0.01	0.06
To do household chores	0.00	0.01	0.00	0.00
For leisure	0.06	0.04	0.17	0.27
Other reasons	0.11	0.09	0.17	0.13
Observations	18	111	157	674

Table 25 summarizes the evolution of the answered reasons between 2013 and 2016. Concerning fathers childless individuals, the share of part-time workers is stable while it

increases by 10 percentage points for mothers. The larger share of childless workers with health problems can probably be explained by their age. For men, it might be due to physical constraints previously observed. The most interesting part of these responses is the large increase in childcare for parents and especially mothers. This seems to explain the large increase in the share of mothers working part-time.

Table 25: The reason to work part-time evolution between 2013 and 2016

	Fathers	Mothers	Childless Men	Childless Women
Part-Time	0.01	0.11	0.00	0.00
To have a complementary job	0.10	-0.07	-0.01	0.03
To study	-0.07	-0.02	-0.08	-0.02
For health reasons	0.10	-0.05	0.10	0.04
Cannot find Full-Time job	-0.01	-0.35	-0.08	-0.07
To care for children	0.22	0.59	-0.01	-0.01
To care for a family member	-0.14	-0.02	0.00	0.01
To do household chores	0.00	-0.01	-0.01	-0.01
For leisure	-0.08	-0.14	0.10	0.00
Other reasons	-0.10	0.06	-0.01	0.03
Observations	+4	+45	-2	+7

6 Discussion

According to the previous results, there is a gap in average labour incomes between men and women in the studied population in 2013 and in 2016. This gap is wider between those who become parents during the period than between those who remain childless. Moreover, this gap increases more between the former ones than between childless individuals. This difference of increase can be partially explained by differences in demographic characteristics between each subgroups but it remains even after comparing similar individuals.

The presence of non-employed individuals in the second period explains most of the labour income variations within each of these subgroups. Moreover, differences of evolution in labour force participation also explain most of the differences in the gender labour income gap evolution between parents and non-parents. More precisely, on the basis of the information available, the larger share of mothers who become unemployed explains most of these variations and the remaining unexplained gap is not significant

at a five percent level.

Then, focusing on the individuals who remain employed between the two periods, a similar gap can be observed between men and women. The gap is also wider between parents than between non-parents. However, within this sub-population, the gap only increases within the parent group, while it remains perfectly stable within the childless group. If we consider individuals similar, in 2013, to fathers, the triple difference is even larger. However, if we consider individuals, in both periods, who are similar to fathers, the triple difference is negative which means that the gender labour income gap decreases through parenthood. This shows that the effect of parenthood on the gender labour income gap is mainly driven by difference in demographic and work characteristics.

Descriptive statistics highlight differences in career evolution with more firm change for fathers and more career interruption for mothers. Working conditions also evolve differently with less physical constraints and unconventional hours for mothers, while fathers seem to have more unconventional schedules and overwork. The decrease in physical constraints might be explained by the increasing home work which consequences can be observed through the larger share of mothers reporting fatigue due to household chores and childcare. The total time and days worked decreases for mothers but also for fathers in a lesser degree. However, the gender labour income gap evolution is driven only by the increase of mothers working part-time, while most of the previous differences does not have a significant impact on labour incomes and hence on the gender labour income gap evolution. Moreover, the unexplained part of the gender labour income gap evolution is no longer significant.

Hence, the increase of the gender labour income gap through parenthood within the employed population can be totally explained by the larger increase of mothers working part-time. This difference of evolution between parents seems to be driven by the fact that mothers are more responsible for child care than men.

These results confirm the positive effect of parenthood on the gender gap in average labour income in France already observed in different articles (Coudin, Maillard and Tô, 2018). Moreover, as in Cukrowska-Torzewska and Matysiak (2020) these results also agree on the absence of significant unexplained parenthood effect after controlling for demographic and work-related variables. Consistently with other studies on French data, working conditions seems to evolve though parenthood in terms of flexibility (Coudin, Maillard and Tô, 2018) and in terms of physical constraints. However, as observed in Felfe (2012), these job amenities does not have a significant impact on the parenthood effect.

Moreover, these results support the theory of gender roles with the bred-winner

father and the child-caring mother. Similarly, as in Lott and Chung (2016), mothers seems to use schedule flexibility and control to meet their family demand with potentially a decrease in labour incomes if they work part-time. On the other hand, fathers seem to use schedule flexibility and control to decrease the number of hours days worked without penalty with an increase in unconventional hours and overwork. This might be explained by a change in household work and in childcare where fathers try to have more leisure time available without wage penalties.

On the other hand the results seems to reject the hypothesis of loss in human capital as an explanation of the gap increase as suggested in several articles (Gangl and Ziefle 2009; Napari 2010). Concerning the last hypothesis of discrimination against mothers (Corell, Benard and Paik 2007), some hints such as the increase in unemployment rate after the birth of the first child and the difficulties to find a full-time job might confirm it. The incapacity to find a job or a full-time job can also be explained by the fact that future mothers look for some job amenities, anticipating their future parenthood, which are incompatible with employers expectations (Herr and Wolfram, 2012; Cha, 2013). Indeed, according to event studies (Hotz, Johansson and Karimi, 2017; Kleven, Landais and Søgaard, 2019) the transition to parenthood and the related behaviour can already be observed more than one year before the birth of the first child. A consequence of this is that we can expect that the previous estimations are biased downward because some future mothers might already have changed their behaviour.

Moreover, this thesis uses a population not representative of the whole French working population with an over-representation of individuals working in the public sector where we might expect a lower motherhood penalty. Similarly, the fact that these surveys focus on working individuals hides the individuals who might return to employment during this period and the related differences in parents behaviours. For example, following the bred-winner model we might expect a larger share of fathers to return to the labour force after the birth of their child. Finally, the low number of observations in subgroups limits the validity of some results. For example the fact that there are only 14 part-time working fathers makes it difficult to develop on the reasons to work part-time effect on the gender labour income gap.

This research might be complemented with the use of the new survey *Conditions de travail 2019* which would allow to compare the trends between parents and non-parents before and after parents have their first child. It can also be completed with the study of the effect of the working conditions on the labour force participation and the impact on the gender wage gap with the non-linear decomposition method introduced in the literature review. Indeed, according to the literature we might expect an indirect effect

of job amenities and working conditions through labour force participation. Another possibility would be to investigate the heterogeneity of these results across the wage distribution. Finally, the decomposition method introduced in this thesis can be refined by using pooling models instead of first-difference ones. This would allow to compare the evolution of coefficients before and after the birth of the first child.

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Appendix

Decomposition proofs

The decomposition calculus

$$
\begin{aligned}
\hat{\Delta}^2(Y) &= (\bar{X}^{A,1}\,\hat{\beta}^{A,1} - \bar{X}^{B,1}\,\hat{\beta}^{B,1}) - (\bar{X}^{A,0}\,\hat{\beta}^{A,0} - \bar{X}^{B,0}\,\hat{\beta}^{B,0}) \\
&= \hat{\Delta}^2(X)\,\hat{\beta}^{A,0} - \hat{\Delta}^2(X)\,\hat{\beta}^{A,0} \\
&\quad + \bar{X}^{A,1}\,\hat{\beta}^{A,1} - \bar{X}^{B,1}\,\hat{\beta}^{B,1} - \bar{X}^{A,0}\,\hat{\beta}^{A,0} + \bar{X}^{B,0}\,\hat{\beta}^{B,0} \\
&= \hat{\Delta}^2(X)\,\hat{\beta}^{A,0} + \bar{X}^{A,0}\,\hat{\Delta}^2(\beta) + \hat{\Delta}^2(X)\,\hat{\Delta}^2(\beta) \\
&\quad - \bar{X}^{A,0}\,\hat{\Delta}^2(\beta) - \hat{\Delta}^2(X)\,\hat{\beta}^{A,1} + \hat{\Delta}^2(X)\,\hat{\beta}^{B,1} - \hat{\Delta}^2(X)\,\hat{\beta}^{B,0} \\
&\quad + \bar{X}^{A,1}\,\hat{\beta}^{A,1} - \bar{X}^{B,1}\,\hat{\beta}^{B,1} - \bar{X}^{A,0}\,\hat{\beta}^{A,0} + \bar{X}^{B,0}\,\hat{\beta}^{B,0} \\
&= \hat{\Delta}^2(X)\,\hat{\beta}^{A,0} + \bar{X}^{A,0}\,\hat{\Delta}^2(\beta) + \hat{\Delta}^2(X)\,\hat{\Delta}^2(\beta) \\
&\quad - \bar{X}^{A,0}\,\hat{\beta}^{A,1} + \bar{X}^{A,0}\,\hat{\beta}^{B,1} + \bar{X}^{A,0}\,\hat{\beta}^{A,0} - \bar{X}^{A,0}\,\hat{\beta}^{B,0} \\
&\quad - \bar{X}^{A,1}\,\hat{\beta}^{A,1} + \bar{X}^{B,1}\,\hat{\beta}^{A,1} + \bar{X}^{A,0}\,\hat{\beta}^{A,1} - \bar{X}^{B,0}\,\hat{\beta}^{A,1} \\
&\quad + \bar{X}^{A,1}\,\hat{\beta}^{B,1} - \bar{X}^{B,1}\,\hat{\beta}^{B,1} - \bar{X}^{A,0}\,\hat{\beta}^{B,1} + \bar{X}^{B,0}\,\hat{\beta}^{B,1} \\
&\quad - \bar{X}^{A,1}\,\hat{\beta}^{B,0} + \bar{X}^{B,1}\,\hat{\beta}^{B,0} + \bar{X}^{A,0}\,\hat{\beta}^{B,0} - \bar{X}^{B,0}\,\hat{\beta}^{B,0} \\
&\quad + \bar{X}^{A,1}\,\hat{\beta}^{A,1} - \bar{X}^{B,1}\,\hat{\beta}^{B,1} - \bar{X}^{A,0}\,\hat{\beta}^{A,0} + \bar{X}^{B,0}\,\hat{\beta}^{B,0} \\
&= \hat{\Delta}^2(X)\,\hat{\beta}^{A,0} + \bar{X}^{A,0}\,\hat{\Delta}^2(\beta) + \hat{\Delta}^2(X)\,\hat{\Delta}^2(\beta) \\
&\quad + \bar{X}^{B,1}\,\hat{\beta}^{A,1} + \bar{X}^{A,1}\,\hat{\beta}^{B,1} - \bar{X}^{B,1}\,\hat{\beta}^{B,1} + \bar{X}^{B,0}\,\hat{\beta}^{B,1} \\
&\quad - \bar{X}^{A,1}\,\hat{\beta}^{B,0} + \bar{X}^{B,1}\,\hat{\beta}^{B,0} - \bar{X}^{B,1}\,\hat{\beta}^{B,1} \\
&= \hat{\Delta}^2(X)\,\hat{\beta}^{A,0} + \bar{X}^{A,0}\,\hat{\Delta}^2(\beta) + \hat{\Delta}^2(X)\,\hat{\Delta}^2(\beta) \\
&\quad + (\bar{X}^{A,1} - \bar{X}^{B,1})\,(\hat{\beta}^{B,1} - \hat{\beta}^{B,0}) \\
&\quad + (\bar{X}^{B,1} - \bar{X}^{B,0})\,(\hat{\beta}^{A,1} - \hat{\beta}^{B,1})
\end{aligned}
\tag{23}
$$

The variance of the decomposition

In this section I reproduce the proof in Jann (2005b). Let's define $u_i \tilde{\mathrm{N}}(\mu_i, \Sigma_i)$ for $i \in \{1, 2\}$ and remind :

$$\begin{aligned}\mathbf{E}[u_i + u_j] =& \mu_i + \mu_j \\ \mathbf{E}[u_i' \, u_j] =& \mu_i \mu_j' + Cov(u_i, u_j) \\ \mathbf{E}[u_i' \, u_i] =& \mu_i \mu_i' + \Sigma_i\end{aligned}$$

Then the whole computation gives :

$$\begin{aligned}\mathbf{E}[(u_1' \, u_2)^2] &= \mathbf{E}[u_1' \, u_2 \, u_2' \, u_1] \\ &= tr\left(\mathbf{E}[u_1 \, u_1' \, u_2 \, u_2']\right) \\ &= tr\left(\mathbf{E}[u_1 \, u_1'] \, \mathbf{E}[u_2 \, u_2'] - Cov(u_1 \, u_1', u_2 \, u_2')\right) \\ &= tr\left([\mu_1 \, \mu_1' + \Sigma_1] \, [\mu_2 \, \mu_2' + \Sigma_2] - Cov(u_1 \, u_1', u_2 \, u_2')\right) \\ &= tr\left(\mu_1 \, \mu_1' \, \mu_2 \, \mu_2' + \mu_1 \, \mu_1' \, \Sigma_2 + \Sigma_1 \, \mu_2 \, \mu_2' + \Sigma_1 \, \Sigma_2 - Cov(u_1 \, u_1', u_2 \, u_2')\right) \\ &= tr(\mu_1 \, \mu_1' \, \mu_2 \, \mu_2') + tr(\mu_1 \, \mu_1' \, \Sigma_2) + tr(\Sigma_1 \, \mu_2 \, \mu_2') + tr(\Sigma_1 \, \Sigma_2) - tr(Cov(u_1 \, u_1', u_2 \, u_2')) \\ &= (\mu_1' \, \mu_2)^2 + \mu_1' \, \Sigma_2 \mu_1 + \mu_2' \, \Sigma_1 \, \mu_2 + tr(\Sigma_1 \, \Sigma_2) - tr(Cov(u_1 \, u_1', u_2 \, u_2')) \\ \mathbf{E}[u_1' \, u_2] &= \mu_1 \mu_2' + Cov(u_1, u_2) \\ \mathbf{E}[u_1' \, u_2]^2 &= (\mu_1 \mu_2' + Cov(u_1, u_2))^2 \\ &= (\mu_1 \mu_2')^2 + 2\mu_1 \mu_2' Cov(u_1, u_2) + Cov(u_1, u_2)^2 \\ V(u_1' u_2) &= \mathbf{E}[(u_1' \, u_2)^2] - \mathbf{E}[u_1' \, u_2]^2 \\ &= \mu_1' \, \Sigma_2 \mu_1 + \mu_2' \, \Sigma_1 \, \mu_2 + tr(\Sigma_1 \, \Sigma_2) \\ &\quad - tr(Cov(u_1 \, u_1', u_2 \, u_2')) - 2\mu_1 \mu_2' Cov(u_1, u_2) - Cov(u_1, u_2)^2\end{aligned}$$

Assuming that u_1 and u_2 are independent, it becomes :

$$V(u_1' u_2) = \mu_1' \, \Sigma_2 \mu_1 + \mu_2' \, \Sigma_1 \, \mu_2 + tr(\Sigma_1 \, \Sigma_2)$$

To simplify the computations, I keep this assumption. However, it might be misleading if the characteristics and the coefficients are correlated. Moreover, the two terms u_1 and u_2 might be composed of different variables. For example, considering $\Delta(I)$ I can

decompose the first term u_1 as :

$$\begin{aligned}
u_1 =& \bar{X}^{A,1} - \bar{X}^{B,1} - \bar{X}^{A,0} + \bar{X}^{B,0} \\
\mathbf{E}[u_1] =& \mathbf{E}[\bar{X}^{A,1} - \bar{X}^{B,1} - \bar{X}^{A,0} + \bar{X}^{B,0}] \\
=& \mathbf{E}[\bar{X}^{A,1}] - \mathbf{E}[\bar{X}^{B,1}] - \mathbf{E}[\bar{X}^{A,0}] + \mathbf{E}[\bar{X}^{B,0}] \\
V(u_1) =& V(\bar{X}^{A,1} - \bar{X}^{B,1} - \bar{X}^{A,0} + \bar{X}^{B,0}) \\
=& V(\bar{X}^{A,1}) + V(\bar{X}^{B,1}) + V(\bar{X}^{A,0}) + V(\bar{X}^{B,0}) \\
& - 2\,Cov(\bar{X}^{A,1}, \bar{X}^{B,1}) - 2\,Cov(\bar{X}^{A,1}, \bar{X}^{A,0}) + 2\,Cov(\bar{X}^{A,1}, \bar{X}^{B,0}) \\
& + 2\,Cov(\bar{X}^{B,1}, \bar{X}^{A,0}) - 2\,Cov(\bar{X}^{B,1}, \bar{X}^{B,0}) - 2\,Cov(\bar{X}^{A,0}, \bar{X}^{B,0})
\end{aligned}$$

And the second term u_2 as :

$$\begin{aligned}
u_2 =& \hat{\beta}^{A,1} - \hat{\beta}^{B,1} - \hat{\beta}^{A,0} + \hat{\beta}^{B,0} \\
\mathbf{E}[u_2] =& \mathbf{E}[\hat{\beta}^{A,1} - \hat{\beta}^{B,1} - \hat{\beta}^{A,0} + \hat{\beta}^{B,0}] \\
=& \mathbf{E}[\hat{\beta}^{A,1}] - \mathbf{E}[\hat{\beta}^{B,1}] - \mathbf{E}[\hat{\beta}^{A,0}] + \mathbf{E}[\hat{\beta}^{B,0}] \\
V(u_2) =& V(\hat{\beta}^{A,1} - \hat{\beta}^{B,1} - \hat{\beta}^{A,0} + \hat{\beta}^{B,0}) \\
=& V(\hat{\beta}^{A,1}) + V(\hat{\beta}^{B,1}) + V(\hat{\beta}^{A,0}) + V(\hat{\beta}^{B,0}) \\
& - 2\,Cov(\hat{\beta}^{A,1}, \hat{\beta}^{B,1}) - 2\,Cov(\hat{\beta}^{A,1}, \hat{\beta}^{A,0}) + 2\,Cov(\hat{\beta}^{A,1}, \hat{\beta}^{B,0}) \\
& + 2\,Cov(\hat{\beta}^{B,1}, \hat{\beta}^{A,0}) - 2\,Cov(\hat{\beta}^{B,1}, \hat{\beta}^{B,0}) - 2\,Cov(\hat{\beta}^{A,0}, \hat{\beta}^{B,0})
\end{aligned}$$

Similarly we find :

$$
\begin{aligned}
V\left(\Delta(PE)\right) =& (\bar{X}^{A,0} - \bar{X}^{B,0})' \left[V(\hat{\beta}^{A,1}) + V(\hat{\beta}^{A,0})\right] (\bar{X}^{A,0} - \bar{X}^{B,0}) \\
&+ (\hat{\beta}^{A,1} - \hat{\beta}^{A,0})' \left[V(\bar{X}^{A,0}) + V(\bar{X}^{B,0})\right] (\hat{\beta}^{A,1} - \hat{\beta}^{A,0}) \\
&+ tr\left([V(\bar{X}^{A,0}) + V(\bar{X}^{B,0})]\,[V(\hat{\beta}^{A,1}) + V(\hat{\beta}^{A,0})]\right) \\
V\left(\Delta(PC)\right) =& (\bar{X}^{A,1} - \bar{X}^{A,0})' \left[V(\hat{\beta}^{A,0}) + V(\hat{\beta}^{B,0})\right] (\bar{X}^{A,1} - \bar{X}^{A,0}) \\
&+ (\hat{\beta}^{A,0} - \hat{\beta}^{B,0})' \left[V(\bar{X}^{A,1}) + V(\bar{X}^{A,0})\right] (\hat{\beta}^{A,0} - \hat{\beta}^{B,0}) \\
&+ tr\left([V(\bar{X}^{A,1}) + V(\bar{X}^{A,0})]\,[V(\hat{\beta}^{A,0}) + V(\hat{\beta}^{B,0})]\right) \\
V\left(\Delta(E)\right) =& \hat{\Delta}^2(X)'\, V(\hat{\beta}^{B,1})\, \hat{\Delta}^2(X) \\
&+ \hat{\beta}^{B,1\prime}\, V(\hat{\Delta}^2(X))\, \hat{\beta}^{B,1} \\
&+ tr\left(V(\hat{\Delta}^2(X))\, V(\hat{\beta}^{B,1})\right) \\
V\left(\Delta(C)\right) =& \bar{X}^{B,1\prime}\, V(\hat{\Delta}^2(\beta))\, \bar{X}^{B,1} \\
&+ \hat{\Delta}^2(\beta)'\, V(\bar{X}^{B,1})\, \hat{\Delta}^2(\beta) \\
&+ tr\left(V(\bar{X}^{B,1})\, V(\hat{\Delta}^2(\beta))\right) \\
V\left(\Delta(I)\right) =& \hat{\Delta}^2(X)'\, V(\hat{\Delta}^2(\beta))\, \hat{\Delta}^2(X) \\
&+ \hat{\Delta}^2(\beta)'\, V(\hat{\Delta}^2(X))\, \hat{\Delta}^2(\beta) \\
&+ tr\left(V(\hat{\Delta}^2(X))\, V(\hat{\Delta}^2(\beta))\right)
\end{aligned}
\tag{24}
$$

With :

$$
\begin{aligned}
V(\hat{\Delta}(X)) &= V(\bar{X}^{A,1}) + V(\bar{X}^{B,1}) + V(\bar{X}^{A,0}) + V(\bar{X}^{B,0}) \\
V(\hat{\Delta}(\beta)) &= V(\hat{\beta}^{A,1}) + V(\hat{\beta}^{B,1}) + V(\hat{\beta}^{A,0}) + V(\hat{\beta}^{B,0})
\end{aligned}
\tag{25}
$$